Rewriting MIMO Channel Capacity for Antenna Configuration Comparison

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***Abstract*— A rewriting of the MIMO channel capacity formula is proposed, in order to capture the direct influence of the array matrix (array configuration plus AoA information). The proposed theoretical framework will eventually allow direct comparison between antenna geometries in terms of channel capacity. Simulations indicate that, for small size arrays (up to 8x8), the configuration has a significant influence on the channel capacity, which is no longer the case for larger arrays.**

Keywords—antenna arrays, spatial channel models, capacity planning, cognitive radio, antenna learning, cognitive antenna system

I. INTRODUCTION

The adoption of multiple antenna techniques has been proved to have a significant impact on the overall performance of wireless communication systems [1], [2], [3], [4]. The improvements are mainly in: the efficient use of the spectrum, minimization of the cost of establishing new wireless networks, enhancement of the quality of service, and the realization of reconfigurable, robust, and transparent operation across multitechnology wireless networks [1], [5], [6].

Although it is the actual RF delivery and reception enabler, the antenna has evolved at a much slower pace than baseband processing techniques. Moreover, we could observe a delay in broad adoption of truly innovative techniques such as adaptive antennas [7]. Even though effective solutions for adaptive antennas exist for more than a decade, they are not yet implemented on a large scale. Wireless planning tools turn into impact calculators, universally used, reasoning in terms of capacity dynamics and return on investment [13]. An automated assessment of the impact of antenna configuration on link capacity becomes necessary. But could this be straight forward? Such an analysis may be based on a comparison between antenna designs/configurations in terms of capacity, for a given environment.

Attempts have been made to compare array configurations in terms of capacity for various environments [8], [9], [10], [11], [12], [19]. Spatial channel models [14], [15], [16], [17] suffer from excessive generalization and are computationally intensive. Also, they do not accurately account for specific, local features of a given area, and do not allow direct comparison of various antenna configurations for that area.

In our approach the antenna is seen as a pivotal element in determining and assessing the quality of a wireless communication link. Actually, the user-perceived quality ultimately relies on the antenna performance. Moreover, until recently, the antenna influence on channel measurements was considered a bias, where now it can be used to a benefit by integrating it into the channel analysis [9], [10], [18]. The antenna may take an active role in characterizing and eventually learning the operation environment by first ensuring high accuracy and reliability in observing the environment [11]. In this paper, we provide the theoretical framework that allows direct comparison between antenna geometries in terms of channel capacity. Our method accounts for: array configuration, angle-of-arrival (AoA), and element correlation. The capacity impact comparison is achieved based only on the H channel matrix, without the need to calculate correlation and power delay profile, by explicitly including the AoA in the capacity calculus.

The paper is structured as follows: Section II introduces the theoretical MIMO capacity function of AoA and array configuration. Capacity simulations discussed in Section III. Conclusions are presented in Section IV.

II. REWRITING MIMO CAPACITY

According to the standard approach, spectral efficiency is evaluated as a function of SNR and of the channel matrix H ($H \in C^{MxN}$) where N and M represent the number of antenna elements at the transmitter (Tx) and receiver (Rx), respectively. The H matrix channel coefficients represent the complex gains of the paths between element j at the receiver and element i at the transmitter, and can be written:

$$h_{ji}(t) = \sum_{k=1}^{D} g_k^{ji} \delta(t - \tau_k) \quad (1)$$

where: D is the total number of taps,

t is the considered time sample, and

g_k^{ji} is the complex gain of the k^{th} path between element j at Rx and element i at Tx.

In the frequency domain (for ω frequency) we may write:

$$H = \begin{bmatrix} h_{11} & h_{12} & \dots & h_{1N} \\ h_{21} & h_{22} & \dots & h_{2N} \\ \dots & \dots & \dots & \dots \\ h_{M1} & h_{M2} & \dots & h_{MN} \end{bmatrix} \in C^{MxN} \qquad (2)$$

The H matrix is calculated for each pilot subcarrier and interpolations are made between a certain subcarrier and the closest pilot. Actually, a separate H matrix is obtained for each subcarrier. If the H matrix is known at the receiver (CSI at the receiver) then the spectral efficiency may be expressed as:

$$C_{MIMO} = \max_{Tr(R_{xx})} \left\{ log_2 \left(det \left(I_M + \frac{E_x}{MN_0} H R_{xx} H^H \right) \right) \right\} \quad (3)$$

where: E_x is the Tx antenna energy feed,

N_0 is the noise variance (of type Zero Mean Circular

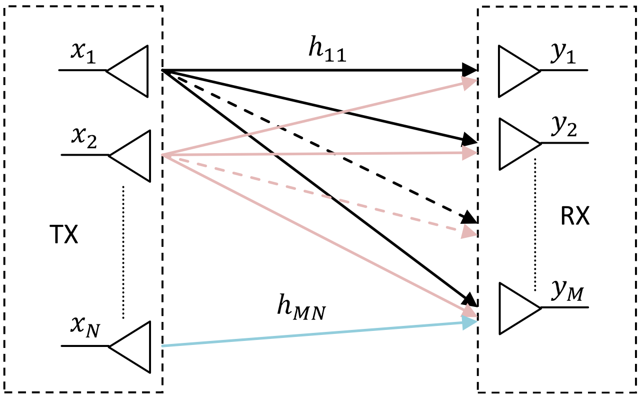

Fig. 1. NxM MIMO channel matrix - complex coefficients h_{ji}.

Symmetric Complex Gaussian (ZMCSCG)), and

R_{xx} is the Tx autocorrelation matrix.

Let us have:

$$x = [x_1\, x_2 \dots x_N]^T \in C^{Nx1},$$

then $R_{xx} = E(xx^H)$

where x^H is the x matrix Hermitian.

We may then write:

$y = [y_1\, y_2 \dots y_M]^T \in C^{Mx1}$ and $y = Hx + z$

where $z = [z_1\, z_2 \dots z_M]^T \in C^{Mx1}$ is the noise at the receiver.

Equation (3) does not account for the Tx or Rx array geometry in an explicit way. The influence of the H matrix is straight forward, whereas a dependence between the H matrix and the array geometry is not apparent. Such a relationship can be established based on the observation that the *array matrix* A may be determined explicitly, accounting for both the array configuration and angles-of-arrival θ_i.

Thus, for a random manifold array we may write [20]:

$$A = \left[a_{(\theta_1)}\, a_{(\theta_2)} \dots a_{(\theta_D)} \right] \in C^{MxD} \quad a_{(\theta_i)} \in C^{Mx1} \qquad (4)$$

where θ_1 , θ_2 , … , θ_D are the angles of arrival of the D incoming waves.

For a uniform linear array (ULA) with M elements we may write $a^T_{(\theta)}$ as a steering vector for angle θ:

$$a^T_{(\theta)} = \left[1\; e^{j\beta d(sin(\theta))} \quad e^{2j\beta d(sin(\theta))} \dots \; e^{(M-1)j\beta d(sin(\theta))} \right] \quad (5)$$

For a uniform rectangular array (URA) with M elements ($\sqrt{M}\; x \sqrt{M}$):

$$a^T_{(\theta)} = \begin{bmatrix} e^{j\beta(dx_1 sin(\theta) + dy_1 cos(\theta))} \; e^{j\beta(dx_2 sin(\theta) + dx_2 cos(\theta))} \dots \\ \dots\; e^{j\beta(dx_M sin(\theta) + dy_M cos(\theta))} \end{bmatrix} \quad (6)$$

where $\beta = \frac{2\pi}{\lambda}$ is the phase constant,

d is the ULA element spacing,

d_x is the x axis consecutive element spacing,

$dx_i = (i-1)d_x$, $= \overline{1, \sqrt{M}}$, and

d_y is the y axis consecutive element spacing,

$dy_j = (j-1)d_y$, $= \overline{1, \sqrt{M}}$.

Writing the channel matrix H as a function of the array matrix A, the influence of the array configuration on the channel capacity (3) is revealed.

Let the received signal y_1 (corresponding to array element 1) be a reference. According to (1) we may write:

$$h_{1i}(t) = \sum_{k=1}^{D} g_k^{1i} \delta(t - \tau_k),\; i = \overline{1, N}. \qquad (7)$$

Also, consider a simple SISO indoor channel where the power delay profile (PDP) follows an exponential model. Each path follows a Rayleigh distribution in terms of delay as long as the power of each wavefront suffers an exponential attenuation according to:

$$\sigma_i^2 = \sigma_0^2 e^{-iT_s/\sigma_\tau}$$

where: σ_0^2 is the power of the first wavefront,

T_s is the sampling time,

σ_τ is the RMS delay spread,

$$\sigma_0^2 = \frac{1 - e^{-T_s/\sigma_\tau}}{1 - e^{-(D+1)T_s/\sigma_\tau}}$$

where: D is the number of resulted taps,

$D = integer\{10T_s/\sigma_\tau\}$.

We may then determine a complex gain matrix G for each path between any i element at the Tx and the reference element at Rx. The gain matrix is:

$$G = \begin{bmatrix} g_1^{11} & g_1^{12} & \dots & g_1^{1N} \\ g_2^{11} & g_2^{12} & \dots & g_2^{1N} \\ \dots & \dots & \dots & \dots \\ g_D^{11} & g_D^{12} & \dots & g_D^{1N} \end{bmatrix} \in C^{DxN} \quad (8)$$

Each column of the G matrix represents the impulse response corresponding to each Tx element.

The gain matrix *G* becomes the key connection between the array matrix *A* and the MIMO channel matrix *H*, so that we can write:

$$H = AG \tag{9}$$

The *H* random channel coefficients are of the form:

$$h_{kj} = \sum_{i=1}^{D} g_i^{1j}\, a_{(\theta_i,k)} \tag{10}$$

where $k = \overline{1, M}$ and $i = \overline{1, N}$ and

$a_{(\theta_i,k)}$ represents the k^{th} element in the column vector $a_{(\theta_i)}$ from *A*.

Equation (3) may now be rewritten as:

$$C_{MIMO} = \max_{Tr(R_{xx})}\left\{log_2\left(det\left(I_M + \frac{E_x}{MN_0} AGR_{xx}G^H A^H\right)\right)\right\} \tag{11}$$

The new form accounts for the array geometry/configuration by including the array matrix *A*. When the reference element is maintained, the gain matrix *G* stays the same. Comparisons in terms of achievable channel capacities can now be made between array geometries like ULA, URA, and UCA, with the same number of elements:

$$C_{MIMO}^{ULA} \neq C_{MIMO}^{URA} \neq C_{MIMO}^{UCA}.$$

In the following Section we take a look at the influence of AoA estimation accuracy on the overall MIMO channel capacity. The AoA estimation accuracy, and implicitly the accuracy of the array matrix *A*, may be assessed based on the Cramer-Rao Bound [21]. The method uses a lower bound (CRB) of an unbiased estimator ($E\{\hat{\theta}\} = \theta$). Thus any estimation method based on MSE will cause an error greater than this limit [20].

The lower band limit CRB can be approximated for high SNR with [21]:

$$CRB_{array} \approx \frac{1}{2KSNR\left|\dot{a}_{(\theta)}\right|^2}$$

where *K* is the number of snapshots used by the estimator,

SNR is the signal to noise ratio for each antenna at the receiver, and

$\dot{a}_{(\theta)}$ is the derivative of the array manifold with respect of teta.

Evaluating $\left|\dot{a}_{(\theta)}\right|^2$ for various array configurations (ULA, UCA, URA) one can obtain the explicit form of the lower limit CRB for each of these.

We may thus establish the limits for the θ angle estimation error for each configuration.

III. SIMULATIONS

After MIMO channel capacity simulations are performed, in order to account for the AoA estimation error and to highlight the direct influence of the array configuration, according to (11). We consider an indoor SISO channel model [16], having the following parameters: sampling time $T_s = 50\, nsec$, RMS delay spread $\sigma_{rms} = 25\, nsec$. This model accounts for 6 distinct paths when calculating the gain matrix *G*, (8).

A *MxN* MIMO channel is obtained based on (9). The array matrix A is generated for uniformly distributed angles of arrival between [-π, π].

For AoA estimation acuracy 2x2 and 16x16 arrays are considerd, and an SNR of 20 dB. Fig. 2 illustrates the CRB variation for a 16x16 array. The maximum accuracy is obtained for θ=0 (perpendicular incoming waves). The estimation error increases with θ, thus a maximum error occurs for side waves. For indoor environments the ULA configuration is suboptimal due to the large angular spread.

For the circular and rectangular array configurations the

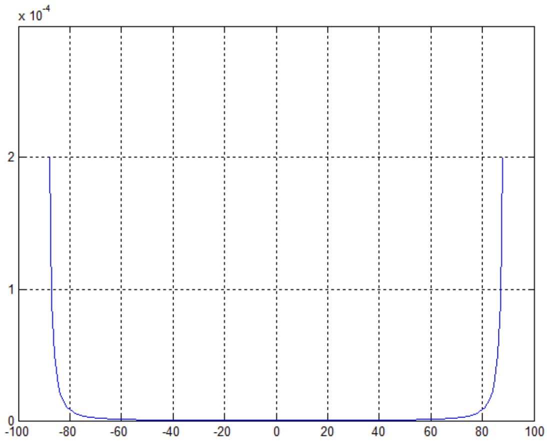

Fig. 2. CRB_{ULA} for *K* = 1, SNR = 20 dB, $\frac{d}{\lambda}$ = 0,5, *M* = 16, *f* = 2.4 GHz

error is constant regardless of the direction of arrival: $CRB_{UCA} = 1{,}11\ 10^{-5}[rad]$ and $CRB_{URA} = 2{,}59\ 10^{-7}[rad]$.

UCA and URA are potentially optimal configurations for indoors, and hybrid configurations may be customized depending on the environment.

Fig. 2 indicates the maximum error for the ULA configuration, which is 0.1146 degrees. For simple channel estimation methods like LS or MMSE – which are not based on EVD matrix decomposition in the receiver – the estimation error increases by approximately 10 dB, ten times greater than the CRB. We may thus expect around one dregree errors.

The AoA estimation error does not influence the ergodic capacity of the MIMO channel.

Fig. 3 illustrates the influence of the Rx array configuration on the channel capacity. 4x4 and 8x8 ULA and UCA configurations may now be compared.

The gain matrix G is generated for a mean SNR = 20 dB. For this SNR value, space diversity is present and a difference between ULA and UCA capacities is noticeable. For a lower SNR (< 10 dB) there is no significant difference between the two configurations, space diversity being reduced.

The better performance of UCA in this case may be due to the initial assumption of uniformly distributed incoming waves. For particular distributions of the incoming waves UCA may not have the same performance. The spatial correlation effect may be also seen in Fig. 3. It is interesting to notice that the array configuration is relevant for a small number of elements (2, 4, and up to 8). The UCA cdf indicates a 0.5 bps/Hz capacity gain compared to ULA, for a 4-element receive array. For an 8-element receive array an increase in capacity is even less noticeable so that the configuration is no longer that relevant.

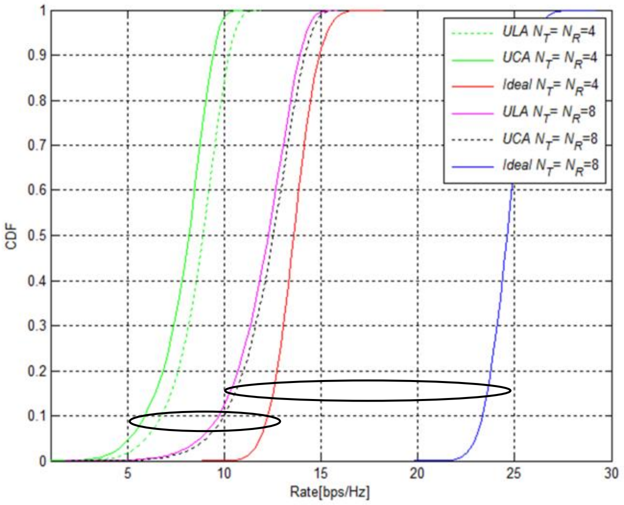

Fig. 3. Capacity CDF for different array configurations: ULA 4x4, UCA 4x4, ULA 8x8, and UCA 8x8. Mean SNR = 20dB.

IV. CONCLUSIONS

A direct relationship between array configuration, AoA and capacity is revealed. This allows for direct comparison, in terms of channel capacity, between different array configurations. Simulations indicate that, for small size arrays (up to 8x8), the array configuration has a significant influence on the channel capacity, which is no longer the case for larger arrays. Advantages of a direct comparison method of various array configurations would be: reduced complexity due to intermediate solutions, reduced delay in broad adoption of truly innovative adaptive antenna techniques, setting premises for antenna learning of the environment.

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