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# New exact bound states solutions for (C.F.P.S.) potential in the case of non commutative three dimensional non relativistic quantum mechanics

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**ABSTRACT**

We obtain here the modified bound-states solutions for central fraction power singular potential (C.F.P.S.) in noncommutative 3-dimensional non relativistic quantum mechanics (NC-3D NRQM). It has been observed that the commutative energy spectra was changed, and replaced degenerate new states, depending on four quantum numbers : $j$, $l$ and $s_z = \pm 1/2$ corresponding to the two spins states of electron by (up and down) and the deformed Hamiltonian formed by two new operators : the first describes the spin-orbit interaction , while the second obtained Hamiltonian describes the modified Zeeman effect (containing ordinary Zeeman effect) in addition to the usual commutative Hamiltonian. We showed that the isotropic commutative Hamiltonian $H_{CFPS}$ will be in non commutative space anisotropic Hamiltonian $H_{NC-CFPS}$.



## 1. Introduction

Recently a considerable effort has been devoted to the study of physics phenomena on commutative and noncommutative space-times ; the study of central physical problems has attracted much attention. The fraction power singular potential and fraction power potential are two exactly solvable like the Columbian and harmonic oscillator in quantum mechanics in two and three dimensional space [1-31]. The central fraction power singular potential has been successfully used in particle physics phenomenology and may be useful in other physics problems [30, 31]. A new concept of space-time, known by noncommutative spaces, represents a hope to obtain a new and profound interpretation at microscopic scales. In this noncommutative space, we extend the standard rules of quantum mechanics to the generalized Heisenberg relation of uncertainty. The formalism of star product, Boopp's shift method and the Seiberg-Witten map were played fundamental roles in this new theory.

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The rich mathematical structure of the noncommutative theory will lead to get a better understanding of physics phenomena at small distances and hopefully will solve above mentioned problems. The physical idea of a noncommutative space will be satisfied by new mathematical product which replaces the old ordinary product by star product between two arbitrary functions $f(x)$ and $g(x)$ noted by $(*)$, the effect of star product change the ordinary product by $\delta\left(f\left(x\right).g\left(x\right)\right)$ [9- 27] :

$$\delta\left(f\left(x\right).g\left(x\right)\right)=-\frac{i}{2}\theta^{ij}\left(\partial_i f\left(x\right)\right)\left(\partial_j g\left(x\right)\right) \tag{1}$$

The parameters $\theta_{ij}$ are an antisymmetric real matrix of dimension square length in the noncommutative canonical-type space. This paper is organized as follows : in the next section we present the central fraction power singular potential in the commutative three dimensional spaces. In section 3 we study the Hydrogen atom with central fraction power singular potential in (NC-3D NRQM); we apply the perturbation theory to deduce the energy levels of electron with two polarizations up and down, also we derive the deformed Hamiltonian of Hydrogen atom with studied potential (C.F.P.S.). The conclusions are given in the last section.

## 2. The (C.F.P.S.) potential in commutative three dimensional NRQM

The stationary reduced Schrödinger equation with central fraction power singular potential depending only on the distance $r$ leads to the following equation for the radial part of wave equation [31] :

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR\left(r\right)}{dr}\right)-\frac{l\left(l+1\right)}{r^2}R\left(r\right)+\left(E-\alpha r^{\frac{2}{3}}-\beta r^{-\frac{2}{3}}-\gamma r^{-\frac{4}{3}}\right)R\left(r\right)=0 \tag{2}$$

Where $E$ and ($\alpha$, $\beta$ and $\gamma$) are the energy spectra and three real numbers, respectively. The wave equation $R\left(r\right)$ has the following ansatz [31] :

$$R\left(r\right)=\exp\left(\frac{3}{4}ar^{\frac{4}{3}}+\frac{3}{2}br^{\frac{2}{3}}\right)\sum_{n=0}a_n r^{2n/3-\nu} \tag{3}$$

Where $a$, $b$ and $v$ are three constants :

$$\begin{array}{rc} a= & -\sqrt{\alpha} \\ b= & \frac{E}{2\sqrt{\alpha}} \\ \nu= & l \end{array} \tag{4}$$

The values $v=l$ assure finite values of $R(r)$ at $r=0$, the energy eigenvalues $E_p^l$ which corresponded $a_p\neq 0$ and $a_{p+1}=a_{p+2}=\cdots=0$ is given by [31] :

$$E_p^l=\pm\sqrt{4\alpha}\left\{\left(\frac{4p}{3}+2l+\frac{7}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}} \tag{5}$$

The various solutions generalized for $p = 0$ and $p = 1$ are determined from the projection of two equations, respectively [31] :

$$
\begin{aligned}
&\Psi^{0}_{l,m}(r,\theta,\phi) = \exp\left(-\tfrac{3}{4}\sqrt{\alpha}r^{\frac{4}{3}} + \tfrac{3}{4}\tfrac{E^{l}_{0}}{\sqrt{\alpha}}r^{\frac{2}{3}}\right) a_0 r^{l} Y_{l,m}(\theta,\phi)\\
&E^{l}_{0} = \pm\sqrt{4\alpha}\left\{\left(2l+\tfrac{7}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}}\\
&\Psi^{1}_{l,m}(r,\theta,\phi) = \exp\left(-\tfrac{3}{4}\sqrt{\alpha}r^{\frac{4}{3}} + \tfrac{3}{4}\tfrac{E^{l}_{1}}{\sqrt{\alpha}}r^{\frac{2}{3}}\right) r^{l}\left(a_0 + a_1 r^{\frac{2}{3}}\right) Y_{l,m}(\theta,\phi)\\
&E^{l}_{1} = \pm\sqrt{4\alpha}\left\{\left(2l+\tfrac{11}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}}
\end{aligned}
\tag{6}
$$

The natural unites $(c = \hbar = 2m = 1)$ and $(\mu = s)$ throughout this paper.

## 3. The (C.F.P.S.) potential in NC 3D NRQM

### 3.1. Noncommutative (C.F.P.S.) Hamiltonian

The first equation for star-product permits us to deduce the star deformed commutators $\left[x'_i \overset{*}{,} x'_j\right]$ :

$$\left[x'_i \overset{*}{,} x'_j\right] = i\theta_{ij} \tag{7}$$

The deformed Hamiltonian operator $H_{NC-CFPS}$ associated with central fraction power singular potential in NC space, will be determined by the following equation :

$$H_{NC-CFPS} = \frac{\overleftrightarrow{p}^{2}}{2m_0} + V_{NC-CFPS}(r') \tag{8}$$

Where $V_{NC-CFPS}(r')$ is the operator of the central fraction power singular potential in NC 3D NRQM. We apply the Boopp's shift method ; we deduce the deformed Schrödinger equation with the (central fraction singular power) potential [22-27] :

$$\left(-\frac{\Delta}{2m_0} + V_{NC\_CFPS}(r')\right)\Psi\left(\overleftrightarrow{r}\right) = E_{NC-CFPS}\Psi\left(\overleftrightarrow{r}\right) \tag{9}$$

Here $E_{NC-CFPS}$ is the energy and $V_{NC\_CFPS}(r')$ is the new potential as a function of operator :

$$V_{NC\_CFPS}(r') = \alpha r'^{\frac{2}{3}} + \beta r'^{-\frac{2}{3}} + \gamma r'^{-\frac{4}{3}} \tag{10}$$

On based to the formulations of the Boopp's shift, the scalar function $\left(\frac{1}{r}\right)$ can be written in the noncommutative three dimensional spaces as [14,21-26] :

$$\frac{1}{r'} = \frac{1}{r} + \frac{\mathbf{L}\boldsymbol{\theta}}{4r^3} \tag{11}$$

Where $\mathbf{L}$ is the angular momentum, which allows to obtaining after a straightforward calculation :

$$\begin{aligned} \alpha r'^{\frac{2}{3}} &= \alpha r^{\frac{2}{3}} - \tfrac{\alpha \mathbf{L}\boldsymbol{\theta}}{6r^{\frac{4}{3}}} \\ \beta r'^{-\frac{2}{3}} &= \beta r^{-\frac{2}{3}} + \tfrac{\beta \mathbf{L}\boldsymbol{\theta}}{6r^{\frac{8}{3}}} \\ \gamma r'^{-\frac{4}{3}} &= \gamma r^{-\frac{4}{3}} + \tfrac{\gamma \mathbf{L}\boldsymbol{\theta}}{3r^{\frac{5}{2}}} \end{aligned} \tag{12}$$

Inserting Eq. (12) into Eq. (10), one obtains :

$$V_{NC\_CFPS}(r') = V_{C\_CFPS}(r) + V_{CFPS\_P}(r) \tag{13}$$

The term $V_{C\_CFPS}(r)$ represents the usual commutative ordinary potential and the supplementary term $V_{CFPS\_P}(r)$ takes the form :

$$V_{CFPS\_P}(r) = \left( -\frac{\alpha}{6r^{\frac{4}{3}}} + \frac{\beta}{6r^{\frac{8}{3}}} + \frac{\gamma}{3r^{\frac{5}{2}}} \right) \mathbf{L}\boldsymbol{\theta} \tag{14}$$

Where $\boldsymbol{\theta} = \mathbf{2}\boldsymbol{\alpha}'\mathbf{S}$, $\boldsymbol{\alpha}'$ is infinitesimal scalar parameter and $\mathbf{s}$ is the spin momentum, then, the new NC Hamiltonian $H_{NC-CFPS}$ will be written as follows :

$$H_{NC\_CFPS} = H_{CFPS} + V_{CFPS\_P}(r) \tag{15}$$

Where $H_{CFPS}$ represent the usual Hamiltonian in ordinary commutative space :

$$H_{CFPS} = -\frac{\Delta}{2m_0} + \alpha r^{\frac{2}{3}} + \beta r^{-\frac{2}{3}} + \gamma r^{-\frac{4}{3}} \tag{16}$$

Considering the noncommutativity as a small perturbation on the structure of the coordinate space, the real parameter $\theta$ is taken very small and our calculations are taken up to the first order in $\theta$. The new added part $V_{CFPS\_P}(r)$ is proportional with the small non-commutative parameter $\theta$, which as a perturbative term. Furthermore, we can rewrite it to the equivalent physical form :

$$V_{CFPS\_P}(r) = 2\alpha' g(r)\ \mathbf{SL} \tag{17}$$

Where the scalar function $g(r)$ is given by :

$$g(r) = -\frac{\alpha}{6r^{\frac{4}{3}}} + \frac{\beta}{6r^{\frac{8}{3}}} + \frac{\gamma}{3r^{\frac{5}{2}}} \tag{18}$$

This allows write the perturbative term $V_{CFPS\_P}(r)$ as :

$$V_{CFPS\_P}(r) = \alpha' g(r) \left(\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2\right) \tag{19}$$

Where $\mathbf{J}$ denote to the total momentum. The operator $g(r)\ \mathbf{SL}$ traduces physically the coupling between spin and orbital momentum. Then, the corresponding NC Hamiltonian $H_{NCCFPS-1}$ will be :

$$H_{NCCFPS-1} = -\frac{\nabla^2}{2m_0} + V_{CFPS}(r) + \theta\left(-\frac{\alpha}{6r^{\frac{4}{3}}} + \frac{\beta}{6r^{\frac{8}{3}}} + \frac{\gamma}{3r^{\frac{5}{2}}}\right)\left(J^2 - L^2 - S^2\right) \tag{20}$$

After a straightforward calculation, we can show that the radial part $R(r)$ of the Schrödinger equation for a solved quantum bound state problem in NC spaces is given by :

$$\begin{array}{l} \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right) - \frac{l(l+1)}{r^2}R(r) + \\ \left(E - \alpha r^{\frac{2}{3}} - \beta r^{-\frac{2}{3}} - \gamma r^{-\frac{4}{3}} - \left(-\frac{\alpha}{6r^{\frac{4}{3}}} + \frac{\beta}{6r^{\frac{8}{3}}} + \frac{\gamma}{3r^{\frac{5}{2}}}\right) L\theta\right) R(r) = 0 \end{array} \tag{21}$$

We know in non relativistic quantum mechanics the triplets $(L_x, L_y, L_z)$ constructs symmetry generators satisfying the Lie algebra and, therefore, ( $\mathbf{J}^2$, $\mathbf{L}^2$, $\mathbf{L}^2$ and $J_z$) is complete set of observables. Then the combined operator $(\mathbf{J}^2 - \mathbf{L}^2 - \mathbf{S}^2)$ will have two eigenvalues $L_U\left(l,\ j = l + \frac{1}{2},\ s\right)$ and $L_D\left(l,\ j = l - \frac{1}{2},\ s\right)$ corresponding (spin up : $j = l + \frac{1}{2}$) and (spin down : $j = l - \frac{1}{2}$), respectively :

$$\begin{array}{ll} L_U\left(l,\ j = l + \frac{1}{2},\ s\right) = & \left(l + \frac{1}{2}\right)\left(l + \frac{3}{2}\right) - l(l+1) - \frac{3}{4} \\ L_D\left(l,\ j = l - \frac{1}{2},\ s\right) = & \left(l - \frac{1}{2}\right)\left(l + \frac{1}{2}\right) - l(l+1) - \frac{3}{4} \end{array} \tag{22}$$

Then, we can form a diagonal matrix of order $(3 \times 3)$ : $H_{NCCFPS-1}$ with diagonal elements $(H_{NC-CFPS})_{11}$, $(H_{NC-CFPS})_{22}$ and $(H_{NC-CFPS})_{33}$ :

$$\begin{array}{l} (H_{NC-CFPS})_{11} = -\frac{\Delta}{2m_0} + V_{CFPS}(r) + \theta g(r)\ L_U\left(l,\ j = l + \frac{1}{2},\ s\right) \text{ for :} j = l + \frac{1}{2} \Rightarrow \text{spinup} \\ (H_{NC-CFPS})_{22} = -\frac{\Delta}{2m_0} + V_{CFPS}(r) + \theta g(r)) L_D\left(l,\ j = l - \frac{1}{2},\ s\right) \text{ for :} j = l - \frac{1}{2} \Rightarrow \text{spindown} \\ (H_{NC-CFPS})_{33} = 0 \end{array} \tag{23}$$

The exact non commutative energies for states : $E_{NU}$ and $E_{ND}$ of an electron with spin up and spin down are determined to be, respectively :

$$\begin{array}{ll} E_{NU} = & E_{p-CFPS} + E_U \\ E_{ND} = & E_{p-CFPS} + E_D \end{array} \tag{24}$$

Where $E_U$ and $E_D$ are the modifications to the energy levels, associated with spin up and spin down. At the first order of parameter $\theta$ and by applying the perturbation theory, $E_U$ and $E_D$ became, respectively :

$$\begin{cases} E_U = \alpha' L_U\left(l,\, j = l + \frac{1}{2},\, s\right) \int \Psi^{(p)*}(\overleftrightarrow{r}) g\left(r\right)\, \Psi^{(p)}\left(\overleftrightarrow{r}\right)\, d\tau \\ E_D = \alpha' L_D\left(l,\, j = l - \frac{1}{2},\, s\right) \int \Psi^{(p)*}\left(\overleftrightarrow{r}\right)\, g\left(r\right)\, \Psi^{(p)}\left(\overleftrightarrow{r}\right)\, d\tau \end{cases} \tag{25}$$

Where $d\tau = r^2 \sin(\theta) d\theta d\varphi dr$ denote to the elementary volumes element in spherical coordinates.

### 3.2. The noncommutative spectra for $p = 0$ for NRQM (C.F.P.S.) potential

The non-commutative modifications of the energy levels, associated with spin up and spin down, in the first order of corresponding $p = 0$ ($E_{0U}$ and $E_{0D}$), are determined using eqs. (6), (18) and (25) to obtain :

$$\begin{aligned} E_{0U} = &\ \theta' \alpha L_U\left(l,\, j = l - \tfrac{1}{2},\, s\right) \int_0^{+\infty} \left[\exp\left(-\tfrac{3}{4}\sqrt{\alpha} r^{\frac{4}{3}} + \tfrac{3}{4}\tfrac{E_0^l}{\sqrt{\alpha}} r^{\frac{2}{3}}\right) a_0 r^l\right]^2 g\left(r\right)\, r^2 dr \\ E_{0D} = &\ \theta' \beta L_D\left(l,\, j = l - \tfrac{1}{2},\, s\right) \int_0^{+\infty} \left[\exp\left(-\tfrac{3}{4}\sqrt{\alpha} r^{\frac{4}{3}} + \tfrac{3}{4}\tfrac{E_0^l}{\sqrt{\alpha}} r^{\frac{2}{3}}\right) a_0 r^l\right]^2 g\left(r\right)\, r^2 dr \end{aligned} \tag{26}$$

Which the equations :

$$\begin{aligned} E_{0U} = &\ \theta' \alpha L_U\left(l,\, j = l + \tfrac{1}{2},\, s\right) a_0^2 \int_0^{+\infty} \exp\left(-\tfrac{3}{2}\sqrt{\alpha} r^{\frac{4}{3}} + \tfrac{3}{2}\tfrac{E_0^l}{\sqrt{\alpha}} r^{\frac{2}{3}}\right) \\ &\left(-\frac{\alpha r^{2l+2-\frac{4}{3}}}{6} + \frac{\beta r^{2l+2-\frac{8}{3}}}{6} + \frac{\gamma r^{2l+2-\frac{5}{2}}}{3}\right) dr \\ E_{0D} = &\ \theta' \beta L_D\left(l,\, j = l - \tfrac{1}{2},\, s\right) a_0^2 \int_0^{+\infty} \exp\left(-\tfrac{3}{2}\sqrt{\alpha} r^{\frac{4}{3}} + \tfrac{3}{2}\tfrac{E_0^l}{\sqrt{\alpha}} r^{\frac{2}{3}}\right) \\ &\left(-\frac{\alpha r^{2l+2\frac{4}{3}}}{6} + \frac{\beta r^{2l+2-\frac{8}{3}}}{6} + \frac{\gamma r^{2l+2-\frac{5}{2}}}{3}\right) dr \end{aligned} \tag{27}$$

The notations;

$$\begin{aligned} &A_1 = -\tfrac{\alpha}{6},\, A_2 = \tfrac{\beta}{6} \\ &A_3 = \tfrac{\gamma}{3},\, \delta = \tfrac{3}{2}\sqrt{\alpha} \text{ and } \varepsilon = -\tfrac{3}{2}\tfrac{E_0^l}{\sqrt{\alpha}} \end{aligned} \tag{28}$$

Eq. (27) to the form (the sum with $\lambda$ from 1 to 3) :

$$\begin{aligned} E_{0U} = &\ \theta' \alpha L_U\left(l,\, j,\, s\right) a_0^2 A_\lambda A^\lambda \\ E_{0D} = &\ \theta' \beta L_D\left(l,\, j,\, s\right) a_0^2 A_\lambda A^\lambda \end{aligned} \tag{29}$$

Where :

$$\begin{aligned}
A^1 &= \int_0^{+\infty} \exp\Big(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\Big)\, r^{2l+2-\frac{4}{3}} dr \\
A^2 &= \int_0^{+\infty} \exp\Big(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\Big)\, r^{2l+2-\frac{8}{3}} dr \\
A^3 &= \int_0^{+\infty} \exp\Big(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\Big)\, r^{2l+2-\frac{5}{2}} dr
\end{aligned} \tag{30}$$

Now, we make the changes; $r^{\frac{4}{3}} = x^{2} \Longrightarrow dr = \frac{3}{2} x dx$, then the above equation reduce to the equivalent form :

$$\begin{aligned}
A^1 &= \frac{3}{2}\int_0^{+\infty} \exp\left(-\delta x^2 - \varepsilon x\right)\, x^{\left(3l+\frac{5}{2}\right)-1} dx \\
A^2 &= \frac{3}{2}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right)\, x^{\left(3l+\frac{1}{2}\right)-1} dx \\
A^3 &= \frac{3}{2}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right)\, x^{\left(3l+\frac{3}{4}\right)-1} dx
\end{aligned} \tag{31}$$

We use the following form of special integral [32] :

$$\int_0^{+\infty} x^{v-1} \exp\left(-\delta x^2 - \varepsilon x\right)\, dx = (2\varepsilon)^{-\frac{v}{2}} \Gamma(v) \exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-v}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \tag{32}$$

We obtain :

$$\begin{aligned}
A^1 &= \tfrac{3}{2}(2\varepsilon)^{-\frac{v}{2}} \Gamma\left(3l+\tfrac{5}{2}\right) \exp\left(\tfrac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{5}{2}\right)}\left(\tfrac{\varepsilon}{\sqrt{2\delta}}\right) \\
A^2 &= \tfrac{3}{2}(2\varepsilon)^{-\frac{v}{2}} \Gamma\left(3l+\tfrac{1}{2}\right) \exp\left(\tfrac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{1}{2}\right)}\left(\tfrac{\varepsilon}{\sqrt{2\delta}}\right) \\
A^3 &= \tfrac{3}{2}(2\varepsilon)^{-\frac{v}{2}} \Gamma\left(3l+\tfrac{3}{4}\right) \exp\left(\tfrac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{3}{4}\right)}\left(\tfrac{\varepsilon}{\sqrt{2\delta}}\right)
\end{aligned} \tag{33}$$

Inserting eq. (33) in (29) we obtain the noncommutative corrections for energy eigenvalue $E_{OU}$ and $E_{OD}$.

### 3.3. The noncommutative spectra for $p = 1$ for NRQM (C.F.P.S.) potential

Now, the non-commutative first-order (in $\theta$) modification of the energy levels $E_{1U}$ and $E_{1D}$, associated with spin up and spin down, respectively, corresponding to $p = 1$ excited states, will be determined from Esq. (7), (18) and (25) :

$$E_{1U} = \alpha'\alpha L_U\left(l,\, j=l+\tfrac{1}{2},\, s\right)\int_0^{+\infty}\left[\exp\left(-\tfrac{3}{4}\sqrt{\alpha}r^{\frac{4}{3}}+\tfrac{3}{4}\tfrac{E_1^l}{\sqrt{\alpha}}r^{\frac{2}{3}}\right)r^l\left(a_0+a_1r^{\frac{2}{3}}\right)\right]^2 g(r)\, r^2dr$$
$$E_{1D} = \alpha'\beta L_D\left(l,\, j=l-\tfrac{1}{2},\, s\right)\int_0^{+\infty}\left[\exp\left(-\tfrac{3}{4}\sqrt{\alpha}r^{\frac{4}{3}}+\tfrac{3}{4}\tfrac{E_1^l}{\sqrt{\alpha}}r^{\frac{2}{3}}\right)r^l\left(a_0+a_1r^{\frac{2}{3}}\right)\right]^2 g(r)\, r^2dr \tag{34}$$

The above relations can be simplified to the following forms :

$$E_{1U} = \alpha' L_U\left(l,\, j=l+\tfrac{1}{2},\, s\right)\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)\begin{pmatrix} B_1r^{2l+\frac{2}{3}}+B_2r^{2l-\frac{2}{3}}+B_3r^{2l+\frac{1}{6}} \\ B_4r^{2l+2}+B_5r^{2l+\frac{2}{3}}+B_6r^{2l+\frac{5}{6}} \\ B_7r^{2l+\frac{4}{3}}+B_8r^{2l-\frac{2}{3}}+B_9r^{2l+\frac{1}{6}}\end{pmatrix}dr$$
$$E_{1D} = \alpha' L_D\left(l,\, j=l-\tfrac{1}{2},\, s\right)\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)\begin{pmatrix} B_1r^{2l+\frac{2}{3}}+B_2r^{2l-\frac{2}{3}}+B_3r^{2l\frac{1}{6}} \\ B_4r^{2l+2}+B_5r^{2l+\frac{2}{3}}+B_6r^{2l+\frac{5}{6}} \\ B_7r^{2l+\frac{4}{3}}+B_8r^{2l-\frac{2}{3}}+B_9r^{2l+\frac{1}{6}}\end{pmatrix}dr \tag{35}$$

Where the covariant notations $B_\mu$ with $(\mu=\overline{1,9})$ are determined from the relations :

$$\begin{array}{ccc} B_1=-\frac{\alpha a_0^2}{6} & B_2=\frac{\beta a_0^2}{6} & B_3=\frac{2a_0a_1\gamma}{3} \\ B_4=-\frac{\alpha a_1^2}{6} & B_5=\frac{\beta a_1^2}{6} & B_6=\frac{\gamma a_1^2}{3} \\ B_7=-\frac{\alpha a_0a_1}{3} & B_8=\frac{a_0a_1\beta}{3} & \text{and } B_9=\frac{2a_0a_1\gamma}{3}\end{array} \tag{36}$$

If we use the contra variant notations $B^\mu$ with $(\mu=\overline{1,9})$ :

$$\begin{array}{ll} B^1=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+\frac{2}{3}}dr, & B^2=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l-\frac{2}{3}}dr \\ B^3=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+\frac{1}{6}}dr, & B^4=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+2}dr \\ B^5=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+\frac{2}{3}}dr, & B^6=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+\frac{5}{6}}dr \\ B^7=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+\frac{4}{3}}dr, & B^8=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l-\frac{2}{3}}dr \\ \text{and} & B^9=\int_0^{+\infty}\exp\left(-\delta r^{\frac{4}{3}}-\varepsilon r^{\frac{2}{3}}\right)r^{2l+\frac{1}{6}}dr \end{array} \tag{37}$$

Now, the same change, the above equation reduces to the equivalent form :

$$\begin{aligned}
B^1 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+\frac{2}{3}} dr \quad & B^2 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l-\frac{2}{3}} dr \\
B^3 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+\frac{1}{6}} dr \quad & B^4 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+2} dr \\
B^5 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+\frac{2}{3}} dr \quad & B^6 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+\frac{5}{6}} dr \\
B^7 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+\frac{4}{3}} dr \quad & B^8 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l-\frac{2}{3}} dr \\
& \text{and} & B^9 &= \int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{2l+\frac{1}{6}} dr
\end{aligned} \tag{38}$$

The convenient mathematical form :

$$\begin{aligned}
B^1 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) x^{\left(l+\frac{11}{6}\right)-1} dx \quad & B^2 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) x^{\left(l+\frac{4}{3}\right)-1} dx \\
B^3 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) x^{\left(3l+\frac{19}{2}\right)-1} dx \quad & B^4 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) x^{\left(3l+\frac{5}{2}\right)-1} dx \\
B^5 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) x^{3l+\frac{15}{4}} dx \quad & B^6 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{\left(3l+\frac{7}{4}\right)-1} dx \\
B^7 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r - \varepsilon r^{\frac{2}{3}}\right) r^{\left(3l+\frac{7}{2}\right)-1} dx \quad & B^8 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{\left(3l+\frac{1}{6}\right)-1} dx \\
& \text{and} & B^9 &= \frac{2}{3}\int_0^{+\infty} \exp\left(-\delta r^{\frac{4}{3}} - \varepsilon r^{\frac{2}{3}}\right) r^{\left(3l+\frac{19}{12}\right)-1} dx
\end{aligned} \tag{39}$$

The above special integral, after a straightforward calculation :

$$
\begin{array}{ll}
B^1 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(l+\frac{11}{6}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(l+\frac{11}{6}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^2 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(l+\frac{4}{3}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(l+\frac{4}{3}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^3 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{19}{2}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{19}{2}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^4 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{5}{2}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{5}{2}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^5 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{15}{4}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{15}{4}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^6 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{7}{4}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{7}{4}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^7 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{7}{2}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{7}{2}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^8 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{1}{6}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{1}{6}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right) \\
B^9 = & \frac{2}{3}(2\varepsilon)^{-\frac{v}{2}}\Gamma\left(3l+\frac{19}{12}\right)\exp\left(\frac{\varepsilon^2}{8\delta}\right) D_{-\left(3l+\frac{19}{12}\right)}\left(\frac{\varepsilon}{\sqrt{2\delta}}\right)
\end{array} \tag{40}
$$

Then, the obtained corrections for $p = 1$ excited states :

$$
\begin{array}{ll}
E_{1U} = & \alpha' L_U\,(l,\, j,\, s)\; B_\mu B^\mu \\
E_{1D} = & \alpha' L_D\,(l,\, j,\, s)\; B_\mu B^\mu
\end{array} \tag{41}
$$

We have used in the above results the Einstein (sum with indies ). For, the stationary state, the two values of $L_U\left(l, j = l + \frac{1}{2}, s\right)$ and $L_D\left(l, j = l - \frac{1}{2}, s\right)$ are equal to and one, while the two values in the first excited states are reduced to *one* and $(-2)$, spin up and spin down, respectively. We summarize the obtained results of energies ($E_{0U}$, $E_{0D}$, $E_{1U}$ and $E_{1D}$) associated with spin up and spin down in the first order perturbation of $\theta$, to the stationary state and the first excited states as follows :

$$
\begin{array}{r}
E_{0U} = \pm\sqrt{4\alpha}\left\{\left(2l+\frac{7}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}} + \theta'\alpha L_U\,(l,\, j,\, s)\; a_0^2 A_\lambda A^\lambda \\
E_{0D} = \pm\sqrt{4\alpha}\left\{\left(2l+\frac{7}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}} + \theta'\alpha L_D\,(l,\, j,\, s)\; a_0^2 A_\lambda A^\lambda \\
E_{1U} = \pm\sqrt{4\alpha}\left\{\left(2l+\frac{11}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}} + \alpha' L_U\,(l,\, j,\, s)\; B_\mu B^\mu \\
E_{1D} = \pm\sqrt{4\alpha}\left\{\left(2l+\frac{11}{3}\right)\sqrt{\alpha}+\beta\right\}^{\frac{1}{2}} + \alpha' L_D\,(l,\, j,\, s)\; B_\mu B^\mu
\end{array} \tag{42}
$$

We can write the commutative central fraction power singular Hamiltonian $H_{CFPS}$ and $H_{SO-CFPS}$ the generated new spin-orbital interaction as :

$$
\begin{array}{r}
H_{CFPS} = \left[-\frac{\Delta}{2m_0} + \alpha r^{\frac{2}{3}} + \beta r^{-\frac{2}{3}} + \gamma r^{-\frac{4}{3}}\right] I_{3\times 3} \\
H_{SO-CFPS} = \theta g(r) \begin{pmatrix} L_U\left(l,\, j = l+\frac{1}{2},\, s\right) & 0 & 0 \\ 0 & L_D\left(l,\, j = l-\frac{1}{2},\, s\right) & 0 \\ 0 & 0 & 0 \end{pmatrix}
\end{array} \tag{43}
$$

The operator $H_{CFPS}$ represents an electron interacted exactly with the central fraction power singular potential in ordinary commutative space, while the matrix $H_{SO-CFPS}$ is the spin-orbit interaction. The new levels are characterized by the quantum numbers $(\mathbf{J}, \mathbf{I})$ and $s_z = \pm 1/2$, contrary to the old levels (commutative space) which are depended only on quantum number $\mathbf{I}$ and the 3-parameters of central fraction power singular potential ( the studied potential) : $\alpha$, $\beta$ and $\gamma$.

### 3.4. The modified Zeeman effect for NRQM (C.F.P.S.) potential :

On another hand, we can draw another physical interpretation for the results of the noncommutativity of the spaces for central fraction power singular potential. If we choose the parameter and the vector of a magnetic field as follows [23]

$$\boldsymbol{\theta} = \varepsilon' \mathbf{B} \text{ and } \theta L_z = \varepsilon' \mathbf{J}\mathbf{B} - \varepsilon H_Z \tag{44}$$

Where $\varepsilon'$ is a real proportionality-constant, $H_Z$ is the usual Zeeman field. Substituting two Eqs. (20) and (22) into eq. (15) leads to the second new NC Hamiltonian $H_{NC-2}$ as :

$$H_{NC-2} = -\frac{\Delta}{2m_0} + V_{CFPS}(r) + H_{mag-cfps} \tag{45}$$

Where the operator $H_{mag-cfps}$ is given by :

$$H_{mag-cfps} = -\varepsilon' g(r)\ H_Z + \varepsilon g(r)\ JB \tag{46}$$

The above operator $H_{mag-cfps}$ represents two physical interactions between the polarized electron and external magnetic field ; the first one is the ordinary Zeeman Effect and the second is the new interaction coupling between the total momentum $\mathbf{J}$ and external magnetic field $\mathbf{B}$. It is easy to see that the classical limit is guaranteed by the condition $(\theta \longrightarrow 0)$ in NC 3D NRQM. The final expression of (NC-3D NRQM) Hamiltonian $H_{NC-CFPS}$ for (C.F.P.S.) potential can be resumed in the diagonal matrix of order $3 \times 3$ as :

$$H_{NC-CFPS} = \begin{pmatrix} \begin{matrix} \theta g(r) L_U\left(l,\, j = l + \frac{1}{2},\, s\right) + \\ H_{CFPS} + H_{mag-cfps} \end{matrix} & 0 & 0 \\ 0 & \begin{matrix} \theta g(r) L_D\left(l,\, j = l - \frac{1}{2},\, s\right) + \\ H_{CFPS} + H_{mag-cfps} \end{matrix} & 0 \\ 0 & 0 & H_{CFPS} \end{pmatrix} \tag{47}$$

It's important to notice that, the noncommutative operator of NC Hamiltonians is changed ; the homage's diagonal elements in commutative Hamiltonian are replaced with different elements :

$$(H_{NC-CFPS})_{11} \neq (H_{NC-CFPS})_{22} \neq (H_{NC-CFPS})_{33} \tag{48}$$

Then, the isotropic commutative Hamiltonian will be in non commutative space anisotropic Hamiltonian.

## 4. Conclusions

We have obtained the modified bound state solutions of the three-dimensional radial Schrödinger equation for central fraction power singular potential in the case of (NC-3D NRQM), the old states are changed radically and replaced by degenerated new states, depending on four quantum numbers $(\mathbf{J}, \mathbf{l})$ and $s_z = \pm 1/2$ corresponding spin up and spin down. The corresponding NC Hamiltonian represented by 3-matrices , $H_{CFPS}$, $H_{SO-CFPS}$ and $H_{mag-cfps}$ : the first represents the interaction of an electron with spin (1/2) in central fraction power singular potential in commutative ordinary space, while the second matrix represents the spin-orbit interaction, the last part of NC Hamiltonians is the interaction between an electron and an external magnetic field.

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