On indexed Summability of a factored Fourier series through Local Property

R.K. Jati[1], S.K.Paikray[2] and U.K.Misra[3]

[1] Research Scholar, Department of Mathematics, Ravenshaw University , Cuttack, Odisha, India
[2] Department of Mathematics , VSS University of Technology, Burla, Odisha, India
[3] Department of Mathematics , NIST , Berhampur, Odisha, India

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ABSTRACT

In this paper we have established a theorem on the local Property of $|N, p_n, \alpha_n; \delta|_k$ summability of factored Fourier series.

Corresponding Author:

S. K. Paikray
Department of Mathematics,
VSS University of Technology, Burla, India
Email Id: spaikray2001@yahoo.com

I. INTRODUCTION

Let $\sum a_n$ be a given infinite series with sequence of partial sums $\{s_n\}$. Let $\{p_n\}$be a sequence of positive real constants such that

$$P_n = \sum_{\nu=0}^{n} p_\nu \to \infty \; as \; n \to \infty \; (P_{-i} = p_{-i} = 0, i \geq 1) \qquad (1.1)$$

The sequence- to - sequence transformation

$$t_n = \frac{1}{P_n} \sum_{\nu=0}^{n} p_{n-\nu} s_\nu \qquad (1.2)$$

defines $|N, p_n|$ - means of the sequence $\{s_n\}$generated by the sequence of coefficients $\{p_n\}$. The series $\sum a_n$ is said to be summable $|N, p_n|_k$, $k \geq 1$, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |t_n - t_{n-1}|^k < \infty \qquad (1.3)$$

For k=1, $|N, p_n|_k$ - summability is same as $|N, p_n|$ - summability.

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When $p_n = 1$, for all n and $k = 1$, $|N, p_n|_k$ - summability is same as $|C,1|$ - summability.

Let $\{\alpha_n\}$ be any sequence of positive numbers. The series $\sum a_n$ is said to be summable $|N, p_n, \alpha_n|_k$, $k \geq 1$, if

$$\sum_{n=1}^{\infty} \alpha_n^{k-1} |t_n - t_{n-1}|^k < \infty \qquad (1.4)$$

Where $\{t_n\}$ is as defined in (1.2).The series $\sum a_n$ is said to be $|\overline{N}, p_n, \alpha_n; \delta|_k, k \geq 1, \delta \geq 0$ summable if

$$\sum_{n=1}^{\infty} \alpha_n^{\delta k+k-1} |t_n - t_{n-1}|^k < \infty \qquad (1.5)$$

For $\delta = 0$, the summability method $|\overline{N}, p_n, \alpha_n; \delta|_k, k \geq 1, \delta \geq 0$, reduces to the summabilty method $|\overline{N}, p_n, \alpha_n|_k, k \geq 1$.

A sequence $\{\lambda_n\}$ is said to be convex if $\Delta^2 \lambda_n \geq 0$ for every positive integer n.

Let $f(t)$ be a periodic function with period 2π and integrable in the sense of Lebesgue over $(-\pi, \pi)$. Without loss of generality we may assume that the constant term in the Fourier series of $f(t)$ is zero, so that

$$f(t) \sim \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t) \qquad (1.6)$$

It is well known that the convergence of Fourier series at t = x is a local property of f (t). (i.e., it depends only on the behavior of f (t) in an arbitrarily small neighbourhood of x) and hence the summability of the Fourier series at t = x by any regular linear method is also a local property of f (t).

II. KNOWN THEOREMS

Dealing with the $|\overline{N}, p_n|_k$ - summability of an infinite series Bor [1] proved the following theorem:

THEOREM 2.1:

Let $k \geq 1$ and let the sequences $\{p_n\}$ and $\{\lambda_n\}$ be such that

$$\Delta X_n = O\left(\frac{1}{n}\right) \qquad (2.1)$$

$$\sum_{n=1}^{\infty} X_n^{k-1} \frac{|\lambda_n|^k + |\lambda_{n+1}|^k}{n} < \infty \qquad (2.2)$$

and

$$\sum_{n=1}^{\infty} (X_n^k + 1)|\Delta\lambda_n| < \infty \qquad (2.3)$$

where $X_n = \frac{P_n}{np_n}$. Then the summability $|\overline{N}, p_n|_k$ of the factored Fourier series $\sum_{n=1}^{\infty} A_n(t)\lambda_n X_n$ at a point can be ensured by the local property.

Subsequently Misra et al [2] proved the following theorem on the local property of $|N, p_n, \alpha_n|_k$ summability of factored Fourier series:

THEOREM 2.2:

Let $k \geq 1$. Suppose $\{\lambda_n\}$ be a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent and $\{p_n\}$ be a sequence such that

$$\Delta X_n = O\left(\frac{1}{n}\right) \qquad (2.2.1)$$

$$\frac{P_{n-r-1}}{P_n} = O\left(\frac{p_{n-r-1}}{P_{n-1}} \frac{P_r}{p_r}\right) \qquad (2.2.2)$$

$$\sum_{n=r+1}^{m+1} (\alpha_n)^{k-1} \frac{p_{n-r}}{P_n} = O\left(\frac{p_r}{P_r}\right) \qquad (2.2.3)$$

$$\sum_{n=1}^{\infty} X_n^{k-1} \frac{|\lambda_n|^k}{n} < \infty \qquad (2.2.4)$$

and

$$\sum_{n=1}^{\infty} X_n^{k-1} \frac{|\Delta\lambda_n|^k}{n} < \infty \qquad (2.2.5)$$

where $X_n = \frac{P_n}{np_n}$. Then the summability $|N, p_n, \alpha_n|_k, k \geq 1$ of the factored Fourier series $\sum_{n=1}^{\infty} A_n(t)\lambda_n X_n$ at a point can be ensured by the local property, where $\{\alpha_n\}$ is a sequence of positive numbers.

In what follows, in the present paper we establish the following theorem on $|N, p_n, \alpha_n, \delta|_k$ -summabilty of a factored Fourier series through its local property.

III. MAIN THEOREM

Let $k \geq 1$. Suppose $\{\lambda_n\}$ be a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent and $\{p_n\}$ be a sequence such that

$$\Delta X_n = O\left(\frac{1}{n}\right) \qquad (3.1)$$

$$\frac{P_{n-r-1}}{P_n} = O\left(\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\right) \qquad (3.2)$$

$$\sum_{n=r+1}^{m+1} (\alpha_n)^{\delta k+k-1} \frac{p_{n-r}}{P_n} = O\left(\frac{p_r}{P_r}\right) \qquad (3.3)$$

$$\sum_{n=1}^{\infty} X_n^{k-1}\frac{|\lambda_n|^k}{n} < \infty \qquad (3.4)$$

and

$$\sum_{n=1}^{\infty} X_n^{k-1}\frac{|\Delta\lambda_n|^k}{n} < \infty \qquad (3.5)$$

where $X_n = \frac{P_n}{np_n}$. Then the summability $|N, p_n, \alpha_n, \delta|_k$, $k \geq 1$ of the factored Fourier series $\sum_{n=1}^{\infty} A_n(t)\lambda_n X_n$ at a point can be ensured by the local property, where $\{\alpha_n\}$ is a sequence of positive numbers .

In order to prove the above theorem we require the following lemma:

IV. LEMMA

Let $k \geq 1$. Suppose $\{\lambda_n\}$ be a convex sequence such that $\sum n^{-1}\lambda_n$ is convergent and $\{p_n\}$ be a sequence such that the conditions **(3.1)-(3.5)** are satisfied. If $\{s_n\}$ is bounded then for the sequence of positive numbers $\{\alpha_n\}$, the series $\sum_{n=1}^{\infty} a_n\lambda_n X_n$ is summable $|N, p_n, \alpha_n, \delta|_k, k \geq 1, \delta \geq 0$.

5. PROOF OF THE LEMMA:

Let $\{T_n\}$ denote the $|N, p_n|$ -mean of the series $\sum_{n=1}^{\infty} a_n\lambda_n X_n$ .Then by definition we have

$$T_n = \frac{1}{P_n}\sum_{v=0}^{n} p_{n-v}\sum_{r=0}^{v} a_r\lambda_r X_r$$

$$= \frac{1}{P_n}\sum_{r=0}^{n} a_r\lambda_r X_r \sum_{v=r}^{n} p_{n-v}$$

$$= \frac{1}{P_n}\sum_{r=0}^{n} a_r P_{n-r}\lambda_r X_r$$

Hence

$$T_n - T_{n-1} = \frac{1}{P_n}\sum_{r=1}^{n} P_{n-r}a_r\lambda_r X_r - \frac{1}{P_{n-1}}\sum_{r=1}^{n-1} P_{n-r-1}a_r\lambda_r X_r$$

$$= \sum_{r=1}^{n}\left(\frac{P_{n-r}}{P_n} - \frac{P_{n-r-1}}{P_{n-1}}\right)a_r\lambda_r X_r$$

$$= \frac{1}{P_n P_{n-1}}\sum_{r=1}^{n}(P_{n-r}P_{n-1} - P_{n-r-1}P_n)a_r\lambda_r X_r$$

$$= \frac{1}{P_n P_{n-1}}\left[\sum_{r=1}^{n-1}\Delta\{(P_{n-r}P_{n-1} - P_{n-r-1}P_n)\lambda_r X_r\}\right]\sum_{v=1}^{r} a_v$$

$$= \frac{1}{P_n P_{n-1}}\left[\sum_{r=1}^{n-1}(p_{n-r}P_{n-1} - p_{n-r-1}P_n)\lambda_r X_r s_r + \sum_{r=1}^{n-1}(P_{n-r-1}P_{n-1} - P_{n-r-2}P_n)\Delta\lambda_r X_r s_r + \sum_{r=1}^{n-1}(P_{n-r-1}P_{n-1} - P_{n-r-2}P_n)\lambda_{r+1}\Delta X_r s_r\right]$$

(By Abel's transformation)

$$=T_{n,1}+T_{n,2}+T_{n,3}+T_{n,4}+T_{n,5}+T_{n,6} \quad \text{(say)}.$$

In order to complete the proof of the theorem by using Minokowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty}(\alpha_n)^{\delta k+k-1}|T_{n,i}|^k<\infty \quad for\ i=1,2,3,4,5,6.$$

Now, we have

$$\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}|T_{n,1}|^k$$

$$=\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}p_{n-r}P_{n-1}\lambda_r X_r s_r\right|^k$$

$$\leq\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\frac{1}{P_n}\left(\sum_{r=1}^{n-1}p_{n-r}|\lambda_r|^k|s_r|^k X_r^k\right)\left(\frac{1}{P_n}\sum_{r=1}^{n-1}p_{n-r}\right)^{k-1}$$

$$=O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k\sum_{n=r+1}^{m+1}(\alpha_n)^{\delta k+k-1}\left(\frac{p_{n-r}}{P_n}\right)$$

$$=O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k\frac{p_r}{P_r} \quad \text{, by (3.3)}$$

$$=O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{ as}$$

$$=O(1)\sum_{r=1}^{m}X_r^{k-1}\frac{|\lambda_r|^k}{r}$$

$$=O(1) \quad as\ m\to\infty, \text{ by (3.4).}$$

Next,

$$\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}|T_{n,2}|^k$$

$$=\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}p_{n-r-1}P_n\lambda_r X_r s_r\right|^k$$

$$\leq\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\frac{1}{P_{n-1}}\left(\sum_{r=1}^{n-1}p_{n-r-1}|\lambda_r|^k|s_r|^k X_r^k\right)\left(\frac{1}{P_{n-1}}\sum_{r=1}^{n-1}p_{n-r-1}\right)^{k-1}$$

$$=O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k\sum_{n=r+1}^{m+1}(\alpha_n)^{\delta k+k-1}\left(\frac{p_{n-r-1}}{P_{n-1}}\right)$$

$$=O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^k\frac{p_r}{P_r} \quad \text{, by (3.3)}$$

$$=O(1)\sum_{r=1}^{m}|\lambda_r|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{ as } X_n=\frac{P_n}{np_n}$$

$$=O(1)\sum_{r=1}^{m}X_r^{k-1}\frac{|\lambda_r|^k}{r}$$

$$=O(1) \quad as\ m\to\infty, \text{ by (3.4).}$$

Further,

$$\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}|T_{n,3}|^k$$

$$=\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-1}P_{n-1}\Delta\lambda_r X_r s_r\right|^k$$

$$\leq\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\frac{1}{P_n}\left(\sum_{r=1}^{n-1}P_{n-r-1}|\Delta\lambda_r|^k|s_r|^k X_r^k\right)\left(\frac{1}{P_n}\sum_{r=1}^{n-1}P_{n-r-1}|\Delta\lambda_r|\right)^{k-1}$$

$$=O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^k\sum_{n=r+1}^{m+1}(\alpha_n)^{\delta k+k-1}\left(\frac{P_{n-r-1}}{P_n}\right)$$

$$\left(Since\ \frac{1}{P_n}\sum_{r=1}^{n-1}P_{n-r-1}|\Delta\lambda_r|\leq\sum_{r=1}^{n-1}|\Delta\lambda_r|=O(1)\right)$$

$$=O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^k\frac{p_r}{P_r} \quad \text{, by (3.3)}$$

$$=O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{ as } X_n=\frac{P_n}{np_n}$$

$$=O(1)\sum_{r=1}^{m}X_r^{k-1}\frac{|\Delta\lambda_r|^k}{r}$$

$$=O(1) \quad as\ m\to\infty, \text{ by (3.5).}$$

Now,

$$\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}|T_{n,4}|^k$$

$$=\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-2}P_n\Delta\lambda_r X_r s_r\right|^k$$

$$\leq\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\frac{1}{P_{n-1}}\left(\sum_{r=1}^{n-1}P_{n-r-2}|\Delta\lambda_r|^k|s_r|^k X_r^k\right)\left(\frac{1}{P_{n-1}}\sum_{r=1}^{n-1}P_{n-r-2}|\Delta\lambda_r|\right)^{k-1}$$

$$=O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^k\sum_{n=r+1}^{m+1}(\alpha_n)^{\delta k+k-1}\left(\frac{P_{n-r-2}}{P_{n-1}}\right) \quad \text{, (as above)}$$

$$= O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^k \frac{p_r}{P_r} \text{, by (3.3)}$$

$$= O(1)\sum_{r=1}^{m}|\Delta\lambda_r|^k X_r^{k-1} \frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n}$$

$$= O(1)\sum_{r=1}^{m} X_r^{k-1}\frac{|\Delta\lambda_r|^k}{r}$$

$= O(1)$ $as\ m\to\infty$, by (3.5).

Again

$$\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}|T_{n,5}|^k$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-1}P_{n-1}\lambda_{r+1}\Delta X_r s_r\right|^k$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{P_{n-r-1}}{P_n}\lambda_{r+1}\Delta X_r s_r\right|^k$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\lambda_{r+1}\Delta X_r s_r\right|^k \text{, by (3.2)}$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\lambda_{r+1}s_r\frac{1}{r}\right|^k \text{, by (3.1)}$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\frac{P_r}{p_r}\lambda_{r+1}s_r X_r\frac{p_r}{P_r}\right|^k \text{, as}$$

$$X_n = \frac{P_n}{np_n}$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}|\lambda_{r+1}|^k|s_r|^k X_r^k\right\}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-1}}{P_{n-1}}\right\}^{k-1}$$

$$= O(1)\sum_{r=1}^{m}|\lambda_{r+1}|^k X_r^k\sum_{n=r+1}^{m+1}(\alpha_n)^{\delta k+k-1}\left(\frac{p_{n-r-1}}{P_{n-1}}\right)$$

$$= O(1)\sum_{r=1}^{m}|\lambda_{r+1}|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n} \text{ and}$$

by **(3.3)**

$$= O(1)\sum_{r=1}^{m}\frac{|\lambda_{r+1}|^k}{r}X_r^{k-1},$$

$= O(1)$ $as\ m\to\infty$, by (3.4).

Finally,

$$\sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}|T_{n,6}|^k$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\frac{1}{P_nP_{n-1}}\sum_{r=1}^{n-1}P_{n-r-2}P_n\lambda_{r+1}\Delta X_r s_r\right|^k$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{P_{n-r-2}}{P_{n-1}}\lambda_{r+1}\Delta X_r s_r\right|^k$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\frac{P_r}{p_r}\lambda_{r+1}\Delta X_r s_r\right|^k \text{, by (3.2)}$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\frac{P_r}{p_r}\lambda_{r+1}s_r\frac{1}{r}\right|^k \text{, by (3.1)}$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left|\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\frac{P_r}{p_r}\lambda_{r+1}s_r X_r\frac{p_r}{P_r}\right|^k \text{, as}$$

$$X_n = \frac{P_n}{np_n}$$

$$= \sum_{n=2}^{m+1}(\alpha_n)^{\delta k+k-1}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}|\lambda_{r+1}|^k|s_r|^k X_r^k\right\}\left\{\sum_{r=1}^{n-1}\frac{p_{n-r-2}}{P_{n-2}}\right\}^{k-1}$$

$$= O(1)\sum_{r=1}^{m}|\lambda_{r+1}|^k X_r^k\sum_{n=r+1}^{m+1}(\alpha_n)^{\delta k+k-1}\left(\frac{p_{n-r-2}}{P_{n-2}}\right)$$

$$= O(1)\sum_{r=1}^{m}|\lambda_{r+1}|^k X_r^{k-1}\frac{p_r}{P_r}\frac{P_r}{rp_r}, \text{as } X_n = \frac{P_n}{np_n} \text{ and}$$

By (3.3)

$= O(1)$ $as\ m\to\infty$, by (3.4).

This completes the proof of the Lemma.

V. PROOF OF THE THEOREM

Since the behavior of the Fourier series, as far as convergence is concerned, for a particular value of x depends on the behavior of the function in the immediate neighborhood of this point only, thus the truth of the theorem is necessarily the consequence of the Lemma.

VI. CONCLUSION

The paper provides an analytic idea in the field of summability theory. In future, the present work can be extended to establish some theorems on different indexed summability factors of Fourier series as well as conjugate series of Fourier series under certain weaker conditions.

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R. K. Jati is Presently working as a research Scholar in the Post Graduate Department of Mathematics, Ravenshaw University, Cuttack, Odisha, India and a faculty member in Dhaneswar Rath Institute of Engineering and Technology, Cuttack, Odisha, India. He has been published around 5 research papers in various national and international journals of repute. Mr. Jati has 8 years of teaching and 5 years of research experience.

Dr. S. K. Paikray is Presently working as a Reader in the Department of Mathematics, VSS University of Technology, Burla, Sambalpur, Odisha, India. Prior to this he had worked as a lecturer in the Post Graduate Department of Mathematics, Ravenshaw University, Cuttack, Odisha, India. Dr. Paikray has 15 years of teaching experiences. Under the guidance of him 5 Ph. D scholars are continuing their research in different Universities. Dr. Paikray has been published around 21 research papers in various national and international journals of repute. The research area of Prof. Dr. Paikray is summability theory, Fourier series, Operations Research and Inventory control.

Prof. U. K. Misra had been working as a faculty member in the Post Graduate Department of Mathematics, Berhampur University for last 30 years. Presently he has been working as a professor of Mathematics in National Institute of Science and Technology, Palur Hills, Berhampur. Under the guidance of him 18 Ph. D scholars and 2 D.Sc. scholars have been awarded their degrees. Presently 8 scholars are working under his supervision. Prof Misra has been published around 100 research papers in various national and international journals of repute. The research area of Prof. Misra is sequence space, summability theory, Fourier series, Operations Research, Inventory control and mathematical modeling. He is a reviewer of Mathematical Review published by American Mathematical Society. Prof Misha has conducted several National and International seminar, conferences and refresher courses sponsored by UGC.