THE MECHANICAL BEHAVIOUR OF MATERIALS IN AUTOMOTIVE ENGINEERING REINFORCED BY STRONG FIBRES

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INTRODUCTION

Today, with the increase of the consumer market, new products have been introduced in order to replace materials such as metals, cement etc., which are very heavy, corrosive and less environmentally friendly. One such material is the fibre reinforced composite. In the past 30 – 40 years fibre composites have been competing with materials such as steel, aluminium and concrete in cars, aircraft, military, buildings, bridges, bicycles and everyday sports goods. A very important aspect of fibre reinforced materials is their mechanical behaviour.

Here is considered orthotropic materials modeled as fibre reinforced materials with one and two families of mechanically equivalent fibres. Constitutive equations employed here are developed for material which is made of unidirectional reinforced thin sheets, whose combinations form model of material. Here we study slowness surfaces, as indicators of dynamical behavior, analytically and numerically to obtain valuable information about wave propagation in arbitrary directions. Degrees of deviations of wave surfaces depend on degrees of anisotropy, and may give valuable information about dynamic deformations.

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The propagation condition in elastic waves propagation is shown by Nayfeh [6]. Failure criterions for materials reinforced by two families of strong fibres are given by Milosavljevic et al. in [5]. Numerical results, for various propagation directions, are given in details by Bogdanovic [1], based on material constants, for one family, measured by the ultrasonic method by Markham [4], and adopted for the case considered here

CONSTITUTIVE EQUATIONS – LINEAR ELASTICITY

The most of dynamical systems are naturally nonlinear. Because of that, it is not easy to find closed solutions of such systems. There we consider infinite domains so that we can omit questions concerned with the nature and interpretation of the correct boundary conditions, as well as the appropriate form of the stress tensor and the associated tractions. It may be shown that the equation describing the initial weak discontinuity, assuming that

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tangent stiffness tensor on both sides of the surface of discontinuity has the same value, has the following form

$$c_{ijkl} n_j n_l p_k = 0\,, \qquad c_{ijkl} = \frac{\partial^2 W}{\partial e_{ij} \partial e_{kl}}\,, \tag{1}$$

where c_{ijkl} represents the tangent stiffness tensor, n_j the unit normal of singular surface, and p_k is the polarization vector. Localization tensor is the second order tensor

$$\Gamma_{ik} = c_{ijkl} n_j n_l\,. \tag{2}$$

The classical localization condition in the considered case may be expressed as

$$\mathbf{det}\,\Gamma_{ik} = \left| c_{ijkl} n_j n_l \right| = 0\,. \tag{3}$$

The equations of motion governing wave propagation in a generally isotropic elastic medium are given by many authors. The equation of motion may be expressed for infinitesimal displacements u_i, Cartesian coordinates x_i, density ρ, stress tensor σ_{ij} and body forces per unit mass f_i, in the form

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{4}$$

where "$,_j$" denotes a partial derivative with respect to x_j and the Einstein summation convention is employed.

Equations of motion have the usual form, considering anisotropic linear elastic material neglecting body forces

$$c_{ijkl} u_{k,lj} - \rho \ddot{u}_i = 0\,, \tag{5}$$

where c_{ijkl} are the components of stiffness tensor of the considered material.

After some transformations, we get Riemann - Christoffel's equation

$$(c_{ijkl} n_j n_l - \rho v^2 \delta_{ik}) p_k = 0 \tag{6}$$

and system of homogeneous equations (6), has nontrivial solutions, provided that

$$\left| \Gamma_{ik} - \rho v^2 \delta_{ik} \right| = 0, \qquad \Leftrightarrow \qquad \left| \Lambda_{ik} - v^2 \delta_{ik} \right| = 0\,, \tag{7}$$

where

$$\Lambda_{ik} = \Gamma_{ik} / \rho = c_{ijkl} n_j n_l / \rho = \lambda_{ijkl} n_j n_l\,. \tag{8}$$

The equation (7), as an eigenvalue problem, leads to three values of phase velocity that correspond to the three polarization vectors $p_k^{(\alpha)}, \alpha = 1,2,3$. The components of Riemann - Christoffel's tensor may be expressed as

$$\begin{aligned}\Gamma_{il} = C_{ijkl} n_k n_j = C_{i11l} n_1 n_1 + (C_{i12l} + C_{i21l}) n_1 n_2 + (C_{i13l} + C_{i31l}) n_1 n_3 \\ + C_{i22l} n_2 n_2 + (C_{i23l} + C_{i32l}) n_1 n_3 + C_{i33l} n_3 n_3\,.\end{aligned}\,, \tag{9}$$

The localization tensor is referred to as the acoustic tensor, in the study of wave propagation. If the tangent stiffness tensor is taken as the elastic stiffness tensor, the eigenvalues of the corresponding acoustic tensor (8) divided by the mass density are squares of the speed of elastic waves propagating in the direction n_i. This equation represents the propagation condition of bulk waves as a set of three homogeneous linear equations. The Riemann-Christoffel's equation may be solved analytically only for the simplest cases of material symmetry.

NUMERICAL ANALYSIS OF SLOWNESS SURFACES FOR MATEIAL REINFORCED BY TWO FAMILIES OF MECHANICALLY EQUIVALENT EXTENSIBLE FIBRES

The material reinforced with two families of continuous fibres has the plane of symmetry tangent to both families of fibres, as the monoclinic symmetry and, therefore, has thirteen independent material constants. When two families of fibres are mechanically equivalent, the material behaves like orthotropic Axes of symmetry along the bisectors of the fibre directions and along the normal to plane tangent to fibres, reducing further number of independent material constants to nine.

The best way when developing constitutive equations for elastic materials is to find an equation for the strain energy density of the material as a function of the strain. The strain energy density, if the material is isotropic, can be a function of strain measures that do not depend on the direction of loading with respect to the material. That the strain energy can be a function of invariants of the strain tensor only that is, combinations of strain components that have the same value in any basis. The strain tensor always has three independent invariants, which could be the three principal strains, or the three fundamental scalar invariants, which are more convenient to use in practice.

Strain energy , for linear elastic materials, may be defined as quadratic of strain in form

$$W = \frac{1}{2} C_{ijkl} \varepsilon_{ij} \varepsilon_{kl}, \qquad (i, j, k, l = 1,2,3). \tag{10}$$

When is material reinforced by two families of mechanically equivalent fibres material behaves as orthotropic and has nine independent material constants. The local fibre directions are denoted by the unit vectors and for bidirectional reinforcement. In that case we say that the vectors and are "mechanically equivalent" if the response is unaltered when and are interchanged. When materials have axes of symmetry along bisectors of the fibre directions and along the normal to plane tangent to fibres, Spencer [7, 8] has shown that the most general quadratic form of expression for strain energy function is

$$\begin{aligned} W = {} & \frac{1}{2}\lambda(tr\mathbf{e})^2 + \mu tr\mathbf{e}^2 + \gamma_1\left[(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{a})^2 + (\mathbf{b}\cdot\mathbf{e}\cdot\mathbf{b})^2\right] + \gamma_2(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{b})^2 + \\ & \gamma_3(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{e}\cdot\mathbf{b})tr\mathbf{e} + \gamma_4\cos 2\phi(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{b})tr\,\varepsilon + \\ & \gamma_5\cos 2\phi(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{e}\cdot\mathbf{b})(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{b}) + \gamma_6(\mathbf{a}\cdot\mathbf{e}\cdot\mathbf{a})(\mathbf{b}\cdot\mathbf{e}\cdot\mathbf{b}) + \gamma_7\left(\mathbf{a}\cdot\mathbf{e}^2\cdot\mathbf{a} + \mathbf{b}\cdot\mathbf{e}^2\cdot\mathbf{b}\right), \end{aligned} \tag{11}$$

where are even functions of , and angle between the two families of fibres.

The elasticity tensor may be calculated by taking double partial derivation of with respect to strain tensor, which leads to the expression for the stiffness tensor as follows

$$
\begin{aligned}
C_{ijkl} = \frac{\partial^2 W}{\partial e_{ij}\partial e_{kl}} &= \left[\lambda\ \delta_{kl} + \gamma_3\left(a_k a_l + b_k b_l\right) + \gamma_4 \frac{1}{2}\left(a_k b_l + a_l b_k\right)\cos 2\phi\right]\delta_{ij} + \mu(\delta_{ik}\delta_{jl} + \delta_{jk}\delta_{il}) \\
&+ \left[\gamma_3\delta_{kl} + 2\gamma_1 a_k a_l + \gamma_6 b_k b_l + \gamma_5 \frac{1}{2}\left(a_k b_l + a_l b_k\right)\cos 2\phi\right] a_i a_j \\
&+ \left[\gamma_3\delta_{kl} + 2\gamma_1 b_k b_l + \gamma_6 a_k a_l + \gamma_5 \frac{1}{2}\left(a_k b_l + a_l b_k\right)\cos 2\phi\right] b_i b_j \\
&+ \frac{1}{2}\left[\gamma_4\delta_{kl}\cos 2\phi + \gamma_5\left(a_k a_l + b_k b_l\right)\cos 2\phi + \gamma_2\left(a_k b_l + a_l b_k\right)\right]\left(a_i b_j + a_j b_i\right) \\
&+ \gamma_7\left[a_r\left(a_j\delta_{rk}\delta_{il} + a_i\delta_{rk}\delta_{jl}\right) + b_r\left(b_j\delta_{rk}\delta_{il} + b_i\delta_{rk}\delta_{jl}\right)\right].
\end{aligned}
\tag{12}
$$

When two families of fibres are initially straight, then the fibre geometry may be described in the Cartesian coordinate system , where is the normal to the plane of the fibres the unit vectors, which represent fibres, may be written as

$$
\begin{aligned}
(a_i) &= (\cos\varphi, \quad \sin\varphi, \quad 0) \\
(b_i) &= (\cos\varphi, \quad -\sin\varphi, \quad 0)\ .
\end{aligned}
\tag{13}
$$

If stiffness tensor is defined then for arbitrary propagation direction may be calculated phase velocities for all three waves, whose reciprocities represent points of corresponding slowness surfaces. In general, it is necessary to calculate wave surfaces numerically. The simplest way of calculation is, if crystallographic axes are known, to coincide axes of symmetry with coordinate axes. Numerical calculation was performed for material with material constants deduced from measurement of unidirectional carbon fibre epoxy resin composite material, measured in [2], with numerical values whereas density is given as

$$
\begin{aligned}
&\lambda = 5{,}65\cdot 10^9\, Nm^{-2},\ \mu = 2{,}46\cdot 10^9\, Nm^{-2},\ \gamma_4 = -1{,}28\cdot 10^9\, Nm^{-2}, \gamma_7 = 3{,}20\cdot 10^9\, Nm^{-2}, \\
&2\gamma_1 = 110{,}45\cdot 10^9\, Nm^{-2},\ \gamma_2 = \gamma_3 = \gamma_5 = \gamma_6 = 0,
\end{aligned}
\tag{14}
$$

Slowness surfaces, for material reinforced by two families of fibres in [3], are calculated in program pack MATLAB.

Case when propagation is in the plane tangent to both families of fibres

In this paper slowness curves are calculated for waves propagating in the plane tangent to both families of fibres, that is in the plane of symmetry. For a fibre inclined for and slowness curves calculated in the plane of the fibres, for considered material for which is given in Figures 1. and 2. In these figures quasi-longitudinal waves are represented with solid lines, whereas two quasi-transversal waves are represented with broken lines.

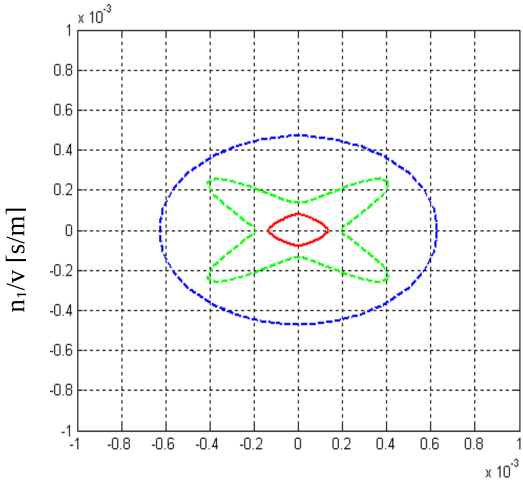

Figure 1 Two families of fibres propagation in the plane of fibres

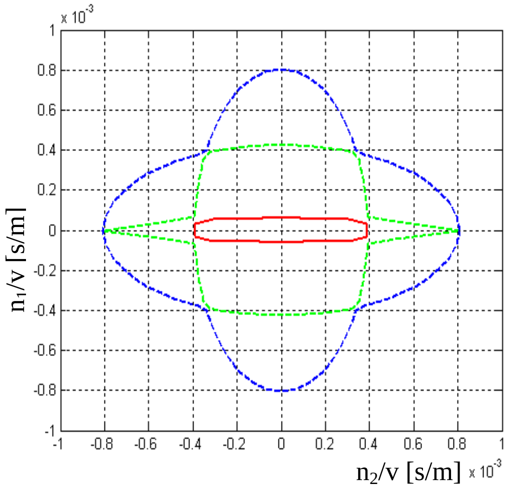

Figure 2 Two families of fibres, propagation in the plane of fibres

For considered material acoustic tensor has been formed, and determined, for different directions of wave propagation.

CONCLUSIONS

In the present paper mechanics of continuum treat material on macroscopic level as an anisotropic continuum and general conclusions about an anisotropic material behavior, in mechanical sense, are drawn from considering of bulk waves propagation. This approach may be used as a first approximation of dynamical behavior of the real parts with anisotropic characteristics. Numerical results show that a coordinate free formulation may give answers about the influence of fibres' direction as well as about the influence of fibres' strength on the wave propagation.

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