

SECTION 8. Architecture and construction

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THE BILINEAR SHEARING STRESS-STRAIN CURVE FOR CONCRETE

Abstract: *The simplified bilinear shearing stress-strain chart of the complete stress-strain curve for concrete in shear and pure torsion has been elaborated on the basis of the experimental research and offered for the reinforced concrete elements' stiffness' calculation.*

Key words: *torsional stiffness, bilinear stress-strain curve for concrete, pure torsion, the secant shearing modulus of concrete.*

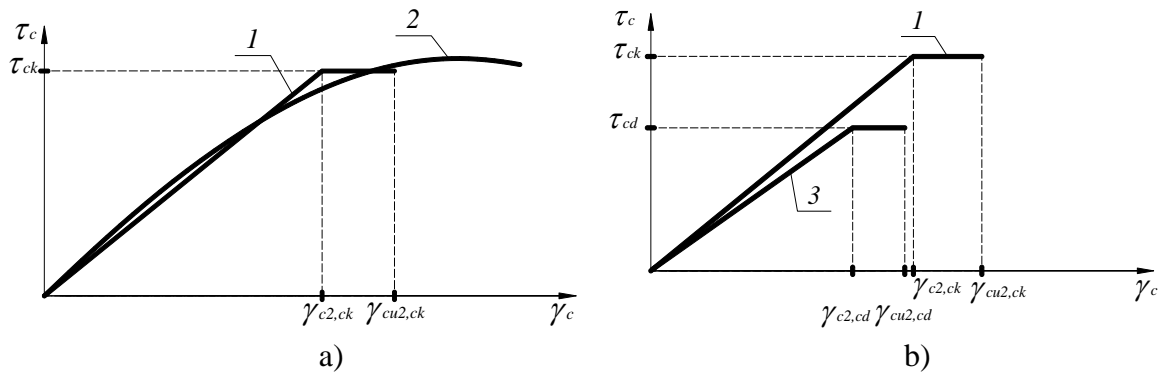
The bending stiffness' studies and mechanical strength characteristics' investigation of RCE under compression-tension are being paid much attention by modern researchers while comparing with the analysis of the torsional effect on the reinforced concrete elements' work. However it should be studied not less because of structures' constant necessity to resist the effect of not only bending and compressing forces but of the torsional ones (for example, in any asymmetrical loading of the spatial structures' elements like bridge superstructures, overlap constructions, as the influence of the forces' redistribution on the side beams causes torsion of the main girder; not to mention spiral staircase and boundary elements of shells and domes etc.). In the issue the building standards and the software packages account the bending stiffness' changing at all the stages of reinforced concrete elements' work, in contrast to the torsional stiffness' reduction owing to the concrete secant shearing modulus' decreasing. And the educational process also avoid shearing stress-strain curve's studying.

The shearing stress-strain curve [21] should be used as the generalized characteristic of the concrete mechanical properties, similar to the compression stress-strain curve [10]. However modern building standards – neither post-Soviet, nor Australian, American, European and others – do not provide design of reinforced concrete structures considering secant shear modulus of concrete [1, 7-9, 12-13, 16-17, 19], while in compression-tension using secant Young's modulus of concrete is prevailing. It can be explained by the lack of the experimental data of the complete shearing stress-strain curve for concrete [4, 6, 10-11, 14-15, 18, 21-22]. The principle of shear and compression stress-strain curve's obtaining for the concrete is common. The difficulty is in the experimental definition of the descending branch's parametric points using traditional experimental settings because of the sudden element's destruction as a result of the ultimate potential deformation energy immediate realization to the impact energy. The essence of the new method of obtaining the shearing stress-strain curve for concrete is in the use of traverse for a supervised efforts' transfer on a concrete model [3, 5]. That's why the experimental research has been conducted, where parametrical points of the curve's descending branch have been defined [2-3, 5, 20]. Complete shearing stress-strain curve has been already explained [2-3, 5], and after the experimental confirmation it is possible to introduce it into design practice.

To simplify the nonlinear shearing stress-strain curve [21] for the engineering estimations the parameters of the bilinear shearing stress-strain chart (fig.1) have been defined by analogy with the compressive-tensile stress-strain diagram [9].

On the fig.1 is noted: 1 – bilinear characteristic stress-strain curve τ - γ for concrete in shear (where the secant shear modulus is defined as the constant, or initial one multiplied by the coefficient of the secant shear modulus' change $G_c = G_c^0 g_c = E_c^0 g_c / [2(1 + \mu_c)]$), according to

[21]); 2 –shearing stress-strain chart for concrete confirmed by the experiments in pure torsion [5]; 3 – bilinear calculation stress-strain chart for concrete under shear τ - γ .



**Figure 1 - a) Calculation and experimental τ - γ dependences;
b) Bilinear stress-strain chart for concrete under shear**

Equivalent critical values of stresses and deformations for the shearing stress-strain curve are prescribed according to the theoretical chart [21] and confirmed experimentally [2-3, 5]. The shearing stresses τ_c of the bilinear shear stress-strain curve should be calculated depending on the relative angular deformations γ_c as

$$\begin{aligned} \tau_c &= G_c \gamma_c \text{ when } 0 \leq \gamma_c \leq \gamma_{c2}; \\ \tau_c &= \bar{\tau}_c \text{ when } \gamma_{c2} \leq \gamma_c \leq \gamma_{cu}, \end{aligned} \tag{1}$$

where ultimate shearing stress is equal to the concrete shearing strength, which depends on the values of compressive f_{ck} and tensile f_{ctk} concrete resistance $\tau_c = \bar{\tau}_c = f_{c,sh} = 0.7 \sqrt{f_{ck} f_{ctk}}$ according to [21].

Accepted notation conventions for the strength and deformability of concrete in shear (similarly to the concrete characteristics in compression according to [8-9]) are given below. *Shearing stresses:* $\tau_{cm} = \tau_{ck} / (1 - 1.64V_c)$ is the average value of the concrete shearing strength, where the constant of variation is equal to 13.5%; τ_{ck} is the characteristic value of the concrete shearing strength; $\tau_{cd} = \tau_{ck} / \gamma_{ct}$ is the calculation value of the concrete shearing strength, where the safety factor is taken as for tension, not compression, as the result of the concrete elements' destruction by reason of the tensile stresses' effect [3, 5]: $\gamma_{ct} = 1.5$ when $\tau_{cd,0.05}$; $\gamma_{ct} = 1.3$ when $\tau_{cd,0.95}$. *Stiffness' characteristics:* G_{cm} is the average value of the initial rigidity modulus of concrete; G_{ck} is the characteristic value of the initial rigidity modulus of concrete; G_{cd} is the calculation value of the initial rigidity modulus of concrete. *Angular deformations:* γ_c is the value of the relative angular deformations for concrete in shear; γ_{c2} is the value of the relative angular deformations for concrete in shear correspondingly to the ultimate shearing stresses τ_{ck} ; γ_{cu} is the value of relative ultimate angular deformations of concrete under shear; angular deformations $\gamma_{c2,ck} = \tau_{ck} / G_{ck}$;

$\gamma_{c2,cd} = \tau_{cd} / G_{cd}$; $\gamma_{cu2,ck} = k_t \times \gamma_{c2,ck}$; $\gamma_{cu2,cd} = k_t \times \gamma_{c2,cd}$, where values of the relative ultimate angular deformations for concrete in shear are accepted after the analysis of the experimental data, according to which the shearing angle increases by $k_t = 24\%$ as compared to the peak point of the curve accordingly to the experiment [5]. When introducing the bilinear chart to the standards on the basis of the experiments' extension one could vary the very value of the k_t coefficient.

For example, the stress-strain curve for the fine-grained concrete C16/20 in shear has been built (fig.2).

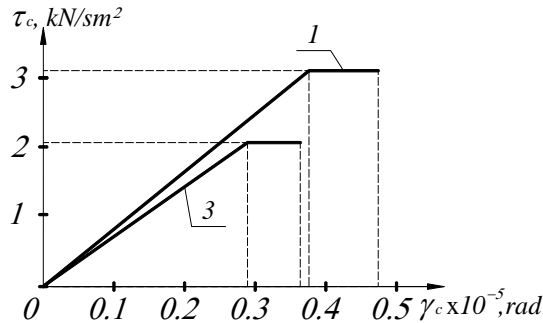


Figure 2 - Bilinear shearing stress-strain chart of the fine-grained concrete C16/20

For the concrete C16/20 according to the experimental data theoretical value of the ultimate shearing stress $\bar{\tau}_c = 0.3091 \text{ kN/sm}^2$, average actual experimental value of the ultimate shearing stress $\bar{\tau}_c = 0.3163 \text{ kN/sm}^2$, the error is 2.27%. The elasticity modulus E_c^0 of heavy-weight concrete is 2700 kN/sm^2 , of fine-grained one is 2200 kN/sm^2 . On the fig.2 the value of the shearing stresses and angular deformations according to the computation by the above mentioned formulas are:

$$\begin{aligned}
 \bar{\tau}_{ck} &= 3.10 \text{ MPa}, \tau_{cd} = 2.07 \text{ MPa}, \tau_{cm} = 3.98 \text{ MPa}; \\
 \gamma_{c2,ck} &= \tau_{ck} / G_{ck} = 0.389, \gamma_{cu2,ck} = 0.24 \times \gamma_{c2,ck} = 0.482, \\
 \gamma_{c2,cd} &= \tau_{cd} / G_{cd} = 0.298, \gamma_{cu2,cd} = 0.24 \times \gamma_{c2,cd} = 0.370.
 \end{aligned} \tag{2}$$

The safety factor's values for the limit state of the 1st and the 2nd principal criteria for concrete (the ultimate limit state (ULS) and the serviceability limit state (SLS), according to the limit state theory) are taken according to the tab.2.1 of the Standard [9].

So, application of the bilinear concrete shearing stress-strain chart while using secant shearing modulus of concrete in order to consider the torsional stiffness' reduction in calculations essentially simplifies estimations comparatively with the usage of the concrete secant shear modulus according to the theoretical stress-strain curve.

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