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A Normalized Cluster Model Wavefunction of ^{5}He using Complex Generator Coordinate Technique

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ABSTRACT : Microscopic theoretical studies of scattering and reaction problems for light and medium mass nuclei have been extensively carried out during the past several years using wavefunctions corresponding to various nuclear models[1]. In this present paper the antisymmetrized wavefunction of ground state of ^{5}He nucleus is constructed with definite spin and parity using the cluster model alongwith Resonating group method RGM and Generator coordinate method GCM. The ^{5}He nucleus in its ground state is considered to consist of one alpha cluster and a neutron cluster. Here we have applied a new approach ie Complex Generator Coordinate Technique CGCT. This technique is applied to the cluster model wavefunction to transform it into antisymmetrized product of single particle wavefunction. In the last the normalization constant is also calculated in order to make the wavefunction to be normalized.

I. INTRODUCTION

The resonating group method (RGM) is used in most cases for nuclei[2,3,4,5,6] which involve two s-shell nuclei. In RGM an antisymmetrized wavefunction is used with the consideration of centre-of-mass motion correctly, nucleon-nucleon exchange effects, which are especially significant when the interacting clusters have similar mass. In cluster model[7][8], we use an independent spatial variable, a set of internal and relative co-ordinates of the clusters. Since, one generally chooses functions of Gaussian dependence for both spatial part of cluster wavefunctions and spatial part of the nucleon-nucleon potential, therefore the multidimensional integrals involve integrands of Gaussian Functions multiplied by polynomials of spatial coordinates. GCM considers the collective motion of the nucleon is the cluster system. The internal structure of each cluster is describe by a translationally invariant harmonic-oscillator shell model function of lowest configuration. The antisymmetrization operator consists of inter cluster and inter cluster and inter cluster exchange terms.

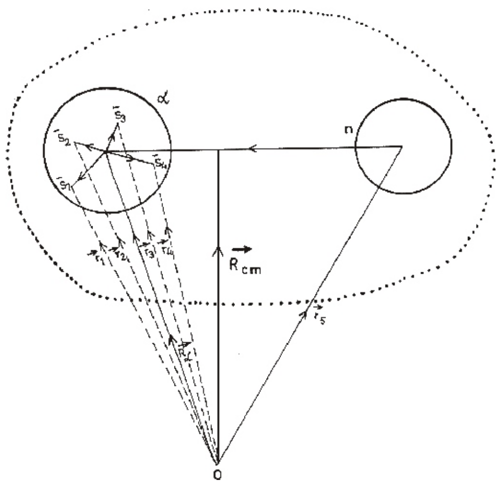

Fig. 1. ^{5}He Cluster Configuration.

II. CLUSTER MODEL WAVEFUNCTION FOR ^{5}He GROUND STATE

The ^{5}He nucleus has two protons and three neutrons. It is well known that the -particle consisting of 2 protons and 2 neutrons is particularly tightly bound and is quite rigid. Hence this and many other physical consideration suggest that 5 particle nucleus in the ground state may be assumed to have a cluster structure consisting of an alpha cluster and a neutron as shown in fig. 1. It may be assumed to be a good approximation that the -cluster remains unexcited and undistorted like free -particle in ^{5}He nucleus.

The spin and parity of ^{5}He in the ground state are J $\frac{3}{2}$, so the cluster model wavefunvtion for ^{5}He ground state can be written as

$$_{\frac{3}{2},M} \quad A \quad (\ ) \quad _{m_l \ m_s \ M} C \ l, m_l, \frac{1}{2} m_s, \frac{3}{2} M \quad _{l_{m_l}}(\bar{R}) \quad _{-m_s} \qquad ...(1)$$

where A is the antisymmetrization operator. ( ) is - cluster wavefunction, it may be written as

$$(\ ) \quad \bar{\ }(\ ) \quad (\ ,\ ) \qquad ...(2)$$

$\bar{\ }$ ( ) denotes the spatial part of the culster internal function and (,) is the spin isospin part. $_{l_{m_l}}$ is the relative motion wavefunciton between -cluuster and the neutron the choice of ℓ is done as $J \quad \frac{3}{2}$ and spin of fifth nucleon is $\frac{1}{2}$, parity is negative so $\ell = 1$ and $m_\ell = 1, 0, -1$.

^{5}He Ground state wavefunction as an Antisymmetrized product of single particle wavefunction using CGCT.

The ^{5}He ground state cluster model wavefunction including the wavefunction $Z(\overline{R}_{cm})$ of the center-of-mass can be written as

$$_{-,m} \quad A[\ (\ )\ _{l_{m_l}}(\overline{R} \quad \overline{r}_5\ )\overline{Z}(R_{cm})] \qquad ...(3)$$

This wavefunciiton is converted into antisymmetrized product of single-particle wavefunction, introducting ans integral representation for , one gets

$$_{\frac{3}{2}M} \quad A[\ (\ ,R\ )\ (\overline{R} \ -R\ )\ (\overline{r}_5 \quad \overline{r}_5\ ) \quad _{n} \ _{l_{m_l}}(\overline{R} \ -\overline{r}_5\ )$$

$$Z_{cm} \ \frac{4\overline{R}\ ' \quad \overline{r}_5}{5}\ ]d\overline{R}\ ' dr_5{}' \qquad ...(4)$$

Now, making use of integral representation of function defined as

$$(\overline{R} \ -\overline{R}\ ')\quad \frac{1}{2}^{3} \ \exp[i\overline{S}\ '.(\overline{R} \ -\overline{R}\ ')]d\overline{S}\ ' \qquad ...(5)$$

and

$$(\overline{r} \ -\overline{r}\ )\quad \frac{}{4}^{3}\ \exp - \frac{}{2}\ \overline{r}_5 - \frac{1}{2}\overline{S}^{\ 2} \quad \frac{}{2}\ \overline{r}_5 - \frac{1}{2}\overline{S}_5^{\ 2} \quad d\overline{S}_5 \qquad ...(6)$$

Next the coordinate transformations, given by the following equations are made,

$$\overline{R} - \overline{r} \quad \overline{R} \qquad \text{(i)}$$

$$\frac{4\overline{R} \quad \overline{r}_5}{5} \quad \overline{R}_{cm} \qquad \text{(ii)}$$

$$\frac{1}{4}\overline{S} \ -1\overline{R} \quad \overline{P} \qquad \text{(iii)} \qquad ...(7)$$

Now,

$$d\overline{R} \quad d\overline{r}_5\ \ d\overline{S}_d \quad |J|\overline{dR}\ d\overline{R}_{cm}\ d\overline{P} \qquad ...(8)$$

where $|J|$ is the Jacobian of the transformation.

Substituting eqs (5), (6), (7), (8) in eq (4) and integrating over $R_{cm}, Ƶ(\bar{R}_{cm})$ is choosen to have a form

$$Ƶ \; (\bar{R}_{cm}) \quad \exp \frac{-5}{2} \; R_{cm}^{2} \qquad \text{...(9)}$$

This can be done since due to the translational invariance of the hamiltonian, one can choose the center of mass wavefunction arbitrarily. By making some more transformations the final wavefunction of ^{5}He in the ground state can be written as

$$_{\frac{3}{2}\frac{3}{2}} \quad \frac{5}{4} \quad A \prod_{i \; 1}^{4} \; _{n} \exp -\frac{}{2}(\bar{r}_i - i\bar{P})^2 \;\; \exp -\frac{}{2}(\bar{r}_5 \quad 4i\bar{P})^2$$

$$_{11}(\bar{R}\;)\exp \quad 2 \;\; \sqrt{5}\;\bar{P} \quad \frac{1}{\sqrt{5}}\bar{R}\;)^2 \;\; d\bar{P}.d\bar{R} \qquad \text{...(10)}$$

The calculation of the normalization constant

The normalization constant can be obtained from the expression (10)

$$\Big\langle \; _{\frac{3}{2}\frac{3}{2}} \Big| \; _{--} \Big\rangle \quad \frac{5}{4}^{\;2} \quad A[\langle \quad |A \quad \rangle \quad \bar{R} \quad _{11}(\bar{R}\;).$$

$$\exp \quad 2 \quad \sqrt{5}\;\bar{P} \quad \frac{1}{\sqrt{5}}\bar{R}^{\;\;2} \quad \exp \quad 2 \;\; \sqrt{5}\;\bar{P} \quad \frac{1}{\sqrt{5}}\bar{R}^{\;\;2} \quad d\bar{R}\; d\bar{R} \;\; \overline{dP}\,\overline{dP} \qquad \text{...(11)}$$

With

$$_{0} \prod_{i \; 1}^{4} \; _{n} \exp -\frac{}{2}(\bar{r}_i - i\bar{P})^2 \;\; \exp -\frac{}{2}(\bar{r}_5 \quad 4i\bar{P})^2 \qquad \text{...(12)}$$

and

$$\langle \;_0 | A \;\; _0 \rangle \qquad \prod_{i \;\; 1}^{4} \;\; _n \exp -\frac{}{2}(\bar{r}_i - i\bar{P})^2 \;\; \exp -\frac{}{2}(\bar{r}_5 \;\;\; 4i\bar{P})^2$$

$$\begin{vmatrix} U_1^{'''} & (1) & U_2^{'''} & (1) & U_3^{'''} & (1) & U_4^{'''} & (1) & U_5^{'''} & (1) \\ U_1^{'''} & (2) & U_2^{'''} & (2) & U_3^{'''} & (2) & U_4^{'''} & (2) & U_5^{'''} & (2) \\ U_1^{'''} & (3) & U_2^{'''} & (3) & U_3^{'''} & (3) & U_4^{'''} & (3) & U_5^{'''} & (3) \\ U_1^{'''} & (4) & U_2^{'''} & (4) & U_3^{'''} & (4) & U_4^{'''} & (4) & U_5^{'''} & (4) \\ U_1^{'''} & (5) & U_2^{'''} & (5) & U_3^{'''} & (5) & U_4^{'''} & (5) & U_5^{'''} & (5) \end{vmatrix}$$

$$d\bar{r}_1 \;\; d\bar{r}_2 \;\; d\bar{r}_3 \;\; d\bar{r}_4 \;\; d\bar{r}_5 \qquad \text{...(13)}$$

where,

$$U_1^{'''} \quad \exp \quad \frac{}{2}(\bar{r}_i \quad i\bar{P})^2 \;\; ; i \quad 1, 2, 3, 4$$

$$U_5^{'''} \quad \exp \quad \frac{}{2}(\bar{r}_5 \quad 4i\bar{P}\;)^2 \qquad \text{...(14)}$$

Saving eq (13) we get,

$$\langle \; | \; \rangle \quad Q_T \frac{^3(\qquad)^5}{} \; \frac{1}{32} \; \frac{1}{}^{\frac{5}{2}} \quad 4^4 \; \frac{1}{15 \;\; ^2 \;\; 15 \;\; ^2 \;\; 34}^{\frac{5}{2}} \qquad \text{...(15)}$$

$$Q_T \quad \frac{N^2}{5!} \; \frac{2}{}^{6} \; \frac{}{}^{\frac{15}{2}} \; \frac{3}{8} \; \frac{5}{}^{2} \; \frac{5}{2(\qquad)}^{3} \qquad \text{...(16)}$$

and

$$N^2 \quad 5!\frac{\sqrt{\;\;\;^7}}{^{13}}. \; \frac{1}{15} \; \frac{1}{32} \; \frac{1}{}^{\frac{5}{2}} \quad 4^4 \; \frac{1}{15 \;\; ^2 \;\; 15 \;\; ^2 \;\; 34}^{\frac{5}{2} \;\; 1} \qquad \text{...(17)}$$

eq (17) shows the numerical value of Normalization constant.

III. CONCLUSION

This form of normalized wavefunction of ^{5}He will be further used to calculate the form factor, r.m.s radius, quadrupole moment cross-section, ionization energy spin-spin interaction, energy levels and other properties of the nucleus. This technique has been successfully used to solve N $^{40}Ca^{13}$ $^{40}C^{14}$, ^{16}O $^{16}O^{15}$, $^{6}Li^{16}$ etc., scattering problems and the results obtained are in good agreement with available experimental data. The use of CGCT is aimed to convert cluster model wavefunction into antisymmetrized product of single particle wavefunctions, which allows the evaluation of matrix elements that arise at the time of calculation of charge form factor, rme radius, quadrupole moment etc. Hence it suggests that the cluster model wavefunction represents the structure of the nuclei to a reasonably good extent.

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