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Balanced Treatment-Control Row-Column Designs

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ABSTRACT: Row-column designs (RCD) are used in experimental situations when the heterogeneity in the experimental material is due to two cross classified sources. In such trials, there may be situations in which the experimenter wants to compare a set of new (test) treatments with an already existing (control) treatment. The experimenter is interested in comparing test treatments with the control with more precision and hence it is unrealistic to replicate the test treatments the same number of times as that of the control. Designs used for such experiments are known as balanced treatment-control row-column (BTRC) designs. With more number of treatments, one may have to conduct the experiment in incomplete rows/columns. In this study, some general methods of constructing BTRC designs in complete/incomplete rows/columns have been developed. Also, a class of BTRC designs with empty nodes has been constructed.

Key words: Row-column designs for test vs. control comparisons, structurally incomplete row-column designs, variance balanced, partially variance balanced

I. INTRODUCTION

For the purpose of error control or error reduction, the principle of blocking is of primary importance. There are situations wherein one needs to block or eliminate heterogeneity in two perpendicular directions. The two blocking systems are referred to as row blocking and column blocking with respect to agricultural field experiments and the resulting designs are referred to as row-column designs (RCD). There are many experimental situations in which the experimenter wants to compare a set of new (test) treatments with already existing (control) treatment(s). It is a common problem in many of the agricultural, biological and industrial experiments. Here, the experimenter is interested in the comparison of new treatments with the control(s) with more precision whereas the comparisons within test treatments or within controls are of less importance. Designs used for such experiments are known as balanced treatment-control row-column (BTRC) designs.

Majumdar and Tamhane (1996) obtained BTRC designs by changing all symbols in a transversal of a Latin square to 0. The transversal of a Latin square of order v is a set of v cells such that each row, column and symbol is represented exactly once in this set (Hedayat and Seiden, 1974). Many classes of row-column designs for comparing a set of test treatments to more than one control treatment, commonly known as balanced bipartite row-column (BBPRC) designs, have been obtained in literature. A review of work done in the area of BBPRC designs is given in Majumdar (1996).

Again, when some of the row-column intersections are having empty nodes, structurally incomplete BTRC designs will be of great utility. In these designs, treatments are applied to a subset of the available experimental units. Need for such design is apparent when the blocking criteria are implemented in sequence where the number of treatments is lesser in the second stage than the first stage. Construction and properties of structurally incomplete RCDs were given by Agrawal (1966), Hedayat and Raghavrao (1975), Ray (1986), Stewart and Bradley (1991), Parsad *et al.* (2003), *etc*.

Some definitions related to the BTRC designs are given below:

Variance Balanced BTRC Designs: A BTRC design for comparing v test treatments with a control treatment (0) is said to be variance balanced if all elementary contrasts

(i) among test treatments are estimated with the same variance ($V_{tt'}$), $t \neq t' = 1,2,\ldots,v$ and

(ii) among test and control treatments are estimated with the same variance (V_{t0}).

Partially Balanced BTRC Designs: A BTRC design for comparing v test treatments with a control treatment (0) is said to be partially balanced if all elementary contrasts

(i) among test treatments that are i^{th} ($i = 1, 2, \ldots, m$) associates to each other are estimated with the same variance ($V_{t_i t_i'}$) ($t_i \neq t_i' = 1, 2, \ldots, n_i$ where n_i is the number of i^{th} associates such that $\sum_{i=1}^{m} n_i = v-1$) and

(ii) among test and control treatments are estimated with the same variance ($V_{t_i 0}$) as long as the test treatments are i^{th} associates to each other.

Structurally Incomplete BTRC Designs: A row-column design for comparing v test treatments with a control treatment is said to be structurally incomplete if there is at least one row-column intersection which does not receive any treatment.

In the subsequent sections, some general methods of construction of structurally complete/incomplete BTRC designs have been explained. The construction method has been illustrated using appropriate examples. The designs obtained are either variance balanced or partially variance balanced.

II. BTRC DESIGNS USING TREATMENT SUBSTITUTION METHOD

Consider a v × v square array of v (= mn) treatments (m odd) such that the array is divided into n box-rows and m box-columns giving rise to mn boxes each of size $m \times n$. The v treatments are arranged in the first box in a natural order. The remaining (m–1) box-columns of the first box-row is obtained by permuting the m rows of the initial box circularly and remaining (n–1) box-rows are obtained by permuting the columns in m box-columns in a circular manner. This arrangement will result in each box-row having distinct box diagonal elements.

Replace the treatments appearing in the box diagonal positions in each box-row by a control treatment 0. The resultant design is a BTRC design in v rows and v columns for comparing v test treatments with a control. Here, each test treatment occurs r_t = v-n times and the control treatment occurs r_0 = nv times.

Example 2.1: Let v = 6 with m = 3 and n = 2. The following arrangement is obtained for 6 treatments in 2 box-rows and 3 box-columns:

1	2	3	4	5	6
3	4	5	6	1	2
5	6	1	2	3	4
2	1	4	3	6	5
4	3	6	5	2	1
6	5	2	1	4	3

Replacing the treatments appearing in the box diagonal positions in each box-row by a control treatment 0, the following BTRC design with 6 test treatments and 1 control treatment in 6 rows and 6 columns having r_t = 4 and r_0 = 12 is obtained:

		Columns					
		i	**ii**	**iii**	**iv**	**v**	**vi**
Rows	**i**	0	0	3	4	5	6
	ii	3	4	0	0	1	2
	iii	5	6	1	2	0	0
	iv	0	0	4	3	6	5
	v	4	3	0	0	2	1
	vi	6	5	2	1	0	0

Variances of estimates of contrasts pertaining to test vs. test and test vs. control treatments have been worked out for $v \leq 15$ by developing a SAS code using PROC IML. These designs are found to be partially variance balanced and hence the average variances of estimates of contrasts due to test vs. test ($\overline{V}_{t_i t_i'}$) and test vs. control ($\overline{V}_{t_i 0}$) treatments were calculated and reported in Table 2.1.

It is seen that the test vs. control treatments have been estimated with less variance than test vs. test treatments.

Table 2.1: BTRC designs using treatment substitution method.

S. No.	v	m	n	r_t	r_0	$\overline{V}_{t_i t_i'}$	$\overline{V}_{t_i 0}$
1	6	3	2	4	12	0.600	0.375
2	9	3	3	6	27	0.375	0.222
3	10	5	2	8	20	0.262	0.181
4	12	3	4	8	48	0.273	0.156
5	14	7	2	12	28	0.170	0.121
6	15	3	5	10	75	0.214	0.120

III. BTRC DESIGNS USING LATIN SQUARE WITH DISTINCT DIAGONAL ELEMENTS

A BTRC design for comparing a set of v test treatments with a control can always be constructed from a v × v Latin square with distinct diagonal entries by replacing the diagonal entries with the control treatment 0. The resultant design has v rows, v columns, $r_t = v–1$ and $r_0 = v$.

Example 3.1: For v = 5, following is a Latin square with distinct diagonals:

1	2	3	4	5
2	3	4	5	1
3	4	5	1	2
4	5	1	2	3
5	1	2	3	4

Replacing the treatments appearing in the diagonal positions by the control treatment 0, following BTRC design for comparing 5 test treatments with a control having $r_t = 4$ and $r_0 = 5$ is obtained:

		Columns				
		i	**ii**	**iii**	**iv**	**v**
Rows	**i**	0	2	3	4	5
	ii	2	0	4	5	1
	iii	3	4	0	1	2
	iv	4	5	1	0	3
	v	5	1	2	3	0

Example 3.2: For v = 8, following is a BTRC design for comparing 8 test treatments with a control having $r_t = 7$ and $r_0 = 8$:

		Columns							
		i	**ii**	**iii**	**iv**	**v**	**vi**	**vii**	**viii**
Rows	**i**	0	2	3	4	5	6	7	8
	ii	5	0	7	8	1	2	3	4
	iii	2	1	0	3	6	5	8	7
	iv	6	5	8	0	2	1	4	3
	v	7	8	5	6	0	4	1	2
	vi	3	4	1	2	7	0	5	6
	vii	8	7	6	5	4	3	0	1
	viii	4	3	2	1	8	7	6	0

Table 3.1 gives the list of parameters of the designs for v ≤ 15 along with the variances pertaining to different groups of treatment comparisons. These designs are variance balanced.

Table 3.1: Designs using Latin square with distinct diagonal elements.

S. No.	v	r_t	r_0	$V_{tt'}$	V_{t0}
1	3	2	3	1.500	1.000
2	4	3	4	0.800	0.633
3	5	4	5	0.556	0.472
4	7	6	7	0.350	0.317
5	8	7	8	0.296	0.272
6	9	8	9	0.257	0.239
7	11	10	11	0.204	0.192
8	13	12	13	0.169	0.161
9	15	14	15	0.144	0.139

IV. STRUCTURALLY INCOMPLETE BTRC DESIGNS

Consider a standard Latin square for v treatments (v being an odd number). Replace the treatments appearing in the forward diagonal positions by the control treatment 0 and Leave the positions in the backward diagonal empty, except the one common to the forward diagonal. This yields a structurally incomplete BTRC design for comparing v-1 test treatments with one control treatment in v rows and v columns. Here, r_t = v–1 and r_0 = v.

Example 4.1: The standard Latin square for v = 7 treatments is as follows:

1	2	3	4	5	6	7
2	3	4	5	6	7	1
3	4	5	6	7	1	2
4	5	6	7	1	2	3
5	6	7	1	2	3	4
6	7	1	2	3	4	5
7	1	2	3	4	5	6

Replacing the forward diagonal elements by 0 and leaving the remaining positions of the backward diagonal elements empty (denoted by *), we get the following BTRC design for 6 test treatments and one control in 7 rows and 7 columns with r_t = 6 and r_0 = 7:

		Columns						
		i	**ii**	**iii**	**iv**	**v**	**vi**	**vii**
Rows	**i**	0	2	3	4	5	6	*
	ii	2	0	4	5	6	*	1
	iii	3	4	0	6	*	1	2
	iv	4	5	6	0	1	2	3
	v	5	6	*	1	0	3	4
	vi	6	*	1	2	3	0	5
	vii	*	1	2	3	4	5	0

The above procedure can be adopted for even number of treatments provided a Latin square with distinct diagonal is available.

A list consisting of parameters of the designs for v ≤ 15 has been prepared along with the computed average variances pertaining to different groups of treatment comparisons (Table 4.1). This class of designs is partially balanced. In all the class of designs obtained above, it is seen that the contrast pertaining to test treatments versus control are estimated more precisely.

Table 4.1: Structurally incomplete BTRC designs.

S. No.	**Test Treatments**	$\mathbf{r_t}$	$\mathbf{r_0}$	$\overline{\mathbf{V}}_{\mathbf{t_i t_i'}}$	$\overline{\mathbf{V}}_{\mathbf{t_i 0}}$
1	2	2	3	1.500	1.000
2	4	4	5	0.573	0.478
3	6	6	7	0.353	0.318
4	8	8	9	0.258	0.240
5	10	10	11	0.204	0.192
6	12	12	13	0.169	0.161
7	14	14	15	0.145	0.136

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