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A Common Fixed Point Theorem Using A-Compatible and S-Compatible Mappings

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ABSTRACT: The aim of this paper is to generalize common fixed point theorem proved by Bijendra singh by introducing the two types of weak reciprocally continuous mappings. The concepts of compatibility and complete metric space are replaced by two different types of weak reciprocally continuous mappings along with some weaker conditions.

Keywords: Fixed point, self maps, compatible mappings, weakly compatible, mappings, associated sequence, reciprocally continuous mappings, A- compatible mapping, S-compatible**.** Mapping, A-Weak reciprocally continuous mappings, S-Weak reciprocally continuous mappings**,**

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I. INTRODUCTION

G. Jungck [1] introduced the concept of compatible maps which is weaker than weakly commuting maps. G.Jungck [1] generalized the concept of compatible mappings by introducing the notion of compatible mappings of type (A). Pathak extended the concept of compatibility to two analogus definitions namely A- compatible and S-compatible. After wards Jungck and Rhoades [4] defined weaker class of maps known as weakly compatible maps.
Pant [2] introduced a new notion of continuity namely reciprocal continuity for a pair of self maps and proved some common fixed point theorems. Further Pant [2] *et al* introduced the concept of weak reciprocally continuity.

1.1. 1 Definitions and Preliminaries Compatible mappings: Two self maps A and S of a metric space (X,d) are said to be compatible mappings [1] if $\lim_{n\to\infty} d(ASx_n, SAx_n) = 0$, whenever $<x_n>$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some t □ X.

1.1. 2 Weakly compatible mappings: Two self maps A and S of a metric space (X, d) are said to be weakly compatible [4] if they commute at their coincidence point. i.e. if Au=Su for some u □ X then ASu = SAu.

1.1.3 Reciprocally continuous mappings: Two self maps A and S of a metric space (X, d) are said to be reciprocally continuous [2] if $\lim_{n\to\infty} ASx_n = At$ and $\lim_{n\to\infty} SAx_n =$ St when ever $<x_n>$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

1.1.4 Weak reciprocally continuous mappings: Two self maps A and S of a metric space (X, d) are said to be Weak reciprocally continuous [11] iff $\lim_{n\to\infty} ASx_n = At$ or $\lim_{n\to\infty} SAx_n = St$ when ever $<x_n>$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.
Now we define the weak reciprocally continuous mappings by introducing into two analogous definitions.

1.1.5 A-Weak reciprocally continuous mapping: Two self maps A and S of a metric space (X, d) are said to be A-Weak reciprocally continuous iff $\lim_{n\to\infty} ASx_n = At$ whenever $<x_n>$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

1.1.6 S-Weak reciprocally continuous mapping: Two self maps A and S of a metric space (X, d) are said to be A-Weak reciprocally continuous iff $\lim_{n\to\infty} SAx_n = St$ when ever $<x_n>$ is a sequence such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t$ for some $t \in X$.

1.1.7 A-Compatible mappings

Two self maps S and T of a metric space (X, d) are A-compatible iff $\lim_{n\to\infty} d(ASx_n, SSx_n) = 0$, when ever $<x_n>$ is a sequence such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = t$ for some $t \in X$.

1.1.8 S-Compatible mappings

Two self maps S and T of a metric space (X, d) are S-compatible iff $\lim_{n\to\infty} d(AAx_n, SAx_n) = 0$, when ever $<x_n>$ is a sequence such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Ax_n = t$ for some $t \in X$.

It is clear that every compatible pair is weakly compatible but its converse need not be true.

Singh and Chauhan [6] proved the following theorem.

2. Theorem (A): Let A, B, S and T be self mappings from a complete metric space (X, d) into itself satisfying the following conditions

$A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ ...(2.1.1)

One of A, B, S or T is continuous ...(2.1.2)

$$\begin{aligned}[d(Ax,By)]^2 \le\ & k_1[d(Ax,Sx)d(By,Ty)+d(By,Sx)d(Ax,Ty)] \\ & +k_2[d(Ax,Sx)d(Ax,Ty)+d(By,Ty)d(By,Sx)]\end{aligned} \quad \ldots (2.1.3)$$

where $0 \le k_1 + 2k_2 < 1, k_1, k_2 \ge 0$

The pairs (A, S) and (B, T) are compatible on X …(2.1.4)

Further if X is a complete metric space …(2.1.5)

Then A, B, S and T have a unique common fixed point in X.

Now we use definition of associated sequence [10] that plays a vital role in proving our theorem

2.1 Associated Sequence: Suppose AB,S and T are self maps of a metric space (X, d) satisfying the condition (2.1.1) Then for an arbitrary $x_0 \in X$ such that $Ax_0 = Tx_1$ and for this point x_1, there exist a point x_2 in X such that $Bx_1 = Sx_2$ and so on. Proceeding in the similar manner, we can define a sequence $<y_n>$ in X such that $y_{2n} = Ax_{2n} = Tx_{2n+1}$ and $y_{2n+1} = Bx_{2n+1} = Sx_{2n+2}$ for $n \ge 0$. We shall call this sequence as an "*Associated sequence of x_0* "relative to the four self maps A, B, S and T.

Now we prove a lemma which plays an important role in our main Theorem.

2.2 Lemma: Let A, B, S and T be self mappings from a complete metric space (X, d) into itself satisfying the conditions (2.1.1) and (2.1.3) Then the associated sequence $\{y_n\}$ relative to four self maps is a Cauchy sequence in X.

Proof: From the conditions (2.1.1), (2.1.3) and from the definition of associated sequence we have

$$\begin{aligned}[d(y_{2n+1},y_{2n})]^2 &= [d(Ax_{2n+1},Bx_{2n})]^2 \\ &\le k_1[d(Ax_{2n+1},Sx_{2n+1})\ d(Bx_{2n},Tx_{2n})+d(Bx_{2n},Sx_{2n+1})\ d(Ax_{2n+1},Tx_{2n}\ ] \\ &\quad +k_2[d(Ax_{2n+1},Sx_{2n+1})\ d(Ax_{2n+1},Tx_{2n})+d(Bx_{2n},Tx_{2n})\ d(Bx_{2n},Sx_{2n+1})] \\ &= k_1[d(y_{2n+1},y_{2n})\ d(y_{2n},y_{2n-1})+0\ ] \\ &\quad +k_2[d(y_{2n+1},y_{2n})\ d(y_{2n+1},y_{2n-1})+0]\end{aligned}$$

This implies

$$d(y_{2n+1}, y_{2n}) \le k_1 \ d(y_{2n}, y_{2n-1}) + k_2\left[d(y_{2n+1}, y_{2n}) + d(y_{2n}, y_{2n-1})\right]$$
$$d(y_{2n+1}, y_{2n}) \le h \ d(y_{2n}, y_{2n-1})$$
$$where \ h = \frac{k_1+k_2}{1-k_2} < 1$$

$$For\ every\ integer\ p > 0, we\ get$$
$$d(y_n, y_{n+p}) \le d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) + \ldots\ldots\ldots\ldots + d(y_{n+p-1}, y_{n+p})$$
$$\le h^n d(y_0, y_1) + h^{n+1} d(y_0, y_1) + \ldots\ldots\ldots\ldots + h^{n+p-1} d(y_0, y_1)$$
$$\le \left(h^n + h^{n+1} + \ldots\ldots\ldots\ldots + h^{n+p-1}\right) d(y_0, y_1)$$
$$\le h^n\left(1 + h + h^2 + \ldots\ldots\ldots\ldots + h^{p-1}\right) d(y_0, y_1)$$

Since h < 1, $h^n \to 0$ as $n \to \infty$, so that $d(y_n, y_{n+p}) \to 0$. This shows that the sequence $\{y_n\}$ is a Cauchy sequence in X and since X is a complete metric space, it converges to a limit, say $z \in X$.
The converse of the Lemma is not true, that is A, B, S and T are self maps of a metric space (X, d) satisfying (2.1.1) and (2.1.3), even if for any $x_0 \in X$ the associated sequence converges, the metric space (X, d) need not be complete. The following example establishes this.

2.3. Example: Let X = (0,1) with d(x,y) = $|x-y|$

$$Ax = Bx = \begin{cases} \frac{1}{6} & if \ 0 < x < \frac{1}{4} \\ \frac{1}{4} & if \ \frac{1}{4} \le x < \frac{1}{2} \end{cases} \qquad Sx = Tx = \begin{cases} \frac{1}{8} & if \ 0 < x < \frac{1}{4} \\ \frac{1}{2} - x & if \ \frac{1}{4} \le x < \frac{1}{2} \end{cases}$$

Then A(X) = B(X)= $\left\{\frac{1}{6}, \frac{1}{4}\right\}$ while S(X) = T(X) = $\left\{\left(\frac{1}{4}, 0\right] \cup \frac{1}{8}\right\}$ so that the conditions $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$ are satisfied. Clearly (X, d) is not a complete metric space. It is easy to prove that the associated sequence $Ax_0, Bx_1 Ax_2, Bx_3, .., Ax_{2n}, Bx_{2n+1} \ldots$, converges to the point $\frac{1}{4}$ if . $\frac{1}{4} \le x < \frac{1}{2}$, but X is not a complete metric space.
Now we generalize the above Theorem (A) in the following form.

3. MAIN THEOREM

3.1 Theorem (B): Let A, B, S and T are self maps of a metric space (X, d) satisfying the conditions
(i). $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$

(ii) $$\left[d(Ax, By)\right]^2 \le k_1\left[d(Ax, Sx)d(By, Ty) + d(By, Sx)d(Ax, Ty)\right]$$
$$+ k_2\left[d(Ax, Sx)d(Ax, Ty) + d(By, Ty)d(By, Sx)\right]$$

for all x,y in x where $0 \le k_1 + 2k_2 < 1, k_1, k_2 \ge 0$

(iii) The pair (A, S) is A-weak reciprocally continuous and A-compatible

or

The pair (A, S) is S- weak reciprocally continuous and S compatible and

(iv) The pair (B, T) is weakly compatible.

(v) For any $x_0 \in X$ The associated sequence relative to four self maps A ,B, S and T such that the sequence Ax_0, Bx_1 Ax_2, Bx_3,.., Ax_{2n}, Bx_{2n+1},….. converges to $z \in X$. as $n \to \infty$.

Then A, B, S and T have a unique common fixed point z in X.

Proof: Using the condition (v),

We have $Ax_{2n} \to z \;, Tx_{2n+1} \to z \;, Bx_{2n+1} \to z \;, Sx_{2n} \to z \quad as\ n \to \infty$ ......(3.1.1)

Case 1:

Since S is weak reciprocally continuous then $\lim_{n\to\infty} SAx_{2n} \to Sz$

$$\text{since the pair } (A,S) \text{ is } S \text{ compatible then } \lim_{n\to\infty} d(SAx_{2n}, AAx_{2n}) = 0 \text{ giving that}$$

$$\lim_{n\to\infty} SAx_{2n} = \lim_{n\to\infty} AAx_{2n} = Sz \qquad ....(3.1.2)$$

Put x = Ax_{2n} y = x_{2n+1} in condition (ii) we have

$$\begin{aligned}\left[d(AAx_{2n}, Bx_{2n+1})\right]^2 \le k_1 &\left[d(AAx_{2n}, SAx_{2n})\ d(Bx_{2n+1}, Tx_{2n+1}) + d(Bx_{2n+1}, SAx_{2n})\ d(AAx_{2n}, Tx_{2n+1}\right] \\ &+ k_2\left[d(AAx_{2n}, SAx_{2n})\ d(AAx_{2n}, Tx_{2n+1}) + d(Bx_{2n+1}, Tx_{2n+1})\ d(Bx_{2n+1}, SAx_{2n}\right]\end{aligned}$$

$$\text{letting } n \to \infty \text{ on both sides and using the conditions } (3.1.1), (3.1.2) \text{ then we get}$$

$$\begin{aligned}\left[d(Sz,z)\right]^2 \le k_1 &\left[d(Sz,Sz)\ d(z,z) + d(z,Sz)\ d(Sz,z)\right] \\ &+ k_2\left[d(Sz,Sz)\ d(Sz,z) + d(z,z)\ d(z,Sz)\right]\end{aligned}$$

$$\left[d(Sz,z)\right]^2 \le k_1 \left[d(Sz,z)\right]^2$$

$$\left[d(Sz,z)\right]^2 (1-k_1) \le 0 \text{ since } 0 \le k_1 + 2k_2 < 1, k_1, k_2 \ge 0 \text{ we have}$$

$$d(Sz,z) = 0 \text{ givng that } Sz = z$$

Put x = z, y = x_{2n+1} in condition (ii) we have

$$\begin{aligned}\left[d(Az, Bx_{2n+1})\right]^2 \le k_1 &\left[d(Az,Sz)d(Bx_{2n+1}, Tx_{2n+1}) + d(Bx_{2n+1}, Sz)d(Az, Tx_{2n+1})\right] \\ &+ k_2\left[d(Az,Sz)d(Az,Tx_{2n+1}) + d(Bx_{2n+1}, Tx_{2n+1})d(Bx_{2n+1}, Sz)\right]\end{aligned}$$

$$\text{letting } n \to \infty \text{ on both sides and using the conditions } Sz = z, (3.1.1), \text{ then we get}$$

$$\begin{aligned}\left[d(Az,z)\right]^2 \le k_1 &\left[d(Az,z)d(z,z) + d(z,z)d(Az,z)\right] \\ &+ k_2\left[d(Az,z)d(Az,z) + d(z,z)d(z,z)\right]\end{aligned}$$

$$\left[d(Az,z)\right]^2 \le k_2\left[d(Az,z)\right]^2$$

$$(1-k_2)\left[d(Az,z)\right]^2 \le 0 \text{ since } 0 \le k_1 + 2k_2 < 1, k_1, k_2 \ge 0 \text{ we have}$$

$$d(Az,z) = 0 \text{ givng that } Az = z$$

Since A(X) $\subseteq$ T(X) implies there exists u$\in$ X such that z = Az = Tu.

To prove Bu = z, put x = z , y = u in condition (ii) we have

$$[d(Az,Bu)]^2 \le k_1[d(Az,Sz)d(Bu,Tu)+d(Bu,Sz)d(Az,Tu)]$$
$$+k_2[d(Az,Sz)d(Az,Tu)+d(Bu,Tu)d(Bu,Sz)]$$

$U \sin g\ Az = Sz = z, and\ z = Az = Tu\ \ we\, have$

$$[d(z,Bu)]^2 \le k_1[d(z,z)d(Bu,z)+d(Bu,z)d(z,z)]$$
$$+k_2[d(z,z)d(z,z)+d(Bu,z)d(Bu,z)]$$

$$[d(Bu,z)]^2 \le k_2[d(Bu,z)]^2$$

$(1-k_2)[d(Bu,z)]^2 \le 0$ Since $0 \le k_1 + 2k_2 < 1,$ *where* $k_1, k_2 \ge 0$ *we have*

$d(Bu,z) = 0$ *giving that* $Bu = z$

Hence we have Az = Sz = Bu = z.
Since (B, T) is weakly compatible Btu = TBu this implies Bz = Tz

Now we prove Bz = z
Put x = x_{2n}, y = z in condition (ii) we have

$$[d(Ax_{2n},Bz)]^2 \le k_1[d(Ax_{2n},Sx_{2n})d(Bz,Tz)+d(Bz,Sx_{2n})d(Ax_{2n},Tz)]$$
$$+k_2[d(Ax_{2n},Sx_{2n})d(Ax_{2n},Tz)+d(Bz,Tz)d(Bz,Sx_{2n})]$$

letting $n \to \infty$ *on both sides and u* sin *g the conditions* $Tz = Bz$, (3.1.1) *then we get*

$$[d(z,Bz)]^2 \le k_1[d(z,z)d(Bz,z)+d(Bz,z)d(z,z)]$$
$$+k_2[d(z,z)d(z,z)+d(Bz,z)d(Bz,z)]$$

$$[d(Bz,z)]^2 \le k_2[d(Bz,z)]^2$$

$(1-k_2)[d(Bz,z)]^2 \le 0$ Since $0 \le k_1 + 2k_2 < 1,$ *where* $k_1, k_2 \ge 0$ *we have*
$d(Bz,z) = 0$ *giving that* $Bz = z.$

Hence $Bz = Tz = z$

Case2:
Since A is weakly reciprocally continuous then $\lim_{n\to\infty} ASx_{2n} \to Az$

sin *ce the pair* (A,S) *is A compatible then* $\lim_{n\to\infty} d(ASx_{2n}, SSx_{2n}) = 0$ *giving that*

$$\lim_{n\to\infty} SAx_{2n} = \lim_{n\to\infty} AAx_{2n} = Az \qquad ...(3.1.3)$$

Put x = Sx_{2n} y = x_{2n+1} in condition (ii) we have

$$[d(ASx_{2n}, Bx_{2n+1})]^2 \le k_1 [d(ASx_{2n}, SSx_{2n})\ d(Bx_{2n+1}, Tx_{2n+1}) + d(Bx_{2n+1}, SAx_{2n})\ d(ASx_{2n}, Tx_{2n+1}\ ]$$
$$+ k_2 [d(ASx_{2n}, SAx_{2n})\ d(ASx_{2n}, Tx_{2n+1}) + d(Bx_{2n+1}, Tx_{2n+1})\ d(Bx_{2n+1}, SSx_{2n}\ ]$$

letting $n \to \infty$ *on both sides and using the conditions* (3.1.1), (3.1.3) *then we get*

$$[d(Az,z)]^2 \le k_1 [d(Az,Az)\ d(z,z) + d(z,Az)\ d(Az,z)\ ]$$
$$+ k_2 [d(Az,Az)\ d(Az,z) + d(z,z)\ d(z,Az)\ ]$$

$$[d(Az,z)]^2 \le k_1\ [d(Az,z)]^2$$

$$[d(Az,z)]^2 (1 - k_1) \le 0 \ \text{ since } 0 \le k_1 + 2k_2 < 1, k_1, k_2 \ge 0 \ \textit{we have}$$

$$d(Az,z) = 0 \ \textit{giving that } Az = z$$

Since A(X) $\subseteq$ T(X) implies there exists v $\in$ X such that z = Az = Tv
Put x = x_{2n}, y = u

$$[d(Ax_{2n}, Bv)]^2 \le k_1 [d(Ax_{2n}, Sx_{2n}) d(Bv, Tv) + d(Bv, Sx_{2n}) d(Ax_{2n}, Tv)\ ]$$
$$+ k_2 [d(Ax_{2n}, Sx_{2n}) d(Ax_{2n}, Tv) + d(Bv, Tv) d(Bv, Sx_{2n})\ ]$$

letting $n \to \infty$ *on both sides and using the conditions* $Tv = Az = z$ *and* (3.1.1) *then we get*

$$[d(z,Bv)]^2 \le k_1 [d(z,z) d(Bv,z) + d(Bv,z) d(z,z)\ ]$$
$$+ k_2 [d(z,z) d(z,z) + d(Bv,z) d(Bv,z)\ ]$$

$$[d(Bv,z)]^2 \le k_2 [d(Bv,z)]^2$$

$$(1 - k_2)[d(Bv,z)]^2 \le 0 \ \text{ since } 0 \le k_1 + 2k_2 < 1, k_1, k_2 \ge 0 \ \textit{we have}$$

$$[d(Bv,z)] = 0 \ \textit{giving that } Bv = z$$

Since (B, T) is weakly compatible implies BTv = TBv $\Rightarrow$ Bz = Tz

Now we prove Bz = z
Put x = x_{2n}, y = z in condition (ii) we have

$$[d(Ax_{2n},Bz)]^2 \le k_1[d(Ax_{2n},Sx_{2n})d(Bz,Tz)+d(Bz,Sx_{2n})d(Ax_{2n},Tz)]$$
$$+k_2[d(Ax_{2n},Sx_{2n})d(Ax_{2n},Tz)+d(Bz,Tz)d(Bz,Sx_{2n})]$$

letting $n \to \infty$ *on both sides and using the conditions* $Tz = Bz$, (3.1.1) *then we get*

$$[d(z,Bz)]^2 \le k_1[d(z,z)d(Bz,z)+d(Bz,z)d(z,z)]$$
$$+k_2[d(z,z)d(z,z)+d(Bz,z)d(Bz,z)]$$
$$[d(Bz,z)]^2 \le k_2[d(Bz,z)]^2$$

$(1-k_2)[d(Bz,z)]^2 \le 0$ Since $0 \le k_1+2k_2 <1$, *where* $k_1,k_2 \ge 0$ *we have*

$d(Bz,z)=0$ *giving that* $Bz = z$.

Hence $Bz = Tz = z$

$B(X) \subseteq S(X)$ implies there exists $w \in X$ such that Bz = z = Sw.
since the pair (A,S) is A compatible then $\lim_{n\to\infty} d(ASx_{2n},SSx_{2n}) = 0$ implies d(ASw, SSw) = 0 implies ASw = SSw implies Az = Sz = z.

Since Az = Bz = Sz = Tz = z, we get z in a common fixed point of A,B,S and T. The uniqueness of the fixed point can be easily proved.

3.2. Remark: From the example given above, clearly the pair (A, S) is S weak reciprocally continuous and S compatible and (B, T) is weakly compatible as they commute at coincident points $\frac{1}{4}$. But the pairs (A, S) and (B,T) are not compatible and not reciprocally continuous.

For this, take a sequence $x_n=\left(\frac{1}{4}+\frac{1}{n}\right)$ for n≥1, then $\lim_{n\to\infty} Ax_n=\lim_{n\to\infty} Sx_n=\frac{1}{4}$ and $\lim_{n\to\infty} ASx_n=\frac{1}{6}$

also $\lim_{n\to\infty} SAx_n=\frac{1}{4}$. So that $\lim_{n\to\infty} d(ASx_n, SAx_n) \neq 0$. Also note that none of the mappings are continuous and the rational inequality holds for the values of $0 \le k_1+2k_2 <1$, *where* $k_1,k_2 \ge 0$. Clearly $\frac{1}{4}$ is the unique common fixed point of A, B, S and T.

3.3 Remark: Theorem (B) is a generalization of Theorem (A) by virtue of the weaker conditions such as S- weak reciprocally continuous and S compatible in the pair (A,S) and (B,T) is weakly compatible, which are weaker conditions than compatibility of the pairs (A, S) and (B ,T) assumed in theorem (A); The continuity of any one of the mappings is being dropped and the convergence of associated sequence relative to four self maps A, B, S and T in place of the complete metric space assumed in theorem(A).

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