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Machine Repair Problem with Mixed Spares, Balking and Reneging

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ABSTRACT: In this paper we consider a machine repair problem with spares, balking and reneging. The failure and repair of machines is assumed to be exponentially distributed. Birth and death process is employed to formulate the mathematical model of the problem. The Chapman-Kolmogorov equations are formulated for steady state, which have been solved recursively. The queue size distribution for the system having N operating units along with the mixed spares of which S_1 are warm and S_2 are cold is established using product type technique. Some special cases are deduced which tally with earlier existing results. To find out the optimal number of spares and repairmen a cost function is also mentioned.

Keywords: Queue size, Machine repair, Mixed spares, Balking, Reneging, Additional repairmen, State dependent Rates, Recursive technique.

I. INTRODUCTION

The performance of any machining system plays a vital role in human life. Due to machine breakdown, there may not only a loss of production but also a loss of cost and inconvenience. To avoid this loss the spare and appropriate repair facility should be incorporated. In view of such a design the standby units play an important role so that the machining system may keep working to provide the desired grade of service all the time. If an on line unit fails, an available standby unit replaces it and the failed machine is sent for immediate repair. A standby unit may be cold standby type which has zero failure rate whereas warm standby has failure rate non-zero and less than the failure rate of an online unit. Today's machining systems are highly sophisticated and complex. These are comprised of a number of complicated parts. Failure of any part(s) or whole machine directly affects the service system being sought. Thus a machining system can become out of order at any stage or can have different reasons or modes for its failure. A number of researchers have contributed in the field of machine repair problem in different frameworks. An $M/E_k/1$ machine repair problem having N identical automatic machines maintained by a single non-reliable service station was studied by Wang and Kuo (1997). Armstrong (2002) suggested preventive maintenance through age repair policies for a range of machine repair problems. Jain *et al.* (2004) made the performance prediction of a two mode failure machine interference model with spare. Performance analysis of state dependent machine repair system with mixed standbys and two modes of failure were done by Jain et al. (2008). Mehrgani *et al.* (2011) studied a manufacturing system tend to failure concerning two machines working in passive redundancy, whose turning out one part experienced two modes of failure and repair. Ke and Wu (2012) examined a machine repair problem with homogeneous machines and standbys under the care of multiple technicians operating a synchronous vacation policy.

Any interruption in the operation of machining systems involved in manufacturing/production overheads a lot to the concerned organization. To tackle this problem and to maintain the system during machine breakdown, standbys play an essential role. The machining systems with emphasis on spares have been stressed by various researchers. Gross and Harris (1985), Sivazlian and Wang (1989) investigated machine repair models with standbys. Gupta and Rao (1996) presented M/G/1 machine interference problem with spares. Jain and Singh (2000) analyzed the reliability of a repairable multicomponent redundant repairable system. Arulmozhi (2002) discussed reliability of an M-out-of-N warm standby with R repair facilities. Jain and Sharma (2004) studied M/M/C interdependent machining system with mixed spares and controlled rates of failure and repair. Jain *et al.* (2004) performed the reliability analysis of redundant repairable system with degraded failure. Wang and Chiu (2006) studied the cost benefit analysis of availability systems with warm standby units and imperfect coverage. A genetic algorithm approach was used by Azaron *et al*. (2009) to solve a multi-objective discrete reliability optimization problem in a dissimilar-unit non-repairable cold-standby redundant system. For a warm standby repairable system consisting of two dissimilar units and one repairman, Yuan and Meng (2011) performed reliability analysis using Laplace transform. Wang *et al.* (2012) analyzed the warm-standby M/M/R machine repair problem with multiple imperfect coverage involving the service pressure condition.

Long waiting lines for the repair may irritate the caretaker of the machines, which may force him to balk or renege from the system. This concept has been considerably incorporated in various machine repair problems. Shawky (1997) introduced the single server machine interference model with balking, reneging and an additional server for longer queues. Ke and Wang (1999) performed the cost analysis of M/M/R machine repair problem with balking, reneging and server breakdowns. Shawky (2000) suggested the machine interference model M/M/C/K/N with balking, reneging and spares. Ke and Wang (2003) discussed the probability analysis of a repairable system with warm standbys, balking and reneging. Sharma *et al.* (2005) proposed loss and delay multiserver queueing model with discouragement and additional server. Hassan et al. (2008) considered a discrete time retrial queue with general retrial times and balking customers. Al-Seedy *et al.* (2009) and Wu and Ke (2010) analyzed M/M/c queue with balking and reneging. A machine repair model with M identical machines and variable servers considering balking was examined by Wang *et al.* (2011). Boudali and Economou (2012) analyzed a single server Markovian queue with catastrophes and suggested optimal and equilibrium balking strategies for it.

When all the spares are used and all the permanent repairmen are busy and a unit breaks down, it is recommended to employ additional removable repair facility so as to avoid the interruption due to failure of machine components. Jain *et al.* (1996, 1998, 2000) and Jain and Singh (2002) have presented excellent works for machine repair problem with balking, reneging and an additional server. Jain *et al.* (2003) developed a repairable system with spares, state dependent rates and additional repairmen. Glazebrook *et al.* (2005) applied index policies for the maintenance of a collection of machines by a set of repairmen. Wang and Huang (2009) examined a single removable and unreliable server in the $\langle p, N \rangle$ policy M/G/1 queue in which the server breaks down as per the Poisson process and the repair time follows an arbitrary distribution. Huang *et al*. (2011) studied the optimal management problem of an M/M/2/K queueing system with controlling arrival and service of a two-removable-server system.

The present study is an extension of the work of Jain *et al.* (2005) by incorporating the discouragement for state dependent M/M/C/K/N machining system with mixed spares and additional removable repairmen. The aim of this study is to develop a profitable comprehensive model by considering steady state analysis for state analysis for machine repair problems. The state probabilities are obtained to establish the system performance indices. The remaining chapter is organized as follows. The model description and terminology used are given in section 2. In section 3, the mathematical analysis has been provided. In section 4, some performance measures are derived. Some special cases are deduced in section 5. In section 6, cost analysis is performed. Finally, the discussion is made in section 7.

II. MODEL DESCRIPTION AND NOTATIONS

Consider mixed multi-components machining system with balking, reneging, spares and additional repairman. For formulating the model mathematically, the following assumptions are made:

- There are M operating, S_1 warm standby and S_2 cold standby units in the system.
- The system will work with at least m operating units where for normal functioning M units are required.
- The life time and repair time of units are assumed to be exponentially distributed.
- The repair facility consists of R permanent repairmen and r additional removable repairmen to maintain the amount of production up to a desired goal. If the number of failed units is more than the permanent repairmen then we employ the additional removable repairmen one by one depending upon work load.
- After repair, the unit will join the standby group. When an operating unit fails, it is replaced by cold standby are unit if available. If all cold standbys are exhausted, then it is replaced by warm standby unit.
- The repairmen repair the failed units in FCFS fashion.
- We assume that β ($0 \le \beta < 1$) is the probability of the unit to join the queue when all permanent repairmen are busy and some standby units are available and β_0 when all standby are exhausted and no additional repairman turns on. When j ($1 \le j \le r$) additional repairmen are working, the balking probability is given by 1-β_j.
- Failed units reneges exponentially with parameter ν when all permanent repairmen are busy and standby units are available. In case when all standby units are exhausted and number of failed units is below and equal to threshold level T, reneging parameter is denoted by ν_0. Unit reneges exponentially with parameter ν_j when all permanent repairmen and j ($1 \le j \le r$) additional repairmen are busy.
- The additional removable repairmen will be available for repair depending upon the number of failed units present in the system according to a prescribed scheme as stated below:
  - When there are n < T failed units, only R permanent repairmen are available for repair them.

- In case of $jT \le n < (j+1)T$, $j = 1, 2,\ldots,r-1$, there are j additional repairmen available to provide repair with rate μ_i. The j^{th} additional repairmen is again removed when queue length drops to $jT-1$, $j = 1,2,\ldots,r$.
- In case of $Rt \le n < M + S_1 + S_2-m$ failed units, all $(R + r)$ repairmen *i.e.* all the permanent and additional repairmen will be busy in the system.

We develop the mathematical model by taking suitable notations which are given below:

M	:	The number of operating units in the machining system.
R	:	The number of permanent repairmen
r	:	The number of additional removable repairmen
S_1	:	The number of warm standby units
S_2	:	The number of cold standby units
λ_1	:	Failure rate of operating units
λ_2	:	Failure rate of a warm standby
β	:	Joining probability of a failed unit in the queue when some standby units are available.
β_j	:	Joining probability of a failed units when all standbys are exhausted and $j(j = 0,1,\ldots,r)$ additional repairmen are turn on.
ν, ν_j	:	Reneging parameters of failed units when a few standbys are available, and no standby is available and $j(j = 0,1,\ldots,r)$ additional repairmen are turn on.
μ	:	Repair rate of permanent repairmen.
μ_f	:	Faster repair rate of permanent repairman when all standbys are exhausted.
μ_i	:	Repair rate of i^{th} $(i = 1,2,\ldots,r)$ additional removable repairman.
$\lambda(n)$, $\mu(n)$	:	State dependent failure rate, repair rate of units when there are n failed units present in the system.
n	:	The number of failed units in the system waiting for their repair including those failed units which are being repaired.
P_n	:	Probability that there n failed units present in the system in steady state.
P_o	:	Probability that there is no failed unit in the system.

III. FORMULATION OF THE PROBLEM

The failure rates and repair rates of the units are state dependent and are given by

$$\lambda(n)=\begin{cases} M\lambda_1+S_1\lambda_2, & 0\le n<R \\ M\lambda_1\beta+S_1\lambda_2, & R\le n<S_1 \\ M\lambda_1\beta+(S_1+S_2-n)\lambda_2, & S_1\le n<S_1+S_2 \\ M\lambda_1\beta_0+(S_1+S_2-n)\lambda_2, & S_1+S_2\le n<T \\ (M+S_1+S_2-n)\lambda_2\beta_j, & jT\le n<(j+1)T,\ j=1,2,...,r-1 \\ (M+S_1+S_2-n)\lambda_2\beta_r, & rT\le n<M+S_1+S_2-m \end{cases} \qquad \ldots(1)$$

and

$$\mu(n)=\begin{cases} n\mu & 0<n\le R \\ R\mu+(n-R)\nu & R<n\le S_1+S_2 \\ R\mu_f+(n-R)\nu_0 & S_1+S_2+1<n\le T \\ R\mu_f+\sum_{i=1}^{j}\mu_f+(n-\overline{R+j}) & jT<n\le(j+1)T,\ \ j=1,2,...,r-1 \\ R\mu_f+\sum_{i=1}^{j}\mu_f+(n-\overline{R+r}) & rT<n\le M+S_1+S_2-m \\ & \end{cases} \quad ...(2)$$

Using appropriate state dependent rates given in (1) and (2) we can write the governing steady state equations as:

$$-(M\lambda_1+S_1\lambda_1)P_0+\mu P_1=0 \quad ...(3)$$

$$-(M\lambda_1+S_1\lambda_2+n\mu)P_n+(M\lambda_1+S_1\lambda_2)P_{n-1}+(n+1)\mu P_{n+1}=0, \qquad 0<n<C \quad ...(4)$$

$$-(M\lambda_1\beta+S_1\lambda_1+R\mu)P_R\ +(M\lambda_1+S_1\lambda_2)P_{R-1}+(R\mu+\nu)P_{R+1}=0 \quad ...(5)$$

$$-[M\lambda_1\beta+S_1\lambda_2+R\mu+(n-R)\nu]P_n+(M\lambda_1\beta+S_1\lambda_2)P_{n-1}+[R\mu+(n+1-R)\nu]P_{n+1}=0, \quad R<n\le S_2 \quad ...(6)$$

$$-[M\lambda_1\beta+(S_1+S_2-n)\lambda_2+R\mu+(n-R)\nu]P_n+[M\lambda_1\beta+(S_1+S_2+1-n)\lambda_2]P_{n-1}$$
$$+[C\mu+(n+1-C)\nu]P_{n+1}=0, \qquad S_2<n<S_2+S_1 \quad ...(7)$$

$$-[M\lambda_1\beta_0+R\mu+(S_2+S_1-R)\nu]P_{Y+S}+(M\lambda_1\beta+\lambda_2)P_{Y+S-1}$$
$$+[R\mu_f+(S_2+S_1+1-R)\nu_0]P_{Y+S+1}=0 \quad ...(8)$$

$$-[M\lambda_1\beta_0+(S_1+S_2-n)\lambda_2+R\mu_f+(n-R)\nu_0]P_n+[M\lambda_1\beta_0+(S_1+S_2+1-n)\lambda_2]P_{n-1}$$
$$+[R\mu_f+(n+1-R)\nu_0]P_{n+1}=0, \qquad S_2+S_1<n<T \quad ...(9)$$

$$-[(M+S_1+S_2-T)\lambda_1\beta_1+R\mu_f+(T-R)\nu_0]P_T+[M\lambda_1\beta_0+(S_1+S_2+1-T)\lambda_2]P_{T-1}$$
$$+[R\mu_f+\mu_1+(T-R)\nu_1]P_{T+1}=0 \quad ...(10)$$

$$-[(M+S_1+S_2-jT)\lambda_1\beta_j+R\mu_f+\sum_{i=1}^{j-1}\ \mu_i+(jT-\overline{R+j-1})\nu_{j-1}]P_{jT}+[(M+S_1+S_2+1-jT)\lambda_1\beta_{j-1}]$$
$$P_{jT-1}+[R\mu_f+\sum_{i=1}^{j}\ \mu_i+(jT+1-\overline{R+j})\nu_j]P_{jT+1}=0, \qquad j=1,2,...,r-1 \quad ...(11)$$

$$-[(M+S_1+S_2-n)\lambda_1\beta_j+R\mu_f+\sum_{i=1}^{j}\ \mu_i+(n-\overline{R+j})\nu_j]P_n+[(M+S_1+S_2+1-n)\lambda_1\beta_j]P_{n-1}$$
$$+[R\mu_f+\sum_{i=1}^{j}\ \mu_i+(n+1-\overline{R+j})\nu_j]P_{n+1}=0, \qquad jT<n<(j+1)T,\ j=1,2,...,r-1 \quad ...(12)$$

$$-[(M+S_1+S_2-rT)\lambda_1\beta_r+R\mu_f+\sum_{i=1}^{r-1}\ \mu_i+(rT+1-\overline{R+r})\nu_{r-1}]P_{rT}$$
$$+\left[(M+S_1+S_2+1-rT)\lambda_1\beta_{r-1}\right]P_{rT-1}+[R\mu_f+\sum_{i=1}^{r}\ \mu_i+(rT+1-\overline{R+r})\nu_r]P_{rT+1}=0 \quad ...(13)$$

$$-[(M+S_1+S_2-n)\lambda_1\beta_r+R\mu_f+\sum_{i=1}^{r}\ \mu_i+(n-\overline{R+r})\nu_r]P_n+\left[M+S_1+S_2+1-n)\lambda_1\beta_r\right]P_{n-1}$$
$$+[R\mu_f+\sum_{i=1}^{r}\ \mu_i+(n+1-\overline{R+r})\nu_r]P_{n+1}=0, \quad rT<n<M+S_1+S_2-m \quad ...(14)$$

$$-[R\mu_f + \sum_{i=1}^{r} \mu_i + (M+S_1+S_2-m-\overline{R+r})\nu_r]P_{M+S_1+S_2-m} + [(m+1)\lambda_1\beta_r]P_{M+S_1+S_2-m-1} = 0 \qquad \ldots(15)$$

The steady state solution of equations (3)-(15) by using the product type solution is obtained as follows:

$$P_n = \begin{cases} \left(\dfrac{M\lambda_1 + S_1\lambda_2}{\mu}\right)^n \dfrac{1}{n!} P_0 & 0 < n \le R \\[2ex] \dfrac{(M\lambda_1\beta + S_1\lambda_2)^{n-R}}{\prod\limits_{k=R+1}^{n} R\mu + (k-R)\nu} A_1 P_0 & R < n \le S_2 \\[2ex] \dfrac{\prod\limits_{i=S_2+1}^{n}[M\lambda_1\beta + (S_1+S_2+1-i)\lambda_2]}{\prod\limits_{k=R+1}^{n}[R\mu + (k-R)\nu]} (M\lambda_1\beta + S_1\lambda_2)^{S_2-R} A_1 P_0, & S_2 < n \le S_2 + S_1 \\[2ex] \dfrac{\prod\limits_{i=S_1+S_2+1}^{n}[M\lambda_1\beta_0 + (S_1+S_2+1-i)\lambda_2]}{\prod\limits_{k=S_1+S_2+1}^{n}[R\mu_f + (k-R)\nu_0]} \dfrac{\prod\limits_{i=S_2+1}^{S_2+S_1} M\lambda_1\beta + (S_2+S_1+1-i)\lambda_2}{\prod\limits_{k=R+1}^{S_2+S_1}[R\mu + (k-R)\nu]} A_2 P_0 & S_2 + S_1 < n \le T \\[2ex] \dfrac{\prod\limits_{i=T+1}^{n}(M+S_1+S_2+1-i)(\lambda_1)^{n-jT}(\beta_j)^{n-jT}}{\prod\limits_{k=T+1}^{n-jT}\left[R\mu_f + \sum\limits_{i=1}^{j}\mu_i + (k-\overline{R+j})\nu_j\right]} \dfrac{1}{\left[\prod\limits_{i=1}^{j-1}\prod\limits_{k=i}^{(i+1)T}\left[R\mu_f + \sum\limits_{i=1}^{T}\mu_i + (k-\overline{R+i})\nu_f\right]\right]} & \\ \times \dfrac{\prod\limits_{i=S_2+S_1+1}^{T}[M\lambda_1\beta_0 + (S_1+S_2+1-i)\lambda_2]\left(\prod\limits_{i=1}^{j-1}(\lambda_1\beta_i)^T\right)}{\prod\limits_{i=S_1+S_2+1}^{T}[R\mu_f + (k-R)\nu_0]} A_3 P_0; \quad jT < n \le (j+1)T & 1 \le j \le r \\[2ex] \dfrac{\prod\limits_{i=T+1}^{n}(M+S_1+S_2+1-i)(\lambda_1\beta_r)^{n-rT}\left(\prod\limits_{i=1}^{r-1}(\lambda_1\beta_i)^T\right)}{\prod\limits_{k=rT+1}^{n}\left[R\mu_f + \sum\limits_{i=1}^{r}\mu_i + (k-\overline{R+r})\nu_j\right]\prod\limits_{k=jT+1}^{rT}\left[R\mu_f + \sum\limits_{i=1}^{r}\mu_i + (k-\overline{R+r})\nu_r\right]} & \\ \times \dfrac{1}{\prod\limits_{i=1}^{r-1}\prod\limits_{k=iT+1}^{(i+1)}\left[R\mu_f + \sum\limits_{i=1}^{T}\mu_i + (k-\overline{R+1})\nu_1\right]} A_4 P_0; \quad rT \le n \le M + S_2 + S_1 - m & \end{cases}$$

...(16)

Where

$$A_1=\left(\frac{M\lambda_1+S_1\lambda_1}{\mu}\right)^C\frac{1}{R!},\qquad A_2=(M\lambda_1\beta+S_1\lambda_1)^{S_2-R}.A_1,$$

$$A_3=\frac{\prod_{i=S_2+1}^{S_2+S_1+1}[M\lambda_1\beta+(S_1+S_2+1-i)\lambda_2]}{\prod_{k=R+1}^{S_2+S_1}[R\mu+(k-R)\nu]}A_2,\qquad A_4=\frac{\prod_{i=S_2+S_1+1}^{T}[M\lambda_1\beta_0+(S_1+S_2+1-i)\lambda_2]}{\prod_{k=S_2+S_1+1}^{T}[R\mu_f+(k-R)\nu_0]}A_3$$

Now we determine P_0, using the normalization condition $\sum_{n=0}^{M+S_1+S_2-m}P_n=1$. Now we get

$$P_0^{-1}=\sum_{n=0}^{R}\frac{(M\lambda_1+S_1\lambda_2)^n}{\mu^n}\frac{1}{n!}+\frac{(M\lambda_1+S_1\lambda_2)^R}{\mu^C}\frac{1}{R!}\sum_{k=R+1}^{S_2}\frac{(M\lambda_1\beta+S_1\lambda_2)^{n-R}}{\prod_{k=R+1}^{n}[R\mu+(k-R)\nu]}$$

$$+A_1\ (M\lambda_1\beta+S_1\lambda_2)^{S_2-R}\sum_{n=S_2+1}^{S_2+S_1}\frac{\prod_{i=S_2+1}^{n}[M\lambda_1\beta+(S_2+S_1+1-i)\lambda_2]}{\prod_{k=R+1}^{n}[R\mu+(k-R)\nu]}$$

$$+\ A_2\frac{\prod_{i=S_2+1}^{S_2+S_1}[M\lambda_1\beta+(S_2+S_1+1-i)\lambda_2]}{\prod_{k=R+1}^{S_2+S_1}[R\mu+(k-R)\nu]}\sum_{n=S_2+S_1+1}^{T}\frac{\prod_{i=S_2+S_1+1}^{n}[M\lambda_1\beta_0+(S_2+S_1+1-i)\lambda_2]}{\prod_{k=S_2+S_1+1}^{n}[R\mu_f+(k-R)\nu_0]}$$

$$+A_3\frac{\prod_{i=S_2+S_1+1}^{T}[M\lambda_1\beta_0+(S_1+S_2+1-i)\lambda_2]\left(\prod_{i=1}^{j-1}(\lambda_1\beta_i)^T\right)}{\prod_{k=S_2+S_1+1}^{T}[R\mu_f+(k-R)\nu_0]}\frac{1}{\left[\prod_{l=1}^{j-1}\prod_{k=iT+1}^{(i+1)T}(C\mu_f+\sum_{i=1}^{l}\mu_i+(k-\overline{R+l})\nu_l\right]}$$

$$\times\sum_{j=1}^{r-1}\sum_{n=jT+1}^{(j+1)T}\frac{\prod_{i=T+1}^{n}(M+S_1+S_2+1-i)(\lambda_1)^{n-jT}(\beta_j)^{n-jT}}{\prod_{k=jT+1}^{n}\left[R\mu+\sum_{i=1}^{i}\mu_i+(k-\overline{R+j})\nu_j\right]}+A_4\frac{\prod_{i=1}^{r-1}(\lambda_1\beta_i)^T}{\prod_{k=jT+1}^{rT}\left[R\mu_f+\sum_{i=}^{r}\mu_i+(k-\overline{R+r})\nu_r\right]}$$

$$\times\frac{1}{\left[\prod_{l=1}^{r-1}\prod_{k=iT+1}^{(i+1)T}\left(R\mu_f+\sum_{i=1}^{l}\mu_i+(k-\overline{R+l})\right)\nu_l\right]}$$

$$\sum_{n=rT+1}^{M+S_1+S_2-m}\frac{\prod_{i=T+1}^{n}(M+S_1+S_2+1-i)(\lambda_1\beta_r)^{n-rT}}{\prod_{k=rT+1}^{n}\left[R\mu_f+\sum_{i=1}^{r-1}\mu_i+(k-\overline{R+r})\nu_r\right]}\qquad j=1,2,\dots,r-1. \qquad \dots(17)$$

IV. SOME PERFORMANCE MEASURES

After obtaining queue size distribution in previous section, now we obtain some system characteristics as follows:

- The expected number of failed units in the system

$$E(n) = \sum_{n=1}^{M+S_1+S_2-m} n\ P_n \qquad \ldots(18)$$

- Expected number of operating units in the system

$$E(O) = M - \sum_{n=S_1+S_2+1}^{M+S_1+S_2-m} (n - S_1 + S_2)\, P_n \qquad \ldots(19)$$

- Expected number of unused cold spare units in the system

$$E(UCS) = \sum_{n=0}^{S_2} (S_2 - n) P_n \qquad \ldots(20)$$

- Expected number of unused warm spare units in the system

$$E(UYS) = S_2 \sum_{n=0}^{S_1} P_n + \sum_{n=S_2+1}^{S_2+S_1} (S_2 + S_1 - n) P_n \qquad \ldots(21)$$

- Expected number of idle permanent repairmen

$$E(I) = \sum_{n=0}^{R-1} (R - n) P_n \qquad \ldots(22)$$

- Expected number of busy permanent repairmen

$$E(B) = R - E(I) \qquad \ldots(23)$$

- Probability of j th ($1 \le j \le r-1$) additional repairman being busy

$$E(AD_j) = \sum_{j=1}^{r-1} j \sum_{n=jT+1}^{(j+1)T} P_n \qquad \ldots(24)$$

- Expected number of busy additional removable repairmen

$$E(BAS) = \sum_{j=1}^{r-1} j \sum_{n=jT+1}^{(j+1)T} P_n + r \sum_{n=rT+1}^{M+S_1+S_2-m} P_n \qquad \ldots(25)$$

V. SPECIAL CASES

Case I: Model with Cold Spares, Balking Reneging and Additional Repairmen.
If $S_1 = 0$ then our model reduces to model with cold spares, balking, reneging and additional repairmen.

Case II: Model with Balking, Reneging and Mixed Spares
By setting r = 0 our model reduces to machining system with mixed spares, balking and reneging.

Case III: If $\beta = 1,\ \nu = 0$ then our model reduces to Moses (2005) model for machine repair problem with mixed spares and additional repairman.

Case IV: For Y = 0, S = 0, $\beta = 0$ $\nu = 0$, we get results for classical machine repair problem discussed by Kleinrock (1985).

VI. COST FUNCTION

Our main aim in this section is to provide a cost function, which can be minimized to determine the optimal number of repairmen and spares. The average total cost is given by

$$E(C) = C_M \sum_{n=0}^{Y+S} MP_n + C_1 E(I) + C_{SC} E(UCS) + C_{sw} E(UYS) + C_B E(B) + \sum_{j=1}^{r} CA_j E(A_j) \qquad \ldots(26)$$

Where,

C_M	=	Cost per unit time of an operating unit when system works is normal mode.
C_1	=	Cost per unit time per idle permanent repairmen.
C_{SC}	=	Cost per unit time for providing a cold spare unit
C_{Sw}	=	Cost for unit time for providing a warm spare unit
C_B	=	Cost per unit time per permanent repairman when he is busy in providing repair.
CAD_j	=	Cost per unit time of j th (j = 1,2,…,r) additional repairman.

VII. DISCUSSION

In this investigation, a machine repair model with balking reneging, spares and additional repairmen has been developed. The machining system under study comprises of warm and cold standby together with a repair facility having both permanent as well as additional repairmen. The provision of multiple standby support and additional repair facility may help the system manager in providing consistent production up to a desired grade of demand in particular when number of failed machines increases. The expressions for several system performance measures and cost function are derived explicitly which can be further used to determine the optimal combination of standbys and repairmen and might be aid system designer to determine appropriate system components at optimum cost subject to availability constraints.

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