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Common Fixed Point Theorem for Banach Space for Four Mapping

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ABSTRACT: In the present paper we establish a common fixed point theorem for non contractive mapping in rational expression in Banach space. Our result is motivated by many authors.

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I. INTRODUCTION AND PRELIMINARIES

It is well known that a Banach space is a linear space which is also in a special way a complete metric space. The combination of algebraic and metric structures opens up the possibility of studying linear transformation of one Banach space into another which has the additional property of being continuous. A normed linear space is a linear space N in which to each vector z, there corresponds a real number denoted by $\|x\|$ and called the norm of x in such a manner that

(i) $\|x\| \geq 0 \ and \ \|x\| = 0 \Leftrightarrow x = 0$

(ii) $\|x + y\| \leq \|x\| + \|y\|$

(iii $\|\alpha x\| = |\alpha|\|x\|$

The non negative real number $\|x\|$ is to be thought of as the length of vector x. If we regard $\|x\|$ as a real function defined on N. It is easy to verify that the normed function is called norm on N. It is easy to verify that the normed linear space N is a metric space w.r.to the metric d defined by d(x, y) = $\|x - y\|$. A Banach space is a complete normed linear space.

II. MAIN RESULT

Theorem 1: Let K be the closed and convex subset of a Banach space X. Let F, G, H and J be the four mapping of K into itself such that

FG = GF, GH = HG, HJ = JH AND JF = FJ …(1)

$F^2 = I$, $G^2 = I$, $H^2 = I$, $J^2 = I$ (where I denotes the identity mapping) …(2)

$$\|F(x) - F(y)\| \leq \alpha \frac{\|GHJ(x) - F(x)\| \|GHJ(y) - F(y)\| + \|GHJ(x) - F(y)\| \|GHJ(y) - F(x)\|}{\|GHJ(x) - F(x)\| + \|GHJ(y) - F(y)\| + \|GHJ(x) - F(y)\| + \|GHJ(y) - F(x)\|}$$
$$+ \beta \frac{\|GHJ(x) - F(x)\| \|GHJ(x) - F(y)\| + \|GHJ(y) - F(x)\| \|GHJ(y) - F(y)\|}{\|GHJ(x) - F(x)\| + \|GHJ(x) - F(y)\| + \|GHJ(y) - F(x)\| + \|GHJ(y) - F(y)\|}$$
$$+ \gamma \|GHJ(x) - GHJ(y)\| \quad \ldots(3)$$

For every x, y Є K and $0 \le \alpha, \beta, \gamma$ such that $7\alpha + 4\beta + 4\gamma < 8$ …(4)

Then there exists at least one fixed point x_0 of F, G, H, and J.

Further if $\alpha + 2\gamma < 2$

Then x_0 is the common fixed point of F, G, H and J.

PROOF: From equation 1 and 2 it follows that $(FGHJ)^2 = I$, (where I is the identity mapping).
We have

$$\|FGHJ(x) - FGHJ(y)\| \le$$

$$\propto \frac{\|(GHJ)^2G(x) - FGHJG(x)\|\|(GHJ)^2G(y) - FGHJG(x)\| + \|(GHJ)^2G(x) - FGHJG(y)\|\|(GHJ)^2G(y) - FGHJG(x)\|}{\|(GHJ)^2G(x) - FGHJG(x)\| + \|(GHJ)^2G(y) - FGHJG(x)\| + \|(GHJ)^2G(x) - FGHJG(y)\| + \|(GHJ)^2G(y) - FGHJG(x)\|}$$

$$+ \beta \frac{\|(GHJ)^2G(x) - FGHJG(x)\|\|(GHJ)^2G(x) - FGHJG(y)\| + \|(GHJ)^2G(x) - FGHJG(y)\|\|(GHJ)^2G(y) - FGHJG(x)\|}{\|(GHJ)^2G(x) - FGHJG(x)\| + \|(GHJ)^2G(x) - FGHJG(y)\| + \|(GHJ)^2G(x) - FGHJG(y)\| + \|(GHJ)^2G(y) - FGHJG(x)\|}$$

$$+ \gamma \|(GHJ)^2G(x) - (GHJ)^2G(y)\|$$

$$\le \propto \frac{\|G(x) - FGHJG(x)\|\|G(y) - FGHJG(x)\| + \|G(x) - FGHJG(y)\|\|G(y) - FGHJG(x)\|}{\|G(x) - FGHJG(x)\| + \|G(y) - FGHJG(x)\| + \|G(x) - FGHJG(y)\| + \|G(y) - FGHJG(x)\|}$$

$$+ \beta \frac{\|G(x) - FGHJG(x)\|\|G(x) - FGHJG(y)\| + \|G(x) - FGHJG(y)\|\|G(y) - FGHJG(x)\|}{\|G(x) - FGHJG(x)\| + \|G(x) - FGHJG(y)\| + \|G(x) - FGHJG(y)\| + \|G(y) - FGHJG(x)\|}$$

$$+ \gamma \|G(x) - G(y)\|$$

Now if G(x) = V and G(y) = W then,

$$\le \propto \frac{\|V - FGHJV\|\|W - FGHJW\| + \|V - FGHJW\|\|W - FGHJV\|}{\|V - FGHJV\| + \|W - FGHJV\| + \|V - FGHJW\| + \|W - FGHJV\|}$$

$$+ \beta \frac{\|V - FGHJV\|\|V - FGHJW\| + \|V - FGHJW\|\|W - FGHJV\|}{\|V - FGHJV\| + \|V - FGHJW\| + \|V - FGHJW\| - \|W - FGHJV\|}$$

$$+ \gamma \|V - W\|$$

Where $(FGHJ)^2 = I$ and $7\alpha + 4\beta + 4\gamma < 8$

Now to show that F, G, H, J has a fixed point x_0 in K.
Therefore let x be a point in the Banach space X, then taking
$Y = \frac{1}{2}(S + I)\,x$
And $t = S(y)$
$U = 2y - t$

$$\|t - u\| \le \|t - x\| + \|x - u\|$$

Now $\|t - u\| = \|S(x) - 2y + t\| = \|S(x) - 2y + S(y)\| = \|2S(y) - 2y\|$

$$\|t - u\| = 2\|y - S(y)\| \qquad \text{…(5)}$$

Again

$$\|t-x\| = \|S(y)-S^2(x)\|$$

$$\|t-x\| \leq \|S(y)-SS(x)\|$$

$$\|t-x\| \leq \alpha \frac{\|y-S(y)\|\|S(x)-S^2(x)\| + \|S(x)-S(y)\|\|y-S^2(x)\|}{\|y-S(y)\| + \|S(x)-S^2(x)\| + \|S(x)-S(y)\| + \|y-S^2(x)\|} + \beta \frac{\|y-S(y)\|\|y-S^2(x)\| + \|S(x)-S(y)\|\|S(x)-S^2(x)\|}{\|y-S(y)\| + \|y-S^2(x)\| + \|S(x)-S(y)\| + \|S(x)-S^2(x)\|}$$

$$+\gamma \|y-s(x)\|$$

$$\|t-x\| \leq \alpha \frac{\|y-S(y)\|\|S(x)-x\| + \|S(x)-S(y)\|\|y-x\|}{\|y-S(y)\| + \|S(x)-x)\| + \|S(x)-S(y)\| + \|y-x)\|}$$

$$+\beta \frac{\|y-S(y)\|\|y-x\| + \|S(x)-S(y)\|\|S(x)-x\|}{\|y-S(y)\| + \|y-x\| + \|S(x)-S(y)\| + \|S(x)-x\|}$$

$$+\gamma \|y-s(x)\|$$

$$\|t-x\| \leq \alpha \frac{\|y-S(y)\|\|S(x)-x\| + \|S(x)-y+y-S(y)\| \left|\frac{1}{2}(S+I)x-x\right\|}{\|y-S(y)\| + \|y-x)| + \|S(x)-S(y)\| + \|S(x)-x)\|}$$

$$+\beta \frac{|y-S(y)\||y-x\| + |S(x)-y+y-S(y)\|\|S(x)-x\|}{\|y-S(y)| + \|S(x)-S(y)\| + \|x-y\| + \|S(x)-x\|} + \gamma \|y-s(x)\|$$

$$\|t-x\| \leq \alpha \frac{\|y-S(y)\|\|S(x)-x\| + \frac{1}{2}\|x-S(x)\|[\|S(x)-y\| + \|y-S(y)\|]}{|y-S(y)| + \|y-x)\| + \|S(x)-S(y)\| + \|S(x)-x)\|}$$

$$+\beta \frac{\|y-S(y)\| \left\|\frac{1}{2}(S+I)x-x\right| + [\|S(x)-y\| + \|y-S(y)\|]|S(x)-x\|}{\|y-S(y)| + \|S(x)-S(y)\| + |y-x\| + \|S(x)-x\|}$$

$$+\gamma \left\|\frac{1}{2}(S+I)x-x-s(x)\right\|$$

$$\|t-x\| \leq \alpha \frac{\|y-S(y)\|\|x-S(x)\| + \frac{1}{2}\|x-S(x)\|\left[\frac{1}{2}\|x-S(x)\| + \|y-S(y)\|\right]}{\|y-S(y)\| + \|y-x)\| + \|S(x)-x)\|}$$

$$+\beta\,\frac{\|y-S(y)\|\frac{1}{2}\|x-S(x)\|+\left[\frac{1}{2}\|x-S(x)\|+\|y-S(y)\|\right]\|x-S(x)\|}{\|y-S(x)\|-\|y-x\|+\|S(x)-x\|}$$

$$+\gamma\frac{1}{2}\|x-s(x)\|$$

$$\|t-x\|\le\alpha\,\frac{\|y-S(y)\|\|x-S(x)\|+\frac{1}{2}\|x-S(x)\|\left[\frac{1}{2}\|x-S(x)\|+\|y-S(y)\|\right]}{2\|x-S(x)\|}$$

$$+\beta\,\frac{\|y-S(y)\|\frac{1}{2}\|x-S(x)\|+\left[\frac{1}{2}\|x-S(x)\|+\|y-S(y)\|\right]\|x-S(x)\|}{2\|x-S(x)\|}$$

$$+\gamma\frac{1}{2}\|x-s(x)\|$$

$$\|t-x\|\le\frac{\alpha}{2}\|y-S(y)\|+\frac{1}{4}\|x-S(x)\|+\frac{1}{2}\|y-S(y)\|$$

$$+\frac{\beta}{2}\|y-S(y)\|+\left[\frac{1}{2}\|x-S(x)\|+\|y-S(y)\|\right]+\gamma\frac{1}{2}\|x-s(x)\|$$

$$\|t-x\|\le\frac{\alpha}{2}\,\frac{1}{4}\|x-S(x)\|+\frac{3}{2}\|y-S(y)\|$$

$$+\frac{\beta}{2}\left[\frac{1}{2}\|x-S(x)\|+\frac{3}{2}\|y-S(y)\|\right]+\gamma\frac{1}{2}\|x-s(x)\|$$

$$\|t-x\|\le\frac{\alpha}{8}\|x-S(x)\|+\frac{3}{4}\|y-S(y)\|$$

$$+\frac{\beta}{4}\left[\|x-S(x)\|-\frac{3}{4}\|y-S(y)\|\right]+\gamma\frac{1}{2}\|x-s(x)\|$$

$$\|t-x\|\le\frac{\alpha}{8}+\frac{\beta}{4}+\frac{\gamma}{2}\|x-S(x)\|+(3/4\,\alpha+3/4\,\beta)\|y-S(y)\| \qquad \ldots(6)$$

Again

$$\|u-x\|=\|2y-t-x\|=\|(S+I)x-S(y)-x\|$$

$$\|u-x\|=\|S(x)-S(y)\|$$

$$\|u-x\| \le \alpha\frac{\|x-S(x)\|\|y-S(y)\| - \|x-S(y)\|\|y-S(x)\|}{\|x-S(x)\|+\|y-S(y)\| - \|x-S(y)\| + \|y-S(x)\|} + \beta\frac{\|x-S(x)\|\|x-S(y)\| + \|y-S(x)\|\|y-S(y)\|}{\|x-S(x)\|+\|x-S(y)\|+\|y-S(x)\|+\|y-S(y)\|} + \gamma\|x-y\|$$

$$\|u-x\| \le \alpha\frac{\|x-S(x)\|\|y-S(y)\| + [\|x-y\|+\|y-S(y)\|]\|y-S(x)\|}{\|x-S(x)\|+\|x-y\|+\|y-S(x)\|}$$

$$+\beta\frac{\|x-S(x)\|[\|x-y\|+\|y-s(y)\|] + \|y-S(x)\|\|y-S(y)\|}{\|x-S(x)\|+\|x-y\|+\|y-S(y)\|}$$

$$+\gamma\left\|x-\frac{1}{2}(S+I)x\right\|$$

$$\|u-x\| \le \alpha\frac{\|x-S(x)\|\|y-S(y)\| + [\frac{1}{2}\|x-S(x)\|\|y-S(y)\|]\frac{1}{2}\|x-S(x)\|}{2\|x-S(x)\|}$$

$$+\beta\frac{\|x-S(x)\|[\frac{1}{2}\|x-S(x)\|+\|y-S(y)\|+\frac{1}{2}\|x-S(x)\|\|y-S(y)\|}{2\|x-S(x)\|}$$

$$+\frac{\gamma}{2}\|x-S(x)\|$$

$$\|u-x\| \le$$

$$\frac{\alpha}{2}\left[\|y-S(y)\|+\frac{1}{4}\|x-S(x)\|+\frac{1}{2}\|y-S(y)\|\right]$$

$$-\frac{\beta}{2}\left[\frac{1}{2}\|x-S(x)\|+\|y-S(y)\|+\frac{1}{2}\|y-S(y)\|\right]+\frac{\gamma}{2}\|x-S(x)\|$$

$$\|u-x\| \le \frac{\alpha}{8}+\frac{\beta}{4}+\frac{\gamma}{2}\|x-S(x)\| + (3/4\,\alpha+3/4\,\beta)\|y-S(y)\| \quad \ldots(7)$$

Now

$$\|t-u\| \le \|t-x\| + \|x-u\|$$

$$\|t-u\| \le \left(\frac{\alpha}{8}+\frac{\beta}{4}+\frac{\gamma}{2}\right)\|x-S(x)\| + (3/4\,\alpha+3/4\,\beta)\|y-S(y)\|$$

$$+\left(\frac{\alpha}{8}+\frac{\beta}{4}+\frac{\gamma}{2}\right)\|x-S(x)\| + (3/4\,\alpha+3/4\,\beta)\|y-S(y)\|$$

$$\|t-u\| \le [\frac{\alpha}{4}+\frac{\beta}{2}+\gamma]\|x-S(x)\| + (3/2\,\alpha+3/2\,\beta)\|y-S(y)\| \quad \ldots(8)$$

Thus from 5, 6, 7 and 8 we have

$$2\|y - S(y)\| \leq \left(\frac{\alpha}{4} + \frac{\beta}{2} + \gamma\right)\|x - S(x)\| + \left(\frac{3}{2}\alpha + \frac{3}{2}\beta\right)\|y - S(y)\|$$

$$\|y - S(y)\| \leq q\,\|x - S(x)\| \qquad \text{…(9)}$$

Where

$$q = \frac{\frac{\alpha}{4} + \frac{\beta}{2} + \gamma}{2 - \frac{3}{2}\alpha - \frac{3}{2}\beta} < 1$$

Since $7\alpha + 4\beta + 4\gamma < 8$

Now let FGHJ = ½(S+I) x, then for every x Є X

$$\|(FGHJ)^2 x - FGHJ\,(x)\| = \|FGHJ(y) - y\| = \left\|\frac{1}{2}(S + I)y - y\right\| = \frac{1}{2}\|y - S(y)\|$$

$$\|(FGHJ)^2 x - FGHJ\,(x)\| \leq \frac{q}{2}\|x - S(x)\|$$

From 9 and by definition of q we claim that $\{(FGHN)^n(x)\}$ is a Cauchy's sequence in X. By completeness $\{(FGHN)^n(x)\}$ converges to some element x_0 in X.

i.e. $\lim_{n \to \infty}(FGHJ)^n(X) = x_0$

$$FGHJ\,(x_0) = x_0$$

Therefore x_0 is a fixed point of FGHJ.

$FGHJ(x_0) = x_0$

So, $GHJ\,(FGHJ)(x_0) = GHJ\,(x_0)$

Also $J\,FGHJ\,(x_0) = J\,(x_0)$

Or $FGH\,(x_0) = J(x_0)$

Now by using above results and equations 1, 2, 3, 4 we have,

$$\|J(x_0) - x_0\| = \|FGH(x_0) - F\,F^2(x_0)\|$$

$$\|J(x_0) - x_0\| \leq$$

$$\alpha \frac{\|GHJGH(x_0) - FGH(x_0)\|\|GHJF^2(x_0) - FF^2(x_0)\| + \|GHJGH(x_0) - FF^2(x_0)\|\|GHJF^2(x_0) - FGH(x_0)\|}{\|GHJGH(x_0) - FGH(x_0)\| + \|GHJF^2(x_0) - FF^2(x_0)\| + \|GHJGH(x_0) - FF^2(x_0)\| + \|GHJF^2(x_0) - FGH(x_0)\|} +$$

$$\beta \frac{\|GHJGH(x_0) - FGH(x_0)\|\|GHJGH(x_0) - FF^2(x_0)\| + \|GHJF^2(x_0) - FGH(x_0)\|\|GHJF^2(x_0) - FF^2(x_0)\|}{\|GHJGH(x_0) - FGH(x_0)\| + \|GHJGH(x_0) - FF^2(x_0)\| + \|GHJF^2(x_0) - FGH(x_0)\| + \|GHJF^2(x_0) - FF^2(x_0)\|} +$$

$$\gamma\|GHJGH(x_0) - GHJF^2(x_0)\|$$

$$\|J(x_0) - x_0\| < \alpha \frac{\|J(x_0) - J(x_0)\| \|x_0 - x_J\| - \|J(x_0) - (x_0)\| \|(x_0) - J(x_0)\|}{\|J(x_0) - J(x_0)\| + \|x_0 - x_J\| - \|J(x_0) - (x_0)\| + \|(x_0) - J(x_0)\|}$$
$$+ \beta \frac{\|J(x_0) - J(x_0)\| \|J(x_0) - (x_0)\| + \|(x_0) - J(x_0)\| \|(x_J) - (x_J)\|}{\|J(x_0) - J(x_J)\| + \|J(x_0) - (x_0)\| + \|(x_0) - J(x_0)\| + \|(x_0) - (x_0)\|}$$
$$+ \gamma \|J(x_0) - (x_0)\|$$

$$\|J(x_0) - x_0\| \le \frac{\alpha}{2} + \gamma \|J(x_0) - (x_0)\|$$

Since $\alpha + 2\gamma < 2$ it follows that $J(x_0) = x_0$

This shows x_0 is a fixed point of J.

$F(x_0) = FGHJ(x_0)$

$F(x_0) = GH(x_0)$

Again

$$\|F(x_0) - x_0\| = \|F(x_0) - FF(x_0)\|$$

$$\|F(x_0) - x_0\|$$
$$\le \alpha \frac{\|GH(x_0) - F(x_0)\| \|GHF(x_0) - F^2(x_0)\| + \|GH(x_0) - FF(x_0)\| \|GHF(x_0) - F(x_0)\|}{\|GH(x_0) - F(x_0)\| + \|GHF(x_0) - F^2(x_0)\| + \|GH(x_0) - FF(x_0)\| + \|GHF(x_0) - F(x_0)\|}$$
$$+ \beta \frac{\|GH(x_0) - F(x_0)\| \|GH(x_0) - F^2(x_0)\| + \|GHF(x_0) - F(x_0)\| \|GHF(x_0) - FF(x_0)\|}{\|GH(x_0) - F(x_0)\| + \|GH(x_0) - F^2(x_0)\| + \|GHF(x_0) - F(x_0)\| + \|GHF(x_0) - FF(x_0)\|}$$
$$+ \gamma \|GH(x_0) - GHF(x_0)\|$$

$$\|F(x_0) - x_0\|$$
$$\le \alpha \frac{\|F(x_0) - F(x_0)\| \|(x_0) - (x_J)\| + \|F(x_0) - (x_0)\| \|(x_0) - F(x_0)\|}{\|F(x_0) - F(x_0)\| + \|(x_0) - (x_J)\| + \|F(x_0) - (x_0)\| + \|(x_0) - F(x_0)\|}$$
$$+ \beta \frac{\|F(x_0) - F(x_0)\| \|F(x_0) - (x_0)\| + \|(x_0) - F(x_0)\| \|(x_0) - (x_0)\|}{\|F(x_0) - F(x_0)\| \|F(x_0) - (x_0)\| + \|(x_0) - F(x_0)\| \|(x_0) - (x_0)\|}$$
$$+ \gamma \|F(x_0) - (x_0)\|$$

$$\|F(x_0) - x_0\| = \left(\frac{\alpha}{2} + \gamma\right) \|F(x_0) - (x_0)\|$$

Which is a contradiction since $\alpha + 2\gamma < 2$

Hence it follows that $F(x_0) = x_0$

But $F(x_0) = GH(x_0)$

$(x_0) = GH(x_0)$

$F(x_0) = F\,GH(x_0)$

$F(x_0) = J(x_0)$

$F(x_0) = (x_0)$

Hence $F(x_0) = (x_0) = J(x_0)$

Also $F(x_0) = G(x_0)$ and $G(x_0) = H(x_0)$

Therefore x_0 is a common fixed point of FGHJ.

Now to show the uniqueness of x_0, we let y_0 be any other common fixed point of FGHJ then by using

$$\|x_0 - y_0\| = \|FGHJ(x_0) - FGHJ(y_0)\|$$

$$\|x_0 - y_0\|$$

$$\leq \alpha \frac{\|GHJ(x_0) - FGHJ(x_0)\|\|GHJ(y_0) - FGHJ(x_0)\| + \|GHJ(x_0) - FGHJ(y_0)\|\|GHJ(y_0) - FGHJ((x_0)\|}{\|GHJ(x_0) - FGHJ(x_0)\|\|GHJ(y_0) - FGHJ(x_0)\| + \|GHJ(x_0) - FGHJ(y_0)\|\|GHJ(y_0) - FGHJ((x_0)\|}$$

$$+\beta \frac{\|GHJ(x_0) - FGHJ(x_0)\|\|GHJ(x_0) - FGHJ(x_0)\| + \|GHJ(y_0) - FGHJ(x_0)\|\|GHJ(y_0) - FGHJ((y_0)\|}{\|GHJ(x_0) - FGHJ(x_0)\| + \|GHJ(x_0) - FGHJ(x_0)\| + \|GHJ(y_0) - FGHJ(x_0)\| + \|GHJ(y_0) - FGHJ((y_0)\|}$$

$$+ \gamma \|GHJ(x_0) - GHJ(y_0)\|$$

$$\leq \alpha \frac{\|x_0 - F(x_0)\|\|y_0 - F(x_0)\| + \|x_0 - F(y_0)\|\|y_0 - F((x_0)\|}{\|x_0 - F(x_0)\| + \|y_0 - F(x_0)\| + \|x_0 - F(y_0)\| - \|y_0 - F((x_0)\|}$$

$$+\beta \frac{\|x_0 - F(x_0)\|\|x_0 - F(y_0)\| + \|y_0 - F(x_0)\|\|y_0 - F((x_0)\|}{\|x_0 - F(x_0)\| + \|x_0 - F(y_0)\| + \|y_0 - F(x_0)\| + \|y_0 - F((x_0)\|}$$

$$+ \gamma \|x_0 - y_0\|$$

Since $F(x_0) = GHJ(x_0)$ *and also* $\|x_0 - F(x_0)\| = 0$ *and* $\|y_0 - F(y_0)\| = 0$

$$= \alpha \frac{\|x_0 - F(y_0)\|\|y_0 - F((x_0)\|}{\|x_0 - F(y_0)\| + \|y_0 - F((x_0)\|} + \gamma \|x_0 - y_0\|$$

$$= \frac{\alpha}{2}\|x_0 - y_0\| + \gamma\|x_0 - y_0\|$$

$$= (\alpha/2 + \gamma)\|x_0 - y_0\|$$

Therefore $\|x_0 - y_0\| \leq (\alpha/2 + \gamma)\|x_0 - y_0\|$

$$\left(1 - \frac{\alpha}{2} - \gamma\right)\|x_0 - y_0\| \leq 0$$

Since $\alpha + 2\gamma < 2$

$$\|x_0 - y_0\| = 0 \Rightarrow x_0 = y_0$$

Hence x_0 is the unique fixed point of F, G, H, and J.

This completes the proof of the theorem.

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