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Application of Meijer Transform in typical integral involving $\tilde{H}$ - Function

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ABSTRACT: In this paper a theorem on Meijer transform given by Agrawal has been used to evaluate a typical integral involving $\tilde{H}$ - Function. Our result is quite general in nature and number of known and new formulae can be obtained as applications section.

Keywords: $\tilde{H}$ - Function, Meijer transform.

Mathematical subject classification. 2011, 33C45

I. INTRODUCTION

The $\tilde{H}$- Function, a generalization of fox's H- Function introduced by Inayat [5, P4107-17] and studied by Bushman and Shrivastava [2,P 4707-10] and other is defined and represented in the following manner.

$$\tilde{H}^{M,N}_{P,Q}\left[(pt)\left|\begin{matrix}[(e_j;E_j,\epsilon_j)_{1.N}],[(e_j;E_j)_{N+1,P}]\\ [(f_j,F_j)_{1,M}],[(f_j,F_j,\tau_j)_{M+1,Q}]\end{matrix}\right.\right]=\frac{1}{2\pi\omega}\int_L\theta(\xi)(pt)^{\xi}\,d\xi,$$

$$=\frac{1}{2\pi\omega}\int_L\left\{\frac{\prod_{j=1}^{M}\Gamma(f_j-F_j\xi)\prod_{j=1}^{N}\Gamma(1\text{-}e_j+E_j\xi)^{\in_j}}{\prod_{j=M+1}^{Q}\Gamma(1-f_j+F_j\xi)^{\tau_j}\prod_{j=N+1}^{P}\Gamma(e_j-E_j\xi)}(pt)^{\xi}\right\}d\xi, \tag{1.1}$$

And the contour L is the line form c-ω∞ to c + ω∞ ,……….. to keep the pole of $\Gamma(f_j - F_j\,\xi)$, j = 1,2………..M to the right of the path and the singularities of $\{\Gamma(1\text{-}e_j+E_j\xi)^{E_j}\}$, j = 1,2,………N to the left of the path . The other details about H – Function can be seen in the paper cited earlier evidently, if we take E_j (j = 1,2,……,N) and J_j (j = M=1,………Q) equal to unity, . The H–Function reduced to well known Fox's H – Function [3].

The following sufficient condition for absolute convergence of the integral defined in equation (1.1) have been recently given by Gupta Jain, Agarawal [4,167-172].

$$\left.\begin{array}{l}\text{(i)}\quad |\arg.(pt)| < \tfrac{1}{2}\,\Omega\pi \ \text{ and }\ \Omega > 0,\\ \text{(ii)}\quad |\arg.(pt)| < \tfrac{1}{2}\,\Omega\pi \ \text{ and }\ \Omega \geq 0,\end{array}\right\} \tag{1.2}$$

And (a) $\tau \neq 0$, and contour L is so chosen that

$$\left.\begin{array}{l}\quad (c\zeta + p + 1) < 0,\\ \text{(b)}\quad \tau = 0, \quad \text{and} \quad \tau + 1 = 0,\end{array}\right\} \tag{1.3}$$

Where

$$\Omega = \sum_{j=1}^{M} F_j + \sum_{j=1}^{N} \in_j E_j - \sum_{j=M+1}^{Q} F_j \tau_j - \sum_{j=N+1}^{P} E \tag{1.4}$$

$$\tau = \sum_{j=1}^{N} \in_j E_j + \sum_{j=N+1}^{P} E_j - \sum_{j=1}^{M} F_j - \sum_{j=M+1}^{Q} F_j \tau_j, \tag{1.5}$$

$$\rho = \left(\sum_{j=1}^{M} f_j + \sum_{j=M+1}^{Q} F_j \tau_j, - \sum_{j=1}^{N} e_j \in_j - \sum_{j=N+1}^{P} e_j \right) + \frac{1}{2}\left(\sum_{j=1}^{N} \in_j - \sum_{j=M+1}^{Q} \tau_j + P - M - N \right), \tag{1.6}$$

Meijer transform of the function f(t) denoted by φ(p) defined as .

$$\Phi\,(p) \frac{k}{r}\, f(t) = \int_0^{\infty} (pt)^{1/2}\, k_v(pt)\, f(t)\, dt. \tag{1.7}$$

Which is reduced to well known Laplace transform when $v = \pm 1/2$,

II. THEOREM

Theorem given by Agrawal [1.p77] on Meijer transform as,

If

$$\Phi\ (p) \frac{k}{\lambda+\mu}\ h\ (t), \tag{2.1}$$

And $$\Psi\ (\lambda, p, s)\ \frac{k}{\lambda} k\ (st)\ h\ (t), \tag{2.2}$$

Then

$$\int_0^\infty \left[t^{\frac{1}{2}(\lambda-\mu-\frac{1}{2})} \left(\frac{\alpha+\beta t}{\alpha t+\beta}\right)^{\frac{1}{2}(\lambda+\mu+\frac{1}{2})} \times (\alpha+\beta t)^{-\frac{1}{2}} \left\{ \sqrt{\frac{(\alpha+\beta t)(\alpha t+\beta)}{t}} \right\} \right] dt$$

$$= 2\alpha^{-\frac{1}{2}} \psi\ (\lambda, \mu, \alpha, \beta), \tag{2.3}$$

Provided; Re $(\boldsymbol{\alpha}) > 0$, Re $(\boldsymbol{\beta}) > 0$,

Integral involved are absolutely convergent and h (t) is independent of s.

We shall make the use of this theorem in evaluating integral involving $\tilde{H}$-Function we evaluate,

$$\Phi(p) \frac{k}{\lambda+\mu} \tilde{H}^{M,N}_{P,Q}\left[(pt) \left| \begin{matrix} [(e_j;E_j,\epsilon_j)_{1.N}],[(e_j;E_j)_{N+1.P}] \\ [(f_j,F_j)_{1.M}],[(f_j,F_j,\tau_j)_{M+1.Q}] \end{matrix} \right. \right]$$

$$\Phi(p) = \int_0^\infty \left\{ (pt)^{\frac{1}{2}} k_{\lambda+\mu}(pt) \tilde{H}^{M,N}_{P,Q}\left[(pt) \left| \begin{matrix} [(e_j;E_j,\epsilon_j)_{1.N}],[(e_j;E_j)_{N+1.P}] \\ [(f_j,F_j)_{1.M}],[(f_j,F_j,\tau_j)_{M+1.Q}] \end{matrix} \right. \right] \right\} dt$$

Using the definition of $\tilde{H}$-Function and changing the order of integration,

We get,

$$\Phi(p) = \frac{1}{2\pi\omega} \int_L \theta(p\xi) p^{\frac{1}{2}} \rho^{\xi} \{ \int_0^\infty (t)^{(\xi+\frac{1}{2})-1} k_{\lambda+\mu}(pt) dt \} d\xi$$

Using the result [2.pp 331],

$$\Phi(p) = \frac{1}{2\pi\omega}\int_L Q(\rho\xi)\{p^{\frac{1}{2}}\rho^{\xi}\rho^{-(\xi+\frac{1}{2})}.2^{-(\xi+\frac{3}{2}-2)}\times$$

$$\Gamma(\frac{\xi}{2}+\frac{3}{4}-\frac{\lambda}{2}-\frac{-\mu}{2})\Gamma(\frac{1}{2}\xi+\frac{3}{4}+\frac{1}{2}\lambda+\frac{1}{2}-\mu)\}\,d\xi.$$

Using the definition of $\tilde{H}$ -Function and adjusting, we get,

$$\Phi(p) = 2^{\frac{-1}{2}}p^{-1}\tilde{H}^{M,N+2}_{P+2,Q}[2\rho p^{-1}\left[\begin{matrix}(\frac{1}{4}+\frac{1}{2}\lambda+\frac{1}{2}\mu,\frac{1}{2},1))(\frac{1}{4}-\frac{1}{2}\lambda-\frac{1}{2}\mu,\frac{1}{2},1)[(e_j;E_j,\epsilon_j)_{1.N},[(e_j;E_j)_{N+1.P}]\\ [(f_j,F_j)_{1.M}],[(f_j,F_j,\tau_j)_{M+1.Q}]\end{matrix}\right]$$

Now we shall evaluate,

$$\psi(\lambda,\mu,\rho,s)\frac{k}{\lambda}k_{\mu}(st)\times\tilde{H}^{M.N}_{P.Q}\left[(\rho t)\left|\begin{matrix}[(e_j;E_j,\epsilon_j)_{1.N}],[(e_j;E_j)_{N+1.P}]\\ [(f_j,F_j)_{1.M}],[(f_j,F_j,\tau_j)_{M+1.Q}]\end{matrix}\right.\right]$$

$$\psi(\lambda,\mu,\rho,s) = \frac{1}{2\pi\omega}\int_0^{\infty}\left\{(pt)^{\frac{1}{2}}k_{\lambda}(pt)k_{\mu}(st).\tilde{H}^{M.N}_{P.Q}\left[(\rho t)\left|\begin{matrix}[(e_j;E_j,\epsilon_j)_{1.N}],[(e_j;E_j)_{N+1.P}]\\ [(f_j,F_j)_{1.M}],[(f_j,F_j,\tau_j)_{M+1.Q}]\end{matrix}\right.\right]\right\}dt$$

Using the definition of $\tilde{H}$-Function and changing the order of integration,

We get,

$$\psi(\lambda,\mu,\rho,s) = \frac{1}{2\pi\omega}\int_L \theta(\xi)p^{\frac{1}{2}}\rho^{\xi}\left\{\int_0^{\infty} t^{(\xi+\frac{1}{2})-1}k_{\mu}(st)k_{\lambda}(\rho t)dt\right\}d\xi \qquad (2.4)$$

Using the result [2. pp 334], we get,

$$\psi(\lambda,\mu,\rho,s)=\frac{1}{2\pi\omega}\int_L \theta(\xi)p^{\frac{1}{2}}\rho^{\xi}2^{(\xi-\frac{1}{2})}.\frac{1}{\Gamma(\xi+\frac{3}{2})}\times$$

$$p^{(\xi+\mu+\frac{3}{2})}s^{\mu}.\Gamma(\frac{1}{2}\xi+\frac{3}{4}+\frac{1}{2}\lambda+\frac{1}{2}\mu),\Gamma(\frac{1}{2}\xi+\frac{3}{4}-\frac{1}{2}\lambda+\frac{1}{2}\mu)\times$$

$$\Gamma(\frac{\xi}{2}+\frac{3}{4}+\frac{1}{2}\lambda+\frac{1}{2}\mu),\Gamma(\frac{1}{2}\xi+\frac{3}{4}-\frac{1}{2}\lambda+\frac{1}{2}\mu)\times$$

$$2F_1[\frac{1}{2}(\xi+\frac{3}{2}+\lambda+\mu),\frac{1}{2}(\xi+\frac{1}{2}-\lambda+\mu);(1-\frac{s^2}{p^2})(\xi+\frac{3}{2})]$$

Expanding $2F_1$, adjusting the term and then changing the order of integration and summation, using the definition of $\tilde{H}$ - Function, we get,

$$\psi(\lambda,\mu,\rho,s)=\sum_{k=0}^{\infty}\frac{2^{\frac{-1}{2}}p^{2}s^{\mu}}{\lfloor k}(1-\frac{s^2}{p^2})^{k}.M\ , \tag{2.5}$$

Where,

$$M\equiv\tilde{H}^{M\ .N+6}_{P+6\ .Q+3}\left[2\rho p\ \middle|\ \begin{matrix}U\\V\end{matrix}\right] \tag{2.6}$$

U and V are sets of parameters.

$$V:(\frac{1}{4}-\frac{1}{2}\lambda-\frac{1}{2}\mu,\frac{1}{2},1),(\frac{1}{4}+\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2},1),(\frac{1}{4}-\frac{\lambda}{2}+\frac{\mu}{2},\frac{1}{2},1),$$

$$(\frac{1}{4}+\frac{\lambda}{2}+\frac{\mu}{2},\frac{1}{2},1)(\frac{1}{4}-\frac{\lambda}{2}-\frac{\mu}{2}-k,\frac{1}{2},1),(\frac{1}{4}+\frac{\lambda}{2}-\frac{\mu}{2}-k,\frac{1}{2},1)[(e_j;E_j,\epsilon_j)_{1,N}],[(e_j;E_j)_{N+1,P}]$$

$$V:[(f_j,F_j)_{1,M}],[(f_j,F_j,\tau_j)_{M+1,Q}],[(\frac{1}{4}-\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2},1)(\frac{1}{4}+\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2})(\frac{-1}{2}-k,1)]$$

Now substituting the value of $\Phi\{\sqrt{(\alpha+\beta t)(\alpha t+\beta)/t}\}$ and Ψ (**λ**, μ, α, **β**) in (2.3) ,we get,

$$\int_0^\infty t^{\frac{1}{2}(\lambda-\mu-\frac{1}{2})}\left(\frac{\alpha+\beta t}{\alpha t+\beta}\right)^{\frac{1}{2}(\lambda+\mu+\frac{1}{2})}(\alpha+\beta t)^{\frac{-1}{2}}.\sqrt{\frac{1}{(\alpha+\beta t)(\alpha t+\beta)}}\times$$

$$\tilde{H}^{M.N+2}_{P+2.Q}\left[(2p)\sqrt{\frac{1}{(\alpha+\beta t)(\alpha t+\beta)}}\middle|\begin{matrix}(\frac{1}{4}+\frac{1}{2}\lambda+\frac{1}{2}\mu,\frac{1}{2}),(\frac{1}{4}-\frac{1}{2}\lambda-\frac{1}{2}\mu,\frac{1}{2})[(f_j,F_j)_{1,M}],[(f_j,F_j,\tau_j)_{M+1,Q}]\\ [(e_j.E_j,\epsilon_j)_{1,N}],[(e_jE_j)_{N+1,P}]\end{matrix}\right]dt$$

$$=\sum_{k=0}^{\infty}\frac{\alpha^{\frac{1}{2}}.\beta^{\mu}}{\lfloor k}\left(1-\frac{\beta^2}{\alpha^2}\right)^k\ \tilde{H}^{M.N+6}_{P+6.Q+3}\left[2\rho\alpha\ \middle|\begin{matrix}u\\ v\end{matrix}\right],\tag{2.7}$$

Where u and v are mentioned in (2.6),

Provided Re (α) > 0, Re (β) > 0, integrals involved are absolutely convergent. Where t is a non zero complex variable, L is Mellin Barnes type contour integral. The integral converges if,

|arg. (pt)| < 1/2Ωπ, p≤0, Ω, p are mentioned in (1.4) and (1.6).

III. APPLICATIONS

(i) When $\epsilon_j=\tau_j=1$ (unity) reduce to Fox's H-Function, [3],

$$\int_0^\infty t^{\frac{1}{2}(\lambda-\mu-\frac{1}{2})}\left(\frac{\alpha+\beta t}{\alpha t+\beta}\right)^{\frac{1}{2}(\lambda+\mu+\frac{1}{2})}(\alpha+\beta t)^{-\frac{1}{2}}.\sqrt{\frac{1}{(\alpha+\beta t)(\alpha t+\beta)}}\times$$

$$H^{M,N+2}_{P+2,Q}\left[2\rho\sqrt{\frac{1}{(\alpha+\beta t)(\alpha t+\beta)}}\middle|\begin{matrix}(\frac{1}{4}+\frac{1}{2}\lambda+\frac{1}{2}\mu,\frac{1}{2}),(\frac{1}{4}-\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2})[(e_j,E_j)_{P,N}]\\ [(f_j,F_j)_{Q,M}]\end{matrix}\right]dt$$

$$=\sum_{k=0}^{\infty}\frac{\alpha^{\frac{1}{2}}.\beta^{\mu}}{\lfloor k}\left(1-\frac{\beta^2}{\alpha^2}\right)H^{M.N+6}_{P+6.Q+3}$$

$$\left[2\rho\alpha\middle|\begin{matrix}u[(e_j,E_j)_{P,N}](\frac{1}{4}-\frac{1}{2}\lambda-\frac{1}{2}\mu,\frac{1}{2},1),(\frac{1}{4}+\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2},1),(\frac{1}{4}-\frac{\lambda}{2}+\frac{\mu}{2},\frac{1}{2},1)(\frac{1}{4}+\frac{\lambda}{2}+\frac{\mu}{2},\frac{1}{2},1)(\frac{1}{4}-\frac{\lambda}{2}-\frac{\mu}{2}-k,\frac{1}{2},1),(\frac{1}{4}+\frac{\lambda}{2}-\frac{\mu}{2}-k,\frac{1}{2},1),\\ v,[(\frac{1}{4}-\frac{1}{2}\lambda-\frac{1}{2}\mu,\frac{1}{2},1),(\frac{1}{4}+\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2},1),(\frac{1}{4}-\frac{\lambda}{2}+\frac{\mu}{2},\frac{1}{2},1)][(f_j,F_j,\tau_j)_{Q,M}]\end{matrix}\right],\tag{3.1}$$

(iii) $E_j = F_j = 1$ and $\epsilon_j = \tau_j = 1$ for all value of j then (2.6) reduces to G − function,

$$\int_0^\infty t^{\frac{1}{2}(\lambda-\mu-\frac{1}{2})}\left(\frac{\alpha+\beta t}{\alpha t+\beta}\right)^{\frac{1}{2}(\lambda+\mu+\frac{1}{2})}(\alpha+\beta t)^{-\frac{1}{2}}.\sqrt{\frac{1}{(\alpha+\beta t)(\alpha t+\beta)}}\times$$

$$G^{M,N+2}_{P+2,Q}\left[2\rho\sqrt{\frac{1}{(\alpha+\beta t)(\alpha t+\beta)}}\,\middle|\left((\frac{1}{4}+\frac{1}{2}\lambda+\frac{1}{2}\mu,\frac{1}{2}),(\frac{1}{4}-\frac{\lambda}{2}-\frac{\mu}{2},\frac{1}{2})[(e_j)][(f_j)]\right)\right]dt$$

$$=\sum_{k=0}^{\infty}\frac{\alpha^{\frac{1}{2}}.\beta^{\mu}}{\lfloor k}\left(1-\frac{\beta^2}{\alpha^2}\right)G^{M.N+6}_{P+6.Q+3}\left[2\rho\alpha\,\middle|\,\begin{matrix}u,[(e_j)]_{1.p}\\ [(f_j)],v\end{matrix}\right], \qquad (3.2)$$

Where U and V mentioned in (3.1)

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