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On Einstein Static Universe

M.S. Borkar

Post Graduate, Department of Mathematics,

Mahatama Jyotiba Phule Educational Campus, Amravati Road, R.T.M. Nagpur University, Nagpur, (MS)

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ABSTRACT : In this note, the Einstein line element for a static universe have been examined through its characteristic vectors and discussed its geometrical and physical behavior. It is noticed that the whole universe is controlled by a single mathematical quantity ρ^4. The Einstein line element admit the false vacuum model but does not admit the stiff fluid model (Zel'dovich, 1962) and radiating model, in case of vectors (α_1, β_2), (α_1, β_3), (α_2, ρ_3). It admits all three types of models for the vectors (α_1, β_4), (α_2, β_4), (α_3, β_4). Also it is conformally flat for all vectors.

Keywords : Einstein metric, characteristic vectors and scalars.

I. INTRODUCTION

Many kinds of space-times have been considered in relativistic theories and each of these space-times is defined by using the line element in some special coordinate system. Tolman [1] introduced the line element of the model of the universe:

$$L : ds^2 = e^{-2g(t)}\left(1+\frac{r^2}{4R^2}\right)^{-2}(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2) + dt^2$$

where $1/R^2$ is a constant and developed his theory of relativistic cosmology basing on this line element. He derived this model L of the universe on the assumption of spatial isotropy and comoving coordinate system.

As there is no real justification to assume that the whole universe has the same properties, one can use the homogeneous model of universe in our studies. On the assumption of spherical symmetry (SS) and the pressure is same everywhere in the universe, the static homogeneous model of the universe is derived by Tolman [1] as

$$ds^2 = -\left(1-\frac{r^2}{R^2}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + dt^2 \quad \text{... (1)}$$

where $(\wedge - 8\pi p) = \frac{1}{R^2}$. This model is called Einstein line element for a static universe. The pressure p and density ρ of perfect fluid in the model (1) is given by

$$8\pi p = e^{-\lambda}\left(\frac{v'}{r}+\frac{1}{r^2}\right) - \frac{1}{r^2} + \wedge \quad \text{... (2)}$$

$$8\pi p = e^{-\lambda}\left(\frac{\lambda'}{r}+\frac{1}{r^2}\right) - \frac{1}{r^2} - \wedge \quad \text{... (3)}$$

$$\frac{dp}{dr} = \frac{(\rho+p)}{2}v' \quad \text{... (4)}$$

The equation (2) and (3) gives

$$\wedge = 4\pi(\rho + 3p) \quad \text{... (5)}$$

$$\frac{1}{R^2} = 4\pi(\rho + p) \quad \text{... (6)}$$

Any spherically symmetric space-time (SSST) is characterized by a set of two mutually orthogonal unit vectors α_i, β_i and nine scalars $\rho^a, a = 1, \wedge 5, \sigma, \bar{\sigma}, k, \bar{k}$. The definition is given in Takeno (1966) [2]. These vectors α_i, β_i appears in the definition are called the characteristic vectors and $\rho^a, a = 1, \wedge 5, \sigma, \bar{\sigma}, k, \bar{k}$ are called the characteristic scalars and a set of these quantities is called characteristic system (CS) of SSST. The theory of CS developed by Takeno (1966) [2] is useful to discuss the models of the universe. The concept of duplex of these orthogonal unit vectors α_i, β_i in SSST have been defined and on the basis of it, the SSST is examined by Karade and Borkar [3]. There are sixteen possible duplexes (α_i, β_j) given in [3] out of which four duplexes (α_i, β_i), i = 1, 2, 3, 4, do not exists. In view of these duplexes of vectors, the Robertson - Walker metric have been studied and discussed its geometrical and physical properties by Borkar [4].

In the present paper we take–up the study of Einstein model (1), through its characteristic quantities, as these quantities describes the behavior of the universe. This is the motive of the present work to put whole universe on these mathematical quantities and interpret them physically. It is noticed that all $\rho^a = 0, a = 1, \wedge 5$ leads to the Einstein flat universe in case of vectors $(\alpha_1,\beta_2),(\alpha_1,\beta_3),(\alpha_2,\beta_3)$. Further it degenerate false vacuum model but do not admit bulk-viscous stiff model (Zel'dovich, 1962 [5]) as well as radiating model for the above vectors. Also it is seen that all components of Ricci tensor are zero for the above vector. In case of vectors $(\alpha_1,\beta_4),(\alpha_2,\beta_4),(\alpha_3,\beta_4)$, the Einstein line element admits all the three type of models [6]: vacuum or false vacuums model, bulk-viscous stiff model (Zel'dovich, 1962 [5]) and radiating model. It is also observed that the Einstein model is conformally flat for all above vectors.

For Einstein line element (1), we have

$$\left.\begin{aligned} g_{11} &= -\left(1-\frac{r^2}{R^2}\right)^{-1}, \\ g_{22} &= \frac{g_{33}}{\sin^2\theta} = -r^2, \\ g_{44} &= 1 \text{ other } g_{ij} = 0 \end{aligned}\right] \quad \dots (7)$$

The surviving components of the Riemannian curvature tensorfor Einstein metric (1) are

$$\left.\begin{aligned} K_{1212} &= \frac{K_{1313}}{\sin^2\theta} = -\left(1-\frac{r^2}{R^2}\right)^{-1}\frac{r^2}{R^2}, \\ K_{2323} &= -\frac{r^4}{R^2}\sin^2\theta \end{aligned}\right] \quad \dots (8)$$

II. THE CHARACTERISTIC QUANTITIES

For (α_1,β_2) the components of Riemannian curvature tensor K_{ijlm} (equation (54) of Karade and Borkar (2000) [3]), for the Einstein line element (1), yield

$$\left.\begin{aligned} \rho^1 &= \frac{4}{R^2} \\ \rho^2 &= \rho^3 = \frac{-4}{R^2} = 0 \\ \rho^4 &= \rho^5 = 0 \end{aligned}\right] \quad \dots (9)$$

These results shows that $\frac{1}{R^2}=0$ and all $\rho^a = 0, a = 1,\wedge 5$.

The similar statement can be made by the vectors $(\alpha_1,\beta_3),(\alpha_2,\beta_3)$.

Theorem 1: The vectors $(\alpha_1,\beta_2),(\alpha_1,\beta_3),(\alpha_2,\beta_3)$ admits all zero $\rho's$ for Einstein metric (1).

For (α_1,β_4)

From the equations (70) of [3], for the Einstein line element (1), we write

$$\rho^1 = \rho^2 = \rho^5 = 0 \text{ and } \rho^3 = -2\rho^4 = \frac{4}{R^2} \quad \dots (11)$$

Also the vectors $(\alpha_2,\beta_4)(\alpha_3,\beta_4)$ yield the same results (10).

Theorem 2: For the Einstein metric (1), $\rho^1=\rho^2=\rho^3=\rho^4=\rho^5=0$ if $\frac{1}{R^2}=0$, for the vectors $(\alpha_1,\beta_4),(\alpha_2,\beta_4),(\alpha_3,\beta_4)$.

The surviving components of the Ricci tensor K_{ij} are

$$\left.\begin{aligned} K_{11} &= -2\left(1-\frac{r^2}{R^2}\right)^{-1}\frac{1}{R^2} \\ K_{22} &= \frac{K_{33}}{\sin^2\theta} = \frac{-2r^2}{R^2} \end{aligned}\right] \quad \dots (11)$$

Theorem 3: All components K_{ij} = 0, for $(\alpha_1,\beta_2),(\alpha_1,\beta_3),(\alpha_2,\beta_3)$, since $\frac{1}{R^2}=0$, in case of metric (1).

The conformal curvature tensor C_{ijlm} takes the form (using the definition of K_{ijlm} based on tensorial equations given in Takano (1966) [2])

$$C_{ijlm} = \rho^1\left(-\alpha_{[i}\alpha_{[l}\beta_{j]}\beta_{m]} + \frac{1}{2}g_{[i[\alpha_{j]}}(\alpha_m - \beta_{j]}\beta_{m]}) + \frac{1}{6}g_{[i[l}g_{j]m]}\right)$$

which gives

$$C_{ijlm}\alpha^i\alpha^l\beta^j\beta^m = \frac{-\rho^1}{12} \quad \dots (12)$$

Theorem 4: All components C_{ijlm} = 0, in case of all vectors.

III. GEOMETRICAL AND PHYSICAL INTERPRETATION

From the above discussion, the vectors (α_i,β_j) can be classified into two categories according to the values of $\rho's$:

Category I : The vectors $(\alpha_1,\beta_2),(\alpha_1,\beta_3),(\alpha_2,\beta_3)$ for which all $\rho's$ are zero and $\frac{1}{R^2}=0$.

Category II : The vectors $(\alpha_1,\beta_4),(\alpha_2,\beta_4),(\alpha_3,\beta_4)$ for which $\frac{1}{R^2}\neq 0.$

For the vectors of category *I*, $\left(\frac{1}{R^2}=0\right).$

All $\rho's$ are zero leads Einstein flat universe. The equation (6) reduces $(\rho+p)=0,$ which degenerate vacuum or false vacuum model. Thus Einstein metric goes over to false vacuum model. The density ρ and pressure *p* takes the form

$$\rho=-\frac{1}{8\pi}\wedge,\ p=\frac{1}{8\pi}\wedge \qquad \text{... (13)}$$

For incoherent matter, there is no pressure exerting on the bodies. So that for free particles (stars), $\frac{1}{R^2}=\wedge=0$ and the Einstein universe goes over to the flat space-time of special relativity.

If $p = \rho$, then (6) yield $p = 0$, $\rho = 0$, which shows that Einstein metric does not admit bulk-viscous stiff model (Zel'do vich, 1962 [5]).

If $\rho = 3p$, then again (6) gives $p = 0$ and $\rho = 0$. Thus the model corresponding to radiating matter does not exist. This shows that Einstein metric does not admit radiating model.

The velocity of light is ±1 and time necessary for light to travel from the origin around the universe and back is

$$t=4\int_0^R\frac{dr}{\sqrt{1-(r^2/R^2)}}=2\pi R$$

and it would be infinite in case of vectors of category I.

All components $K_{ij} = 0$ in case of vectors of category I. This agreed with the fact that the Einstein metric (1) is a solution of Einstein empty space-time field equations.

For the vectors of category II *i.e.*, for vectors $(\alpha_1,\beta_4),(\alpha_2,\beta_4),(\alpha_3,\beta_4),$ we have $\rho^1=\rho^2=\rho^5=0$ and $\rho^3=-2\rho^4=\frac{4}{R^2}.$

In term of this value of ρ^4, the equations (5) and (6) becomes

$$(\wedge-8\pi p)=-\rho^4/2 \text{ and } 4\pi(\rho+p)=-\rho^4/2.$$

This suggests that the whole universe is controlled by a single mathematical quantity ρ^4 which has a value $-2/R^2$. The expressions for pressure and density of the material filling the model are

$$4\pi p=\left(\frac{\wedge}{2}+\frac{\rho^4}{4}\right) \text{ and } 4\pi p=-\left(\frac{\wedge}{2}+\frac{3\rho^4}{4}\right) \qquad \text{... (14)}$$

If $\rho^4=0,$ then from above expressions, $(\rho + p) = 0$ degenerate vacuum or false vacuum model. If $\wedge=-\rho^4$, then equation (14) gives $p = \rho$ and then Einstein model becomes bulk-viscous stiff model (Zel'dovich, 1962 [5]). If $\wedge=-\rho^4/2$ then it yield $\rho = 3p$ gives radiating model. In case of incoherent matter, we have

$$4\pi\rho=\frac{1}{R^2}=\wedge=-\rho^4/2$$

The velocity of light is

$$\frac{dr}{dt}=\pm\sqrt{1+(r^2\rho^4/2)}$$

and the time necessary for light to travel from the origin around the universe and back comes out to be $2\pi\sqrt{-2/\rho^4}$.

The Einstein metric is conformally flat in case of all vectors.

The values of the scalars $\sigma,\bar{\sigma},k,\bar{k}$ are compiled in following table:

Scalars → Vectors ↓	σ	$\bar{\sigma}$	k	$\bar{k}$
(α_1,β_2)	0	$-\frac{1}{r}$	$-\frac{1}{r},0$	$-\frac{\cot\theta}{\sqrt{-r^2}},0$
(α_1,β_3)	0	$-\frac{1}{r}$	$-\frac{1}{r},0$	0
(α_1,β_4)	0	0	0	$-\sqrt{\left(1-\frac{r^2}{R^2}\right)}\frac{1}{r}$
(α_2,β_3)	0	$-\cot\theta$	0	0
(α_2,β_4)	0	0	$-\frac{\cot\theta}{r}$	0
(α_3,β_4)	0	0	0	0

IV. CONCLUSIONS

The whole universe is controlled by a single mathematical quantity ρ^4. In case of the vectors of Category I, Einstein line element becomes flat. It admit vacuum or false vacuum model. It does not admit bulk-viscous stiff model (Zel'dovich, 1962 [5]) as well as radiating matter.

For the vectors of category II, Einstein line element admits vacuum or false vacuum model, bulk-viscous stiff model (Zel'dovich, 1962 [5]), and also radiating model depending upon the values of ρ^4. $\rho^4 = 0$ degenerate vacuum or false vacuum model. $\wedge = -\rho^4$ gives bulk-viscous stiff model (Zel'dovich, 1962 [5]). $\wedge = -\rho^4/2$ shows radiating model. In case of incoherent matter, we have $4\pi\rho = \frac{1}{R^2} = \wedge = -\rho^4/2.$

The Einstein metric is conformally flat in case of all vectors.

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