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On Spherically Symmetric Charged Perfect Fluid Distribution in General Relativity

M.H. Hajare* and M.S. Borkar*

**Department of Mathematics,*

Mahatma Jyotiba Phule Educational Campus, Amravati Road, Rashtrasant Tukdoji Maharaj Nagpur University, Nagpur, (MH)

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ABSTRACT : In this note, we have examined the spherically symmetric charged perfect fluid distribution through its characteristic vectors and discussed its geometrical and physical properties. We claim that our results are generalization of the results obtained by Pandey and Tiwari [9]. It is observed that $b_4 = 0$, the acceleration is always directed in radial direction and fluid flow is uniform in t-direction and when $a_1 = 0$, then the expansion is time-dependent only for constant value of $(\rho^3 + 2\rho^4)$. Our results agreed with the results of Pandey and Tiwari [9] that the expansion f is time-dependent only for constant value of $(\rho^3 + 2\rho^4)$.

Keywords : Spherically symmetric space-time, Triples of orthogonal unit vectors charged perfect fluid, Electromagnetic field.

I. INTRODUCTION

The whole universe is spherically symmetric in nature and therefore it is important to study the spherically symmetric space-time (SSST). Many researchers are working on spherically symmetric space-time at national and international level. At international level, Takeno (1966) [1] developed the theory of spherically symmetric space-times (SSST) on the ground of invariant quantities containing two orthogonal unit vectors and deduced its geometrical and physical properties. Karade and Borkar [2] have studied spherically symmetric space-time by assuming various duplexes of orthogonal unit vectors α_i, β_i and discussed its geometrical and physical properties. Further the various aspects of spherically symmetric space-time with physical significance have been discussed by Borkar and Karade [3, 4]. The analytical invariants of orthogonal spherically symmetric space-time have been studied by Borkar and Hajare [5] by assuming the various triples of orthogonal unit vectors α_i, β_i. In it, it is shown that there are 48-number of triples of orthogonal unit vectors α_i and β_i out of which 24 reduces to the duplexes given in Karade and Borkar [2]. The remaining 24-number of non-reducing triples have been studied and discussed their geometrical and physical properties in the note of Borkar and Hajare [5]. For these non-reducing triples, it is observed that the components of curvature tensor K_{ijlm} satisfy the relation $Kspsq = K_{rprq} = K_{rsrs} = K_{pqqp} = 0$, where $p \neq q \neq r \neq s$ and $p, q, r, s = 1, 2, 3, 4$. The detail investigations of general spherically symmetric space-time have been carried-out by using these triples by Borkar and Hajare [6, 7]. These triples are playing an important role in analyzing the space-time and in the discussion of physical and geometrical properties of space-time. Therefore in this paper, an attempt has been made to examine the spherically symmetric charged perfect fluid distribution through the triples of its orthogonal unit vectors α_i and β_i.

A solution of Einstein's field equations have been obtained by Roy and Bali [8] which is known as conformally flat non-static spherically symmetric perfect fluid distributions in general relativity. Thereafter Pandey and Tiwari [9] have solved Einstein's field equations and obtained conformally flat spherically symmetric charged perfect fluid distribution in general relativity. In this note, we consider the conformally flat spherically symmetric charged perfect fluid in general relativity [9]), in our discussion and studied it through the theory of triples of orthogonal unit vectors developed by Borkar and Hajare [5].

We consider the line-element of Pandey and Tiwari [9],

$$ds^2 = \frac{1}{(a+b)^2} \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \ \ d\phi^2 - dt^2 \right) \quad \text{... (1)}$$

where a is function of r only and b is function of t only and the pressure and density of the fluid in it are

$$8\pi p = 3 \left(a_1^2 - b_4^2 \right) + (a+b) \left(2b_{44} - a_{11} - \frac{3a_1}{r} \right) \quad \text{... (2)}$$

$$8\pi \in = 3 \left(b_4^2 - a_1^2 \right) + 3 \ (a+b) \left(a_{11} + \frac{a_1}{r} \right) \quad \text{... (3)}$$

(Here and hereafter the subscript-1 and subscript-4 denotes the derivatives with respect to r and t respectively).

For the completeness, we re-write the results of Pandey and Tiwari [9] as follows:

The flow vector v^4 is given by

$$v_4 = \frac{1}{(a+b)} \quad \text{... (4)}$$

The reality conditions $\in + p > 0$ and $\in + 3p > 0$ are

$$(b_{44} + a_{11}) > 0$$

and $$\left(a_1^2 - b_4^2 \right) + (a+b) \left(2b_{44} - \frac{a_1}{r} \right) > 0 \quad \text{... (5)}$$

The non-vanishing component of electromagnetic field tensor F_{ij} in the model (1) is given by

$$F_{14} = (a+b)^{-3/2} \left(\frac{a_1}{r} - a_{11} \right)^{1/2} \quad \text{... (6)}$$

The current density ρ is

$$\rho = -\frac{(a+b)^3}{r^2} \frac{\partial}{\partial r}\left[r^2 (a+b)^{-3/2} \left(\frac{a_1}{r} - a_{11} \right)^{1/2} \right] \text{... (7)}$$

The non-vanishing component of acceleration vector $\dot{v}_i = v_{i|j} v^j$ is

$$\dot{v}_1 = \frac{-a_1}{(a+b)} \quad \text{... (8)}$$

(Here vertical bar '|' denotes covariant derivative).

The expression for expansion ϕ , rotation and shear tensor are

$$\phi = v^i_{|i} \quad \text{... (9)}$$

$$-\omega_{ij} = \frac{1}{2}\left(v_{i|j} - v_{j|i} \right) + \frac{1}{2}\left(\dot{v}_i v_j - \dot{v}_j v_i \right) \quad \text{... (10)}$$

$$\sigma_{ij} = \frac{1}{2}\left(v_{i|j} + v_{j|i} \right) + \frac{1}{2}\left(\dot{v}_i v_j + \dot{v}_j v_i \right) - \frac{\phi}{3}\left(g_{ij} + v_i v_j \right) \quad \text{... (11)}$$

respectively. Pandey and Tiwari [9], in their note shown that the expansion has the value $\phi = 3b_4$, which is time-dependent only and all components of rotation and shear are zero.

The purpose of this paper is to connect these physical quantities with the mathematical quantities ρ's given in Borkar and Hajare [5] on the basis of triples of orthogonal unit vectors α_i and β_i and seeking the relation between them and discuss geometrical and physical properties of space-time. For completeness we recall the definition of triples of orthogonal unit vectors α_i and β_i (given in [5]) as

Definition: A set $\{\alpha_i, \beta_i\}$ of orthogonal unit vectors α_i, β_j in SSST is called triple if any two different components of α_i are nonzero with only one nonzero component of β_i or there is only one nonzero component of α_i with any two different nonzero components of β_i. It is denoted by $(\beta_p, \alpha_q, \beta_r)$ or $(\alpha_r, \beta_p, \beta_q)$. $p, q, r = 1, 2, 3, 4$, $p \neq q$.

There are total 48-number of triples of orthogonal unit vectors as

$$\begin{aligned}
&(\alpha_1,\alpha_2,\beta_1),(\alpha_1,\alpha_2,\beta_2),(\alpha_1,\alpha_2,\beta_3),(\alpha_1,\alpha_2,\beta_4),(\alpha_1,\alpha_3,\beta_1),(\alpha_1,\alpha_3,\beta_2),\\
&(\alpha_1,\alpha_3,\beta_3),(\alpha_1,\alpha_3,\beta_4),(\alpha_1,\alpha_4,\beta_1),(\alpha_1,\alpha_4,\beta_2),(\alpha_1,\alpha_4,\beta_3),(\alpha_1,\alpha_4,\beta_4),\\
&(\alpha_2,\alpha_3,\beta_1),(\alpha_2,\alpha_3,\beta_2),(\alpha_2,\alpha_3,\beta_3),(\alpha_2,\alpha_3,\beta_4),(\alpha_2,\alpha_4,\beta_1),(\alpha_2,\alpha_4,\beta_2),\\
&(\alpha_2,\alpha_4,\beta_3),(\alpha_2,\alpha_4,\beta_4),(\alpha_3,\alpha_4,\beta_1),(\alpha_3,\alpha_4,\beta_2),(\alpha_3,\alpha_4,\beta_3),(\alpha_3,\alpha_4,\beta_4),\\
&(\alpha_1,\beta_1,\beta_2),(\alpha_2,\beta_1,\beta_2),(\alpha_3,\beta_1,\beta_2),(\alpha_4,\beta_1,\beta_2),(\alpha_1,\beta_1,\beta_3),(\alpha_2,\beta_1,\beta_3),\\
&(\alpha_3,\beta_1,\beta_3),(\alpha_4,\beta_1,\beta_3),(\alpha_1,\beta_1,\beta_4),(\alpha_2,\beta_1,\beta_4),(\alpha_3,\beta_1,\beta_4),(\alpha_4,\beta_1,\beta_4).
\end{aligned} \quad (12)$$

out of which 24-triples namely

$$\begin{aligned}
&(\alpha_1,\alpha_2,\beta_1),(\alpha_1,\alpha_2,\beta_2),(\alpha_1,\alpha_3,\beta_1),(\alpha_1,\alpha_3,\beta_3),(\alpha_1,\alpha_4,\beta_1),(\alpha_1,\alpha_4,\beta_4),\\
&(\alpha_2,\alpha_3,\beta_2),(\alpha_2,\alpha_3,\beta_3),(\alpha_2,\alpha_4,\beta_2),(\alpha_2,\alpha_4,\beta_4),(\alpha_3,\alpha_4,\beta_3),(\alpha_3,\alpha_4,\beta_4),\\
&(\alpha_1,\beta_1,\beta_2),(\alpha_2,\beta_1,\beta_2),(\alpha_1,\beta_1,\beta_3),(\alpha_3,\beta_1,\beta_3),(\alpha_1,\beta_1,\beta_4),(\alpha_4,\beta_1,\beta_4),\\
&(\alpha_2,\beta_2,\beta_3),(\alpha_3,\beta_2,\beta_3),(\alpha_2,\beta_2,\beta_4),(\alpha_4,\beta_2,\beta_4),(\alpha_3,\beta_3,\beta_4),(\alpha_4,\beta_3,\beta_4).
\end{aligned} \quad (13)$$

reduces to duplex of orthogonal unit vector α_i and β_i (for details one may refer Borkar and Hajare [5]). For the sake of convenience, we recall the definition of duplex of orthogonal unit vector α_i and β_i (given in [2]) as.

Definition: A pair (α_p, β_q) of orthogonal unit vectors α_p, β_q in SSST is called duplex if only nonzero component of α_i is its p^{th} component and the only nonzero component of β_j is its q^{th} component. It is denoted by (α_p, β_q); $p, q = 1, 2, 3, 4$.

The remaining 24 non-reducing triples of orthogonal unit vectors from (12) are

$$\begin{aligned}
&(\alpha_1,\alpha_2,\beta_3),(\alpha_1,\alpha_2,\beta_4),(\alpha_1,\alpha_3,\beta_2),(\alpha_1,\alpha_3,\beta_4),(\alpha_1,\alpha_4,\beta_2),(\alpha_1,\alpha_4,\beta_3),\\
&(\alpha_2,\alpha_3,\beta_1),(\alpha_2,\alpha_3,\beta_4),(\alpha_2,\alpha_4,\beta_1),(\alpha_2,\alpha_4,\beta_3),(\alpha_3,\alpha_4,\beta_1),(\alpha_3,\alpha_4,\beta_2),\\
&(\alpha_3,\beta_1,\beta_2),(\alpha_4,\beta_1,\beta_2),(\alpha_2,\beta_1,\beta_3),(\alpha_4,\beta_1,\beta_3),(\alpha_2,\beta_1,\beta_4),(\alpha_3,\beta_1,\beta_4),\\
&(\alpha_1,\beta_2,\beta_3),(\alpha_4,\beta_2,\beta_3),(\alpha_1,\beta_2,\beta_4),(\alpha_3,\beta_2,\beta_4),(\alpha_1,\beta_3,\beta_4),(\alpha_2,\beta_3,\beta_4).
\end{aligned} \quad (14)$$

which are playing the important role in the study of spherically symmetric charged perfect fluid distribution and in the discussion of its geometrical and physical properties. This is the motive of present work to study the geometrical and physical properties of the space-time (1) in view of triples of orthogonal unit vector α_i and β_i, in this note.

Out of these 24 triples, four triples namely $(\alpha_1, \alpha_4, \beta_2)$, $(\alpha_1, \alpha_4, \beta_3)$, $(\alpha_2, \beta_1, \beta_4)$, $(\alpha_3, \beta_1, \beta_4)$ do not exist. According to the behaviour of characteristic scalar ρ's (given in [2,5]), the remaining 20 triples can be classified into three categories.

Category I : $(\alpha_1,\alpha_2,\beta_3),(\alpha_1,\alpha_2,\beta_4),(\alpha_1,\alpha_3,\beta_2),(\alpha_1,\alpha_3,\beta_4),$
$(\alpha_3,\beta_1,\beta_2),(\alpha_4,\beta_1,\beta_2),(\alpha_2,\beta_1,\beta_3),(\alpha_4,\beta_1,\beta_3).$

Category II : $(\alpha_2,\alpha_4,\beta_1),(\alpha_2,\alpha_4,\beta_3),(\alpha_3,\alpha_4,\beta_1),(\alpha_3,\alpha_4,\beta_2),$
$(\alpha_1,\beta_2,\beta_4),(\alpha_3,\beta_2,\beta_4),(\alpha_1,\beta_3,\beta_4),(\alpha_2,\beta_3,\beta_4).$

Category III : $(\alpha_2,\alpha_3,\beta_1),(\alpha_2,\alpha_3,\beta_4),\ (\alpha_1,\beta_2,\beta_3),(\alpha_4,\beta_2,\beta_3).$

II. Characteristic Quantities:

We are studying the space-time (1), in relation to the triples corresponding to the different categories I, II, III of vectors. For the line-element (1), the components of curvature tensor K_{ijlm} are

$$K_{1212}=\frac{ra_1}{(a+b)^3}-\frac{r^2a_1^{\,2}}{(a+b)^4}+\frac{r^2b_4^{\,2}}{(a+b)^4}+\frac{r^2a_{11}}{(a+b)^3}$$

$$K_{1313}=\sin^2\theta K_{1212}$$

$$K_{1414}=\frac{a_1^{\,2}}{(a+b)^4}-\frac{a_{11}}{(a+b)^3}-\frac{b_4^{\,2}}{(a+b)^4}+\frac{b_{44}}{(a+b)^3}$$

$$K_{2323}=\sin^2\theta\left[\frac{2r^3a_1}{(a+b)^3}-\frac{r^4a_1^{\,2}}{(a+b)^4}-\frac{r^4b_4^{\,2}}{(a+b)^4}\right]$$

$$K_{2424}=\frac{r^2a_1^{\,2}}{(a+b)^4}-\frac{ra_1}{(a+b)^3}-\frac{r^2b_4^{\,2}}{(a+b)^4}+\frac{r^2b_{44}}{(a+b)^3}$$

$$K_{3434}=\sin^2\theta K_{2424}$$

$$K_{2124}=\frac{r^2a_1b_4}{(a+b)^3}-\frac{r^2a_1b_4}{(a+b)^4}\quad K_{3134}=\sin^2\theta K_{2124}\qquad \text{... (15)}$$

The components of curvature tensor K_{ijlm} in view of the definition of SSST given in [1, 5] for the line-element (1) for the triple $(\alpha_1, \alpha_2, \beta_3)$ of category I are

$$4K_{1212}=\frac{r^2}{(a+b)^4}\left(\overset{2}{\rho}+2\overset{4}{\rho}\right)$$

$$4K_{1313}=\frac{r^2\sin^2\theta}{(a+b)^4}\left[\frac{r^2}{1+r^2}\left(\overset{1}{\rho}+\overset{2}{\rho}\right)+\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)\right]$$

$$4K_{1414}=-\frac{1}{(a+b)^4}\left[\frac{r^2}{1+r^2}\overset{2}{\rho}+2\overset{4}{\rho}\right]$$

$$4K_{1213}=\frac{r^2\sin\theta}{2(a+b)^4\sqrt{-(1+r^2)}}\overset{5}{\rho}$$

$$4K_{2123}=\frac{r^4\sin\theta}{2(a+b)^4\sqrt{-(1+r^2)}}\overset{5}{\rho}$$

$$4K_{2323}=\frac{r^4\sin^2\theta}{(a+b)^4}\left[\frac{1}{1+r^2}\left(\overset{1}{\rho}+\overset{2}{\rho}\right)+\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)\right]$$

$$4K_{2424}=-\frac{r^2}{(a+b)^4}\left[\frac{1}{1+r^2}\overset{2}{\rho}+2\overset{4}{\rho}\right]$$

$$4K_{3132}=\frac{r^4\sin^2\theta}{(1+r^2)(a+b)^4}\left(\overset{1}{\rho}+\overset{2}{\rho}\right)$$

$$4K_{3434}=\frac{-r^2\sin^2\theta}{(a+b)^4}\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)$$

$$4K_{4142}=\frac{-r^2}{2(1+r^2)(a+b)^4}\overset{2}{\rho}$$

$$4K_{4143}=\frac{-r^2\sin\theta}{2(a+b)^4\sqrt{-(1+r^2)}}\overset{5}{\rho}$$

$$4K_{4243}=\frac{-r^2\sin\theta}{2(a+b)^4\sqrt{-(1+r^2)}}\overset{5}{\rho}\qquad \text{... (16)}$$

From (15) and (16), by straightforward calculation, we obtained

$$\overset{1}{\rho}=\overset{2}{\rho}=\overset{5}{\rho}=0$$

$$\overset{3}{\rho}+2\overset{4}{\rho}=\frac{-4(a+b)^4}{r^2\sin^2\theta}K_{3434}$$

$$=4\left[\left(b_4^{\,2}-a_1^{\,2}\right)+(a+b)\left(\frac{a_1}{r}-b_{44}\right)\right]\qquad \text{... (17)}$$

For the remaining triples of category I, we get the same results (17).

Theorem 1: For the triples of category I, for the space-time (1), we have

$$\overset{1}{\rho}=\overset{2}{\rho}=\overset{5}{\rho}=0$$

$$\overset{3}{\rho}+2\overset{4}{\rho}=4\left[\left(b_4^{\,2}-a_1^{\,2}\right)+(a+b)\left(\frac{a_1}{r}-b_{44}\right)\right]$$

For the triples of category II, likewise we can deduced the following relations:

$$\overset{1}{\rho}=\overset{2}{\rho}=\overset{5}{\rho}=0$$

$$\overset{3}{\rho}+2\overset{4}{\rho}=4\left[\left(b_4^{\,2}-a_1^{\,2}\right)+(a+b)\left(\frac{a_1}{r}+a_{11}\right)\right]\qquad \text{... (18)}$$

Theorem 2: For the space-time (1), for the triples of category II, we have

$$\overset{1}{\rho} = \overset{2}{\rho} = \overset{5}{\rho} = 0$$

$$\overset{3}{\rho} + 2\overset{4}{\rho} = 4 \left[\left(b_4^2 - a_1^2 \right) + (a+b) \left(\frac{a_1}{r} + a_{11} \right) \right]$$

Theorem 3: For the triples of category III, for the space-time (1), we write

$$\overset{1}{\rho} = \overset{2}{\rho} = \overset{5}{\rho} = 0$$

$$\overset{3}{\rho} + 2\overset{4}{\rho} = 4 \left[\left(b_4^2 - a_1^2 \right) + (a+b) \left(a_{11} - b_{44} \right) \right] \quad \text{... (19)}$$

Let us write the results of pressure, density and other physical quantities, in view of category wise vectors.

For the vectors of category I

Using equation (17), the equations (2) and (3) give the pressure and density as

$$8\pi p = -\frac{3}{4} \left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) - (a+b) \left(a_{11} + b_{44} \right) \quad \text{... (20)}$$

$$8\pi \in = \frac{3}{4} \left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) + 3 (a+b) \left(a_{11} + b_{44} \right) \quad \text{... (21)}$$

respectively.

The reality conditions $\in + p > 0$ and $\in + 3p > 0$ lead

$$\left(b_{44} + a_{11} \right) > 0 \text{ and } -\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) > 0 \text{ or } \left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) > 0. \quad (22)$$

From (6), the component of electromagnetic field tensor F_{14} have the form

$$F_{14} = \frac{1}{2(a+b)^2} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) - 4\left(b_4^2 - a_1^2 \right) + 4(a+b)\left(b_{44} - a_{11} \right) \right]^{1/2} \quad 23$$

The current density ρ (using (7) becomes

$$\rho = \frac{(a+b)}{4} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) - 4(b_4^2 - a_1^2) + 4(a+b)(b_{44} - a_{11})^{1/2} \right] \left[\frac{3a_1}{(a+b)} + \frac{a_{111}}{\left(\frac{a_1}{r} - a_{11} \right)} - \frac{3}{r} \right] \quad (24)$$

The acceleration vector $\dot{v}_1$ (from equation (8), will be

$$\dot{v}_1 = \frac{-r}{4(a+b)^2} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) + 4 \left(a_1^2 - b_4^2 \right) + 4 (a+b) b_{44} \right] \quad (25)$$

From the expression (9), the expansion ϕ of the model takes the value

$$\phi = \frac{3}{4} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) - 4(a+b) \left(\frac{a_1}{r} - b_{44} \right) + 4a_1^2 \right]^{1/2} \quad \text{... (26)}$$

The components of rotation ω_{ij} (from (10) and its shear σ_{ij} (from (11) all goes to zero.

For the triples of the category II

We deduced, the pressure (p) and density $(\in)$ as

$$8\pi p = -\frac{3}{4} \left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) + 2(a+b)(a_{11} + b_{44}) \quad \text{... (27)}$$

$$8\pi \in = \frac{3}{4} \left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) \quad \text{... (28)}$$

The reality conditions $\in + p > 0$ and $\in + 3p > 0$ give $\left(b_{44} + a_{11} \right) > 0$

and $$-\frac{3}{2} \left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) + 6(a+b)\left(a_{11} + b_{44} \right) > 0 \quad \text{... (29)}$$

Electromagnetic field tensor F_{14} is

$$F_{14} = \frac{1}{2(a+b)^2} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) - 4\left(b_4^2 - a_1^2 \right) - 8a_{11}(a+b) \right]^{1/2} \quad (30)$$

The current density ρ becomes

$$\rho = \frac{(a+b)}{4} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) - 4\left(b_4^2 - a_1^2 \right) - 8a_{11} \right]^{1/2} \left[\frac{3a_1}{(a+b)} + \frac{a_{111}}{\left(\frac{a_1}{r} - a_{11} \right)} - \frac{3}{r} \right] \quad \text{... (31)}$$

The acceleration vector becomes

$$\dot{v}_1 = \frac{-r}{4(a+b)^2} \left[\left(\overset{3}{\rho} + 2\overset{4}{\rho} \right) + 4\left(a_1^2 - b_4^2 \right) - 4a_{11} \right] \quad \text{... (32)}$$

The expression for expansion ϕ, rotation and shear tensor are

$$\phi\sqrt{b_4} = \frac{3}{4}\left[\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)b_4 - 4(a+b)\left(\frac{a_1}{r}+a_{11}\right)b_4 + 4b_4a_1^2\right]^{1/2} \quad (33)$$

$$\omega_{ij} = 0$$

$\sigma_{ij} = 0$ respectively.

For the triples of the category III

The pressure and density are given by

$$8\pi p = -\frac{3}{4}\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) + (a+b)\left(2a_{11} - b_{44} - \frac{3a_1}{r}\right) \quad \text{... (34)}$$

$$8\pi \in = \frac{3}{4}\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) + 3\ (a+b)\left(b_{44} + \frac{a_1}{r}\right) \quad \text{... (35)}$$

The reality conditions $\in + p > 0$ and $\in + 3p > 0$ give $(b_{44} + a_{11}) > 0$

and $$-\frac{3}{2}\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) + 3\ (a+b)\left(2a_{11} - \frac{2a_1}{r}\right) > 0 \quad \text{... (36)}$$

Components of electromagnetic field tensor F_{14} is

$$F_{14} = \frac{1}{2(a+b)^2}\left[\begin{matrix} -\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) + \left(b_4^2 - a_1^2\right) \\ +4(a+b)\left(\frac{a_1}{r} - b_{44}\right)\end{matrix}\right]^{1/2} \quad (37)$$

The current density ρ becomes

$$\rho = \frac{(a+b)}{4}\left[-\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) + \left(b_4^2 - a_1^2\right) + 4(a+b)\left(\frac{a_1}{r} + b_{44}\right)\right]^{1/2}$$

$$\left[\frac{3a_1}{(a+b)} + \frac{a_{111}}{\left(\frac{a_1}{r} - a_{11}\right)} - \frac{3}{r}\right] \quad \text{... (38)}$$

The acceleration vector is

$$\dot{v}_1 = \frac{-1}{4(a+b)^2}\left[-\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) + 4b_4^2 + 4(a+b)\left(a_{11} - b_{44}\right)\right] \quad \text{... (39)}$$

The expression for expansion ϕ, rotation and shear tensor are

$$\phi = \frac{3}{4}\left[\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) - 4(a+b)\left(\frac{a_1}{r} - b_{44}\right) + 4a_1^2\right]^{1/2} \quad \text{... (40)}$$

$$\omega_{ij} = 0$$

$\sigma_{ij} = 0$ respectively.

III. GEOMETRICAL AND PHYSICAL FEATURES:

In the model (1), the mathematical quantity $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)$ plays an important role to study the geometrical and physical properties of the model. From the equations (20) and (21), it is observed that the considered model (1), in general relativity exists, when

$$-4(a+b)\left(a_{11}+b_{44}\right) < \left(\overset{3}{\rho}+2\overset{4}{\rho}\right) < -\frac{4}{3}(a+b)\left(a_{11}+b_{44}\right) \quad (41)$$

There is an acceleration in both direction radial as well as time and it is zero when

$$\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) = -4\left[\left(a_1^2 - b_4^2\right) + (a+b)b_{44}\right] \quad \text{... (42)}$$

Further, the model is expanding in *x*-direction with respect to time.

For the vector of category I, It is seen that, $p = 0$, $\in = 0$, when $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) = -\frac{4}{3}(a+b)\left(a_{11}+b_{44}\right)$

and $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) = -4(a+b)\left(a_{11}+b_{44}\right)$ respectively. This gives vacuum model for the above values of $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)$.

For the vectors of category II, charged perfect fluid distribution model (1) exists, when

$$0 < \left(\overset{3}{\rho}+2\overset{4}{\rho}\right) < \frac{8}{3}(a+b)\left(a_{11}+b_{44}\right) \quad \text{... (43)}$$

and acceleration of fluid flow is in radial as well as time direction and it is zero when

$$\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) = 4\left[\left(b_4^2 - a_1^2\right) + a_{11}\right] \quad \text{... (44)}$$

The model is expanding in *x*-direction without shearing and rotating.

It is observed that for the vector of category II, we have $p = 0$, $\in = 0$, when $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right) = \frac{8}{3}(a+b)\left(a_{11}+b_{44}\right)$

and $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)=0$ respectively. This shows that model (1) is vacuum for the above values of $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)$.

For the vectors of category III, the model (1) exists, for

$$-4(a+b)\left(b_{44}+\frac{a_1}{r}\right)<\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)<\frac{4}{3}(a+b)\left(2a_{11}-b_{44}-\frac{3a_1}{r}\right) \quad \text{... (45)}$$

The acceleration is directed in radial as well as in time direction, and it is zero when

$$\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)=4\left[(a+b)\left(a_{11}-b_{44}\right)+b_4{}^2\right] \quad \text{... (46)}$$

Further, this model is expanding in *x*-direction with respect to time.

For these vectors of category III, we find $p = 0$, $\in = 0$, when $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)=\frac{4}{3}(a+b)\left(2a_{11}-b_{44}-\frac{3a_1}{r}\right)$

and $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)=-4(a+b)\left(b_{44}+\frac{a_1}{r}\right)$ respectively, from which it is clear that the model is vacuum for the above values of $\left(\overset{3}{\rho}+2\overset{4}{\rho}\right)$.

It is realized that our results are generalizations of the results obtained by Pandey and Tiwari [9]. When $b_4 = 0$, the acceleration is always directed in radial direction and fluid flow is uniform in t-direction and when $a_1 = 0$ then the expansion is time-dependent only for constant value of $(\rho^3 + 2\rho^4)$. This agreed with the remark of Pandey and Tiwari [9].

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