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MINIMIZING CARDINALITY OF THE SET OF RASTER POINTS FOR ORTHOGONAL PACKING PROBLEM

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Abstract. We consider the *N-dimensional orthogonal packing feasibility problem* (OPP-N). Given a set of N-dimensional rectangular items, OOP-N is to decide whether all items can be orthogonally packed into the given rectangular container. We construct a method which finds equivalent packing for given orthogonal packing under certain criterion.

Keywords: orthogonal packing problem; raster points; linear programming.

1. INTRODUCTION

We consider following problem: m N-dimensional parallelepiped items $R = \{R_i\}$, $i \in \{1,\dots,m\}$ with sizes $R_i = (r_i^1,\dots,r_i^N)$ are to be packed into a container $\widetilde{S} = (\widetilde{S}_1,\dots,\widetilde{S}_N)$, which is also N-dimensional orthogonal parallelepiped. The *N-dimensional orthogonal packing feasibility problem* (OPP-N) asks whether all the items R can be orthogonally packed into the container $\widetilde{S}$ without rotations. This problem is a main procedure of solving many discrete optimization problems [1,2,3]. Such as cutting and packing problems, resource-constrained project scheduling problem etc. All these problems are NP-hard.

The approach of F. Clautiaux [4] gives on the best computational results at this time. Author investigates the raster model of the orthogonal packing problem, which was represented as integer linear programming problem with pseudopolinomial number of variables. It is noticed, that effectively of algorithm working depends on both the number of considered items and the ratio of their sizes. For example, the solution of the problem with sizes *R₁={(20,20),(21,21),(22,22),(23,23)}* and $\widetilde{S}$ = *(50,50)* from the algorithmic point of view simpler than the solution of the problem with *R₁={(20,20),(20,20),(20,20),(20,20)}* and $\widetilde{S}$ = *(50,50)*. It can be explained by the fact that on the first case has many symmetric variants in contrast to the second case.

2. EQUIVALENT SETS

Lets given set *(l,L),* consist of vector $l \in R_+^n$ and number $L \in R_+$. Define the set of binary vectors

$$P_{\le}(l,L) := \{a : \sum a_i l_i \le L, a \in \{0,1\}^n\},$$

$$P_{>}(l,L) := \{a : \sum a_i l_i > L, a \in \{0,1\}^n\}.$$

Using this definition, we can take the definition of the equivalent sets from [3].

Definition 1. Set (l,L) is equivalent to $(\hat{l},\hat{L})$ if $P_{\le}(l,L) = P_{\le}(\hat{l},\hat{L})$.

Lets known some vector $w \in R_+^n$. Consider the 0-1 knapsack problem, where L is the size

$$K(l,w,L) = \max(\sum a_i w_i, a \in P_{\le}(l,L)).$$

The following theorem was proved:

Theorem 1. The set (l,L) is *equivalent* the set $(\hat{l},\hat{L})$ if and only if $K(l,\hat{l},L) < \hat{L}$ and $K(\hat{l},l,\hat{L}) < \hat{L}$.

Theorem 1 gives simple condition to verify the equivalence of two any sets.

In case, when $L = \hat{L}$ is equivalent, the set of all possible sets, which is equivalent to (l,L) is represented as:

$$\prod(l,L) := \{\pi \in R_+^n : \sum \pi_i a_i \le L,\ \sum \pi_i \breve{a}_i > L,$$
$$a \in P_{\le}(l,L),\ \breve{a} \in P_{>}(l,L)\}.$$

The set $\prod(l,L)$ is convex polytope and elements of it, may be given with using the methods of linear programming generation on the base of the column generation method. Similar conversion for different type of integer programming problem is discussed in [5].

The reduced raster points set

Definition 1. The reduced raster points set for (l, L) is

$$R(L, \mathbf{l}) = \left\{ \sum_{j=1}^{m} l_j a_j : \sum_{j=1}^{m} l_j a_j \le L, \mathbf{a} \in \{0,1\}^m \right\}.$$

Example 1.

Let be $L = 20$ $n = 10$ $l = \{4, 5, 6, 7, 7, 8, 11, 13, 13, 14\}$ then $R(L,l) = \{0, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$ and $|R(L,l)| = 18$.

Using a fitness function to set $\prod(l, L)$ we can generate an equivalent size with certain properties.

For example, the a fitness functions are follow:

1) $k^1 = \min(\max)(\sum_i \pi_i : \pi \in \Pi(l, L))$ is minimal (maximal) total size.

2) $k^2 = \min(\pi_1 - \pi_N : \pi \in \Pi(l, L))$ is minimal variance of lengths.

3) $k^3 = \min(\hat{L} : \pi \in \Pi(l, L), \pi \in Z_+^n)$ is minimal length of the object L where sizes of items are integer.

Example 2.

Given: $L = 20$, $n = 10$, $l = \{ 4, 5, 6, 7, 7, 8, 11, 13, 13, 14 \}$.

The reduce raster points is $R(L, \mathrm{l}) = \{ 0, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20 \}$ and $|R(L, \mathrm{l})| = 18$.

Using the fitness function k^3, we have a new items size

$L = 14$, $n = 10$, $l = \{3, 3, 4, 5, 5, 6, 8, 9, 9, 10 \}$.

$R(L, \mathrm{l}) = \{0, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$, $|R(L, \mathrm{l})| = 13$.

The orthogonal packing problem

Lets $r^j = (r_j^1, \ldots, r_j^m)$.

Defenition 2. We will say, that problem $(R, \widetilde{S})$ is equivalent to $(\hat{R}, \widehat{S})$, if for $i = \overline{1, N}$ the sets $(r^i, \widetilde{S}_i)$ and $(\widehat{r}^i, \widehat{S}_i)$ is equivalent.

Theorem 2. If problems $(R, \widetilde{S})$ and $(\hat{R}, \widehat{S})$ is equivalent, then the solution of one of them may be transformed to the solution of another one.

From the theorem 2 follows, that the sets (1) may be used for the constructing of equivalent problems. It allows to choose the most successful problem statement from the class of equivalence by using the addition of objective function to the restrictions (1). For example, the criterion of choice may have the following representation:

Computational results

For the testing was generated the set of following instances for three-dimensional case ($N=3$): container is represented by cube with sizes $S_1 = S_2 = S_3 = 1000$. The number of items m is belong to the set {10, 15}, approximate waste value e is from the {0 %, 2 %, ..., 40 %}. Maximal ratio $r_{\max}$ between dimensions of each item is belong to the set {1, 3, 20}. Obviously, if $r_{\max} = 1$, then we have a set of cubes. For each class (m, e, $r_{\max}$) was generated ten instances. For more detail of generation method see [1].

Table 1. Minimizing the number of the raster points (k^3)

Type of tests	Number of inst.	Num. of inst where min RRP	Number of RRP before / after
Claut.	672	56	1.01
LCPS	1260	24	1.001
$n = 10$	900	850	1.447
$n = 15$	900	6	1.001

RRP is reduced raster points.

For generated test set was obtained the following results.

1. The main aim for the first criterion (the maximization of the total sum of elements in each dimension) was getting the equivalent problem with the bigger total volume of items.
   The lower bound by volume for the obtained problem was increased (from 0 till 1) for **20.5 percents** of tests.
2. The main aim for second criterion (the minimization of variance of items) was getting the equivalent problem with the less number of raster point. (Table 1)

4. CONCLUSION

The method, proposed in this paper, allows building the set of equivalent problem for the given one. Thereby the opportunity of getting the most successful statement of the problem from the equivalent class was obtained. Also the criterion of choice for input data was proposed.

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