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Amplitude variation in fractional multiple-interference on N-beams

Variación de la amplitud de interferencia múltiple fraccional de N-haces

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ABSTRACT

Multiple periodic patterns by light interference are a complicated problem from an experimental point of view. We report a theoretical model of the incidence of N beams of light at one point in space. Two components of the electric field are projected: azimuthal (angle formed with the projection of the vector in space on the *x-y* plane) and zenithal (angle of the incident vector with the *z*-axis). As a result, azimuthal intensity and zenithal intensity are found. The case of fractional amplitude or fractional number of beams is analyzed. When the phase is modified, the process allows us to visualize the change of geometry. Quasi-lineal patterns to maximum and minimum concentric circles are observed. Some applications of these patterns can be found in sensors and engraving in some lithography processes.

RESUMEN

Los patrones periódicos múltiples por interferencia de luz son un problema complicado desde un punto de vista experimental. En este trabajo reportamos un modelo teórico de incidencia de N haces de luz sobre un punto en el espacio. Dos componentes del campo eléctrico son proyectados: azimutal (ángulo formado con la proyección del vector en el espacio sobre el plano *x-y*) y zenital (ángulo del vector incidente con el eje *z*). En consecuencia, encontramos la intensidad azimutal y zenital. Analizamos el caso de amplitud fraccional o número de haces fraccionario. Cuando el cambio de fase es modificado, nos permite visualizar el proceso de cambio de la geometría. Observamos patrones cuasi-lineales, circulares concéntricos de máximos y mínimos. Algunas aplicaciones de estos patrones pueden encontrarse en sensores y grabado en procesos de litografía.

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Palabras clave:
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INTRODUCTION

Periodic patterns in nature are often critical to some species of animals or insects for survival reasons. For example, the colors in butterfly wings often reflect light, which can frighten predators wrongly assuming this is a poisonous species. Similar structures and patterns have been found in diatoms. Diatoms are unicellular, eukaryotic, photosynthetic algae of an enormous ecological importance on this planet that display a diversity of patterns and structures at the nano-to-millimeter scale (Gordon, Losic, Tiffany, Nagy & Sterrenburg, 2008). From the physical point of view, these are very interesting due to their multifarious possible properties such as anti-reflection (Li, Zhang & Yang, 2010), and super-hydrophobia (Wu *et al*., 2011), and they have been found in different disciplines such as solid-state physics, biology, Faraday waves and nonlinear optics (Staliunas, Longhi & De Valcárcel, 2002). A variety of applications have been developed such as organic light-

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emitting diode (OLED), solar cells, and self-cleaning surfaces (Liu *et al.*, 2012). To produce these patterns, different configurations have been developed such as direct laser writing (Lin, Chen, Niu, Jiang, Wang & Sun, 2010), lithography (Levenson, Viswanathan & Simpson, 2010), two optical beams interference patterns (Menezes, Cescato, De Calvalho & Braga, 2006), and multiple beam interference (Lin, Rivera, Poole & Chen, 2006). Recently, the interference of N beams has been studied theoretically (Jiménez-Ceniceros, Trejo-Durán, Alvarado-Méndez & Castaño, 2010). In this paper, two sets of polarization vectors are considered on the basis of the electric field selected symmetry: zenithal and azimuthal polarization. Different periodic patterns were found, depending on the azimuthal or zenithal angle. However, these periodic patterns correspond to an N beams integer, and the fractional beams case has not been explored. In this paper, we show the case of fractional interference between 6 and 7 beams that allows us to understand how periodic patterns are formed.

Mathematical model

Let us consider a set of N beams impinging radially on a point onto a screen (figure 1), with the same coherence degree, amplitude and phase. Each n wave is considered a plane wave,

$$\vec{\Psi}=A_n\exp[i(\vec{k}_n.\vec{\rho}-\omega t+\epsilon_n)], \quad (1)$$

where $\hat{\rho}=x\hat{\imath}+y\hat{\jmath}+z\hat{k}$; the n-total waves are,

$$\vec{\Psi}_N=\sum_{n=1}^{N}\vec{\Psi}_n=\exp(-i\omega t)\sum_{n=1}^{N}A_n\exp[i(\vec{k}_n.\vec{\rho})+\epsilon_n)], \quad (2)$$

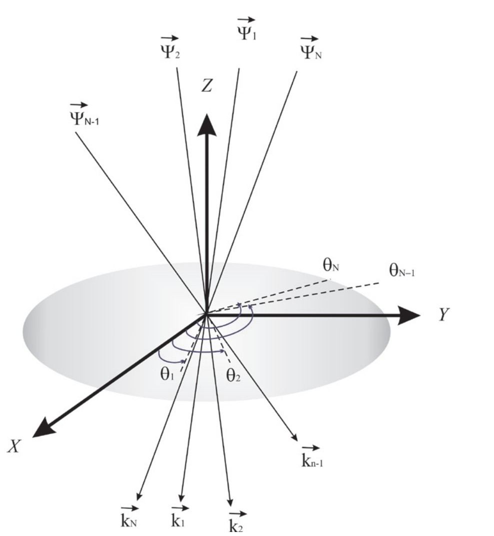


Figure 1. N light beams coincide radially on a point, with cylindrical symmetry. A few light beams of the ensemble are shown.
Source: Authors own elaboration.

The intensity of the electric field is average temporal of the square module. We have azimuthal (the angle of the incident beam with the z-axis,ϕ) and zenithal (the angle formed with the projection of the vector in space on the x-y,θ) symmetry. Mathematically, the conditions of these angles are $\phi_n=\phi$; and $\theta_n=2\pi n/N$; we use spherical coordinates due to the symmetry of the incidence beams. The mathematical expression of the intensity is,

$$I_N=\sum_{m=1}^{N}\sum_{n=1}^{N}\vec{A}_m.\vec{A}_n Re\{exp[i(\vec{k}_m-\vec{k}_n).\vec{\rho}+(\epsilon_m-\epsilon_n])\}, \quad (3)$$

where $\vec{k}_m=kcos\theta_m sin\phi\hat{\imath}+ksin\theta_m sin\phi\hat{\jmath}-ksin\theta\hat{k}$; with algebra we can obtain the intensity with azimuthal polarization (Jiménez-Ceniceros *et al.*, 2010),

$$(\vec{k}_m-\vec{k}_n).\vec{\rho}=2ksin\theta\sin\left[\tfrac{\pi(m-n)}{N}\right]\left\{\sin\left[\tfrac{\pi(m+n)}{N}\right]x-\cos\left[\tfrac{\pi(m+n)}{N}\right]y\right\}, \quad (4)$$

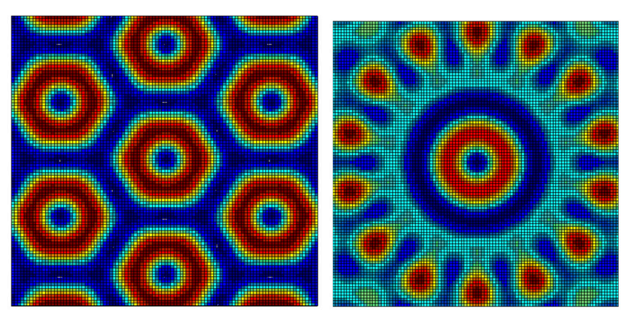
$$I_N=(||)=A^2cos^2(\phi)\sum_{m=1}^{N}\sum_{n=1}^{N}cos\left[\tfrac{2\pi(m+n)}{N}\right]cos\left[2k(C_{m,n}x-Q_{m,n}y\right]. \quad (5)$$

In this configuration, the m-components of the polarizations are radially distributed in the r-θ plane, which we called the radial component.

Numerical simulations

Equation (5) shows the general mathematical model of interference.

The radial component, I_N $(||)$, for the case of 6 interference beams (figure 2a) hexagonal patterns is formed. These patterns are very interesting because they show rotational symmetry, but not translational. Figure 2b shows a formation of rings in the center surrounded by bright spots with rotational symmetry. The zenithal angle is $\phi=4\pi/9$



(a) (b)

Figure 2. Interference pattern of 6 beams: (a) I_6 $(||)$ and (b) I_7 $(||)$. The azimuthal angle is $\phi=4\pi/9$.
Source: Authors own elaboration.

The case of the fractional number of the beams is possible to explore of the numerical point of view. The change of geometry can be found by selecting the fractional number of beams. In this paper we only analyze the radial component, $I_N(||)$. Figure 3 shows the sequence of interference patterns to 6.2, 6.4, 6.6, 6.8 beams. Figure 3(a) corresponds to N = 6.2, the hexagonal pattern presenting oscillations in amplitude. At the center of each hexagon, a dark hole is observed. Figure 3(b) shows the case N = 6.4, and the central hexagon is distorted using two maximum intensities. The same behavior is observed using two aligned neighboring hexagons. The top two hexagons are similar to each other, but the distribution of the energy is not symmetric. The same case is observed for the two lower hexagons. Figure 3(c) represents the N = 6.6 case and the filamentation for each hexagon is observed. The three aligned hexagons are filamented in two new spots. Each of the upper and lower hexagons start developing filamentats into three new beams. It is also interesting to note that the upper and lower new structures have maximum intensities. In other words, the energy distribution is asymmetric. Figure 3(d) shows the case of N = 6.8. The three aligned hexagons are split into two new beams symmetrically, but a remnant of energy interacts with the two new spots, and a circular structure is formed. This will give rise to new spots of beams with circular symmetry as figure 2(b) shows. These three aligned originals hexagons was rotated 17.5° and the central hexagon was converted into a rhombus, while the central spot changes from a circular to an elliptical shape.

To analyze how the ring is formed, numerical precision in the fractional N value is necessary. Figure 4 corresponds to N = 6.9995 and the same value of the azimuthal angle $\phi = 4\pi/9$. The central spots are surrounded by remnants of energy from the other spots of light. We finally obtain the energy distribution shown in figure 2b, when N = 7.

CONCLUSIONS

We have demonstrated theoretically the interference of N beams with azimuthal and zenithal components. The case of fractional amplitude or fractional number of the beams allows us to understand transient states between different geometries corresponding to integer values. The transition of the hexagon periodic patterns to one ring surrounded by circular spots of light is due to amplitude variations and energy interchange. The azimuthal angle affects the energy exchange.

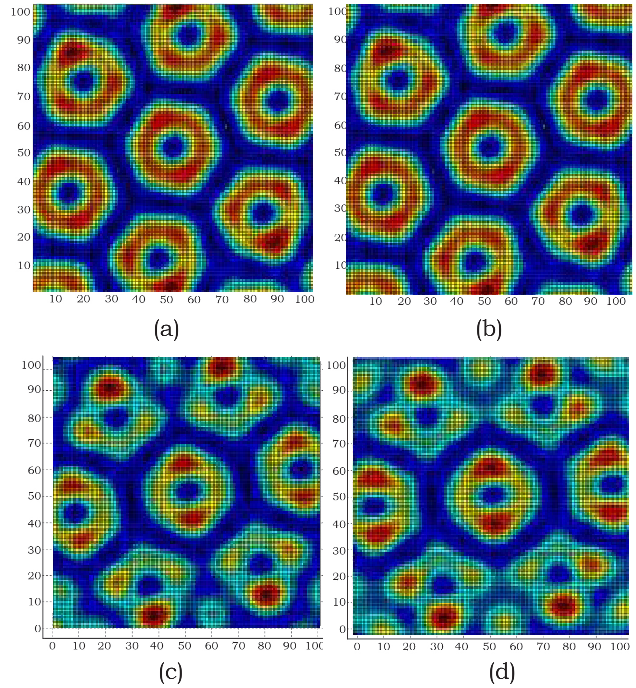


Figure 3. Fractional beams interference. (a) N = 6.2; (b) N = 6.4; (c) N = 6.6, and N = 6.8 beams. The azimuthal angle is $\phi = 4\pi/9$ in all cases.
Source: Authors own elaboration.

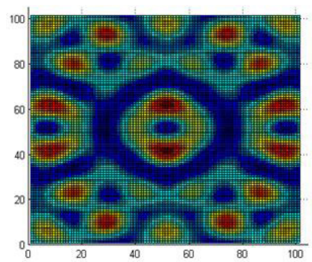


Figure 4. Ring formation around of the central spots using N = 6.9995 and $\phi = 4\pi/9$.
Source: Authors own elaboration.

REFERENCES

Gordon, R., Losic, D., Tiffany, M. A., Nagy, S. S. & Sterrenburg, F. A. S. (2008). The glass Menagerie: diatoms for novel applications in nanotechnology. *Trends in Biotechnology*, *27*(2), 116-127.

Jiménez-Ceniceros, A., Trejo-Durán, M., Alvarado-Méndez, E. M. & Castaño, V. (2010). Extinction zones and scalability in N-beam interference lattices. *Optics Communications*, *283*(3), 362-367.

Levenson, M. D., Viswanathan, N. S. & Simpson, R. A. (2010). Improving resolution in photolithography with a phase-shifting mask. *Optics Express*, *29*(12), 568-580.

Li, Y., Zhang, J. & Yang, B. (2010). Antireflective surfaces based on biomimetic nanopillared arrays. *Nano Today*, *5*(2), 117-127.

Lin, X. F., Chen, Q. D., Niu, L. G., Jiang, T., Wang, W. Q. & Sun, H. B. (2010). Mask-free production of integrable monolithic micro logarithmic axicon lenses. *Journal of Lightway Technology*, *28*(8), 1256-1260.

Lin, Y., Rivera, D., Poole, Z. & Chen, P. (2006). Five-beam interference pattern controlled through phases and waves vectors for diamond like photonic crystals. *Applied Optics*, *45*(31), 7971-7976.

Liu, Y.-F., Feng, J., Bi, Y.-G., Song, J.-F., Jin, Y., Bai, Y., Chen, Q.-D. & Sun, H.-B. (2012). Omnidirectional emission from top-emitting organic light-emitting devices with microstructured cavity. *Optics Letters*, *37*(2), 124–126.

Menezes, J. W., Cescato, L., De Calvalho, E. J. & Braga, E. S. (2006). Recording different geometries of 2D hexagonal photonic crystals by choosing the phase between beam interference exposures. *Optics Express*, *14*(19), 8578-8583.

Staliunas, K., Longhi, S. & De Valcárcel, G. C. (2002). Faraday patterns in Bose-Einstein Condensates. *Physics Review Letters, 89*(21), 210406.

Wu, D., Wu, S.-Z., Chen, Q.-D., Zhang, Y.-L., Yao, J., Yao, X., Niu, G. L., Wang, J.-N., Jiang, L. & Sun, H.-B. (2011). Curvature-driven reversible in situ switching between pinned and roll-down superhydrophobic states for water droplet transportation. *Advanced Materials*, *23*(4), 545–549.