THE RESEARCH AND STUDY PATHS IN THE SECONDARY SCHOOL: THE CASE OF THE POLYNOMIAL FUNCTIONS OF THE SECOND DEGREE

Viviana Carolina Llanos, María Rita Otero
National University of the Centre of the Province of Buenos Aires, Argentina
National Council of Scientific and Technical Researches (CONICET)
E-mail: vcllanos@exa.unicen.edu.ar, motero@exa.unicen.edu.ar

Abstract

This analysis introduces the pedagogy of research and questioning the world within the Math classrooms at secondary schools in Argentina by using Research and Study Paths (RSP). The RSP have been proposed by Chevallard in the Anthropologic Theory of Didactic (ATD) to face the mechanistic model of teaching mathematics in secondary school (Chevallard, 2004). The RSP has been previously conceived as part of this research and could enable to cover the syllabus of the three last years of the secondary level (students aged 14 to 18 years old). The RSP has been carried out in courses intentionally selected by the researcher. Two implementations have been performed once a year during three years; with 163 students participating in the whole research. The results of introducing RSP into the classroom, the characteristics of the Mathematical Organization (MO) and the advantages and disadvantages of teaching mathematics based on the pedagogy of research and questioning the world are described in the present work.
Key words: *functions, pedagogy of research and questioning the world, polynomial functions of second degree, research and study paths, secondary school.*

Introduction

The present research aims at providing alternatives to the problems that characterize the teaching of mathematics at secondary school in Argentina. The dominant didactic model is "reproductive", mechanistic and proposes a fragmented study of the mathematical knowledge.

The Research and Study Paths (RSP) (Chevallard, 2004) allow to develop the *Pedagogy of research and questioning the world* in the classroom. This research introduces an RSP in typical lessons at secondary school in Argentina in a local and experimentally controlled way.

In order to introduce it to the reader, the general characteristics of RSP are briefly described, being the generative question Q_0: *How to operate with any curves, knowing only its graphical representation and the unit in the axes?* (Otero, Llanos, 2011). From this question the whole RSP is generated. The production of a response to Q_0 generates some derivative questions relative to the different lessons that would to some extent cover the curriculum of last three years of Secondary School in Argentina (14 to 18-year-olds). Therefore, the results are described from one of the derivative questions, enabling the reconstruction of the MO of the polynomial functions of the second degree.

Theoretical Frame and Research Questions

The Anthropological Theory of Didactics (ATD) (Chevallard, 1999) and more specifically the notion of Research and Study Path (RSP) (Chevallard, 2007, 2009) are adopted in this research.

In any RSP there is a generative question Q_0 which originates and inspires the process of study. This question $Q_{0,}$ should allow the enunciation of another derivatives $(Q_i)_{1\leq i\leq n}$. The different MOs are generated from every Q_i, as response to the questions that have aroused from their construction. The scheme that Chevallard (2009) called *developed herbartian* [*S*(*X; Y; Q*) ➦ $\{R_1^\Diamond, R_2^\Diamond, ..., R_n^\Diamond, O_{n+1}, ..., O_m\}$] ➥ $R^\heartsuit$, enables to interpret the following: the RSP must be organized around a generative question (Q_0); the didactic system S(*X*; *Y*; *Q*) is composed by an *X* group of students; the aids to the study provided by a group of teachers *Y* or a teacher {*y*}; and the ♥ of the whole process, given by *Q*. This system allows the constitution of a didactic means $M = \{R_1^\Diamond, R_2^\Diamond, ..., R_n^\Diamond, O_{n+1}, ..., O_m\}$, which includes answers which have not been originated in the same class, as for example, the use of textbooks in the classroom, internet connected PCs, teacher notes and teacher support. On the other hand, there are O_j works, theories and praxeologies available, from which the responses $R^\Diamond$ can be deconstructed, and decide which the components that allow for the constructions of $R^\heartsuit$ are.

From the ready-made answers and those which are generated in the didactic medium; the elaboration, validation and institutionalization of a response $R^\heartsuit$ as result of the process of study are expected to be obtained.

The research questions are: What are the characteristics of the proposed RSP? What mathematical organizations in the secondary school curriculum in Argentina does it allow to meet?

Methodology of Research

The investigation begins when students start the 4th year of high school and continue on the following year, i.e. in the 5th year. The study was conducted in three cohorts, with two parallel courses each at the same institution. In total N = 163 students participated in the research, given that the courses where the research was carried out were examples of ordinary lessons at secondary school level, and therefore, every student in the course was involved in the investigation. In each class, there were between 25-30 students, so that in the six implementations presented the number of students increased as indicated. In all cohorts the course teachers took part of the investigation.

In each class, all written protocols for every student are obtained. At the end of the class, the teacher obtains the written productions of the students, organized in the study groups in each class. Later, the different works are scanned and returned to the students on the following meeting. Thus, the records obtained out of the student-generated answers, provide an important empirical base for the research team. Moreover, the lessons are recorded in audios and field notes are taken. Given the total number of audio recordings obtained, only those ones allowing the researcher to gain information which has not been duly justified in the student-written protocols, are transcribed. The audio recordings enable the teacher/researcher to recover information from the on-going processes in the class which have not been recorded by any other written means, i.e. by the students´ protocols or the researchers´ notes.

Participant and non-participant observation was carried out from the collaboration with colleagues from the research team. In addition, the teacher/researcher elaborated field notes before and after the class. These records allow not only to know to what extent the objectives proposed in each meeting have been achieved, but also to describe the decisions that have been taken throughout the proposed course of study, which had not been previously taken into consideration in the class. Then, the results obtained from the implementation of RSP are analyzed out of the volume generated by the written protocols, the audio recordings and the observations made.

RSP Characteristics: Possible Paths and Its Insertion in the Classroom

RSP begins with the study of the following question Q_0: *How to operate with any curves, knowing only its graphical representation and the unit in the axes?* (Llanos, Otero, 2012, 2013a; Otero, Llanos, 2011). From the Q_0 it is possible to reconstruct the RMO in algebraic functions provided that those can be obtained by doing operations with other curves of the same kind. The trigonometric functions, exponential and logarithmic ones which also correspond to the curriculum of three last years of secondary school, cannot be directly covered by the generative question in the proposed RSP; though it is possible to adapt the techniques reconstructed within it to other cases, and to construct the OMs out of the other questions deriving from the RSP, as shown by Farfán and Rosales in their research on the study of logarithmic functions.

So as to know the curves and operations presented by the students to develop the RSP, the initial problem is therefore reduced to their selection. This will establish the possible paths that could be generated with Q_0 in the implementation context, resulting from the available knowledge in the students. Figure 1 shows a description of the different MOs that can be studied and a detail of the derived questions arising from each lesson in the selected courses.

Initially, and as a means to introduce the RSP in the class, the teacher presents the generative question to the students. A first outcome is relative to the decisions taken by the students, regarding which curves they suggest and which operations they are going to perform with them. These answers generate the possible paths, which in the present analysis, are expressed in the problem of the multiplication of two straight lines as the addition and subtraction are given by the availability of their knowledge.

This article presents some of the results of the problem of multiplication of two straight lines. Moreover, a demonstration of other protocols is presented in which the students generalize the previously constructed techniques to multiply more than two straight lines, straight lines and parabolas or several parabolas; also the quotient between polynomial functions. As follows, the different derivative questions resulting from Q_0, enable the students to reconstruct the MOs of the polynomial functions of the second degree, of the polynomial and of the rational functions. In all cases, the problem proposed needs the recovery of geometric techniques before the algebraic ones. The mid course between analytic and synthetic geometry achieved from the start, enables to reconstruct, visualize and justify the essential characteristics of the graphic representation of functions. This result cannot be obtained if started within the analytic framework. The relevance of achieving an interaction between the synthetic and analytic framework provides the results with a superior level of justification, otherwise not achieved by directly starting within the analytic framework.

ISSN 1822-7864

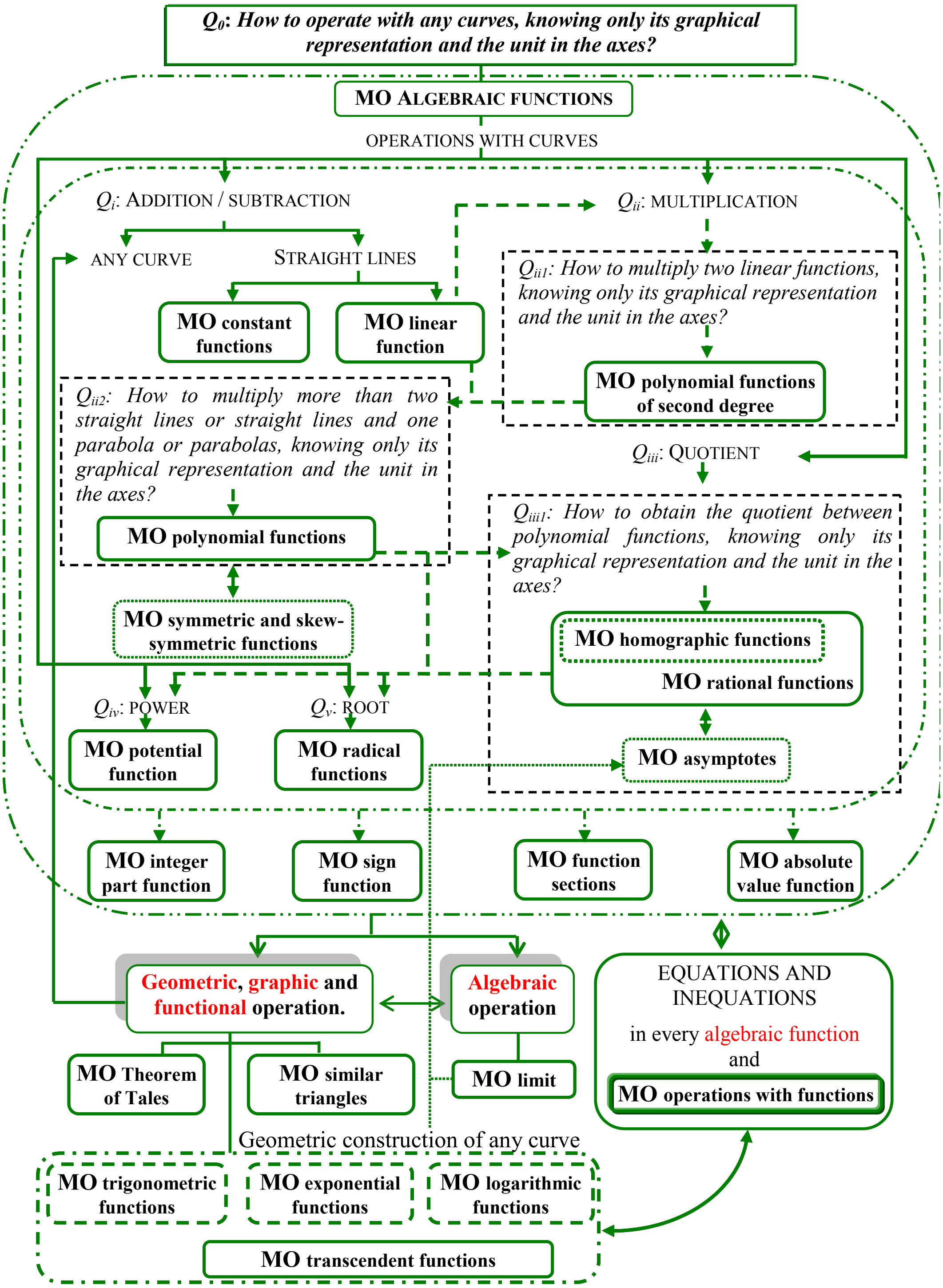

Figure 1: Description of the derived questions developed in the lessons, and identification of the possible MOs to be reconstructed.

Therefore, a brief description is presented of the results of introducing the RSP from the Q_1: *How to multiply two straight lines, knowing only its graphical representation and the unit in the axes?* The teacher initially suggests constructing a graphic representation of polynomial functions of second degree, considering the different pairs of straight lines. The questions suggested at this first stage are of the following type:

The f and g functions are presented in the graphic representation in Figure 2. All straight lines A//B//C//D, are perpendicular to the axis x. Function.

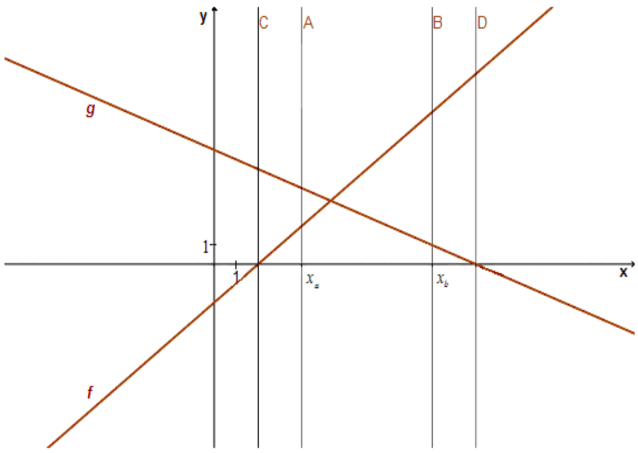

Figure 2: Graphics corresponding to functions f and g in situation 1.

(a) Which is the most reasonable graphic for *h*? What characteristics from graphic *h* could you justify?

(b) For every x_a y x_b equidistant from the zeros in each function, $\overline{CA} = \overline{BD}$. Is it correct to say that $h(x_a) = h(x_b)$? Could you justify it?

From this provided situation, the students have constructed a graphic representation *h*. The construction of the characteristics of the curve and the geometric techniques required, correspond to a long-term process, and initially presenting many obstacles. At the beginning, students feel the information available is not enough in order to solve the problem. On the other hand, the teacher cannot provide an answer. The implementation of RSP compels the students organized in four-people groups to elaborate and construct the results. The role of the teacher is limited to directing the process of study. The mathematical techniques that allow justifying the characteristics of the curves (Llanos, Otero, 2012, 2013b) are obtained from the information provided in the problem itself: the representation of the straight lines and the units.

The outcome allows justifying that the main difficulties for students are relative to the generation of geometrical techniques. The identification of the signs, the outstanding points constructed from one and zeros, do not present an obstacle. However, the test for the symmetry of the curve corresponding to item (*b*) needs a more advanced reconstruction process. As a means of describing the results obtained in the path generated by Q_1, we have selected the protocol corresponding to student A153, as shown in Figure 3.

According to this protocol, we can observe the way in which A153 analyses the signs of the curve, constructs the zeros and other outstanding points, analyses that $h(x_a) = h(x_b)$ for the equidistant points of the zeros in the straight lines $h(x_a) = h(x_b)$; and obtains a representation for *h*. In order to analyze that the test of symmetry is valid to any pair of points that equidistant from the zeros, this student has reproduced the construction in the x_e x_e y x_f points that he has selected. Consequently, symmetric points are obtained. The vertex is constructed from the student´s inference process. The representation shows that the curve has a maximum in the ordinate, which corresponds to the symmetric axes, being this problem analyzed in the following situations, also by means of the recurrence to geometry.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Volume 52, 2013

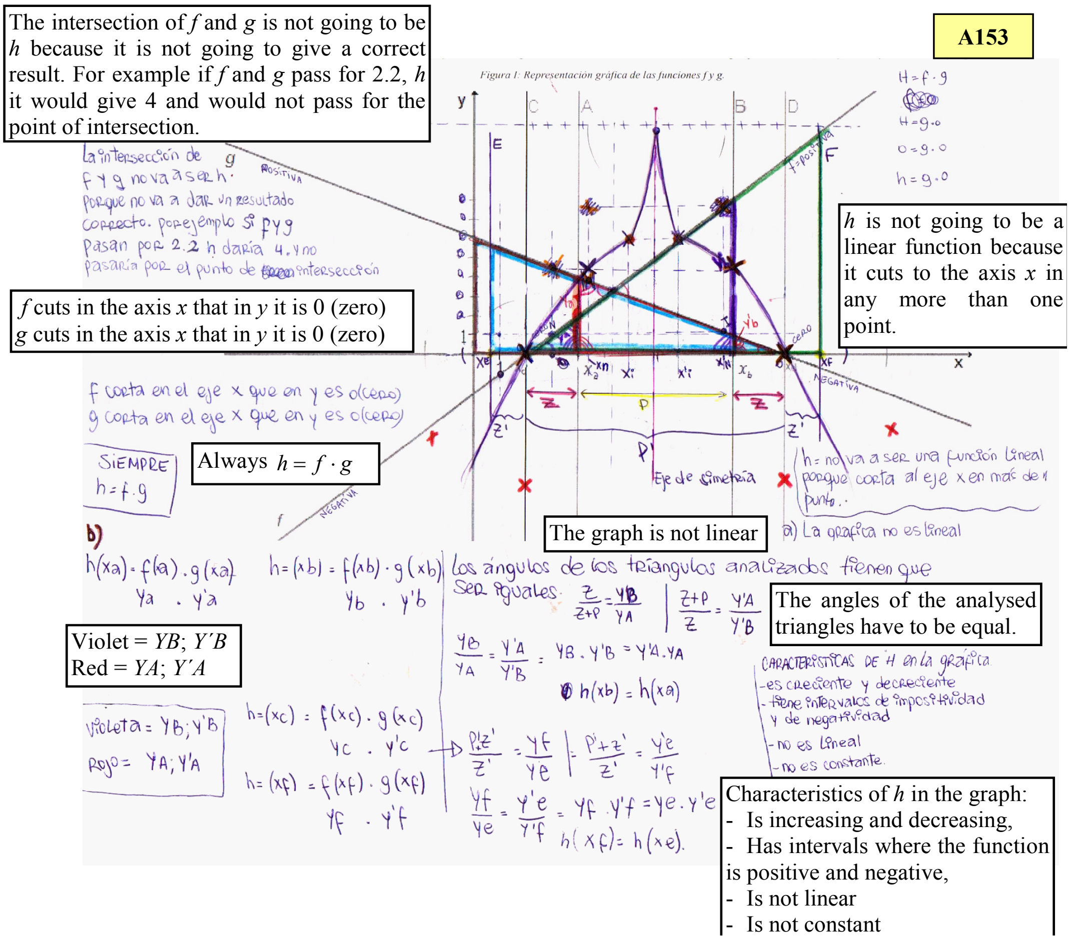

Figure 3: Protocol corresponding to student A153.

Then, the task of constructing a curve for *h* is also presented. It consists of obtaining the vertex by construction while some properties of the zeros in the polynomial functions of the second degree are analyzed. Apart from changing the straight lines that are multiplied, two questions are included:

(c) Which triangles need to be constructed in order to calculate the multiplication between *f* and *g* in the symmetry axis, providing the unit is the side of one of the triangles?

(d) Providing *g* function is moved so as to coincide with *f* in the intersection of the axis *x*: Which would be the curve for *h* and which characteristics would it present?

Thus, the student obtains an approximate representation for *h*, when the two straight lines display a negative gradient and different zeros. So as to graphic *h,* he constructs outstanding points, analyzes the signs, demonstrates that the curve is symmetric and attempts to construct the vertex; as presented in item *c*. Other points in the representation corresponding to item *d* are also obtained (where the straight lines have coinciding zeros) from an adaptation of the technique generated for the vertex. The student also carries out the test by the symmetry for both representations and justifies the characteristics of the curve by the construction of the outstanding points. Then, the properties of the zeros in the polynomial functions of the second degree are analyzed. The results obtained by student A153 are represented in Figure 4.

ISSN 1822-7864

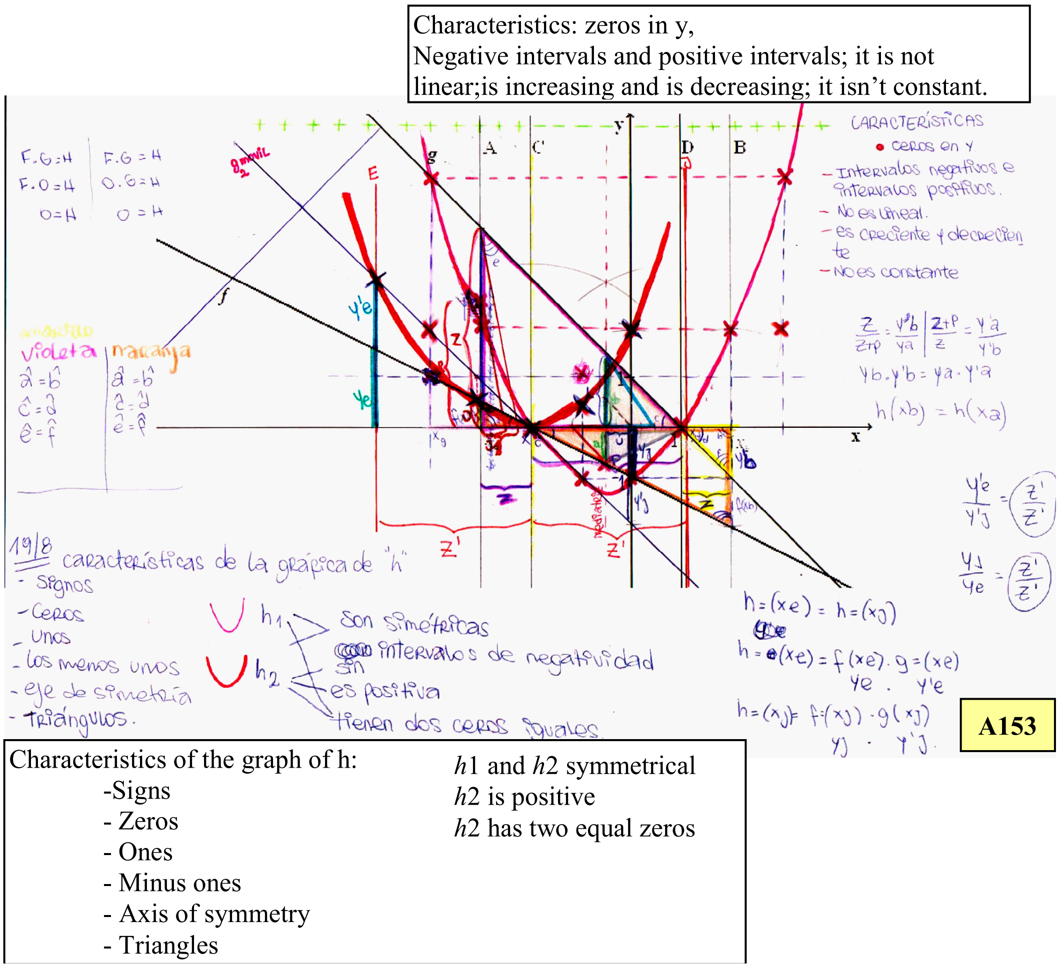

Figure 4: Protocol corresponding to student A153.

A similar situation, where the information about the coordinates of some points has been stated, allows the students can construct the characteristics of the equivalent analytic representations for the polynomial functions of the second degree, as shown in Figure 5.

In order to represent *h*, the characteristics constructed in the previous situations are recovered. Then, the protocol corresponding to student A113 is selected as it facilitates the description of how the vertex technique was constructed not only analytically, but also from the construction of similar triangles.

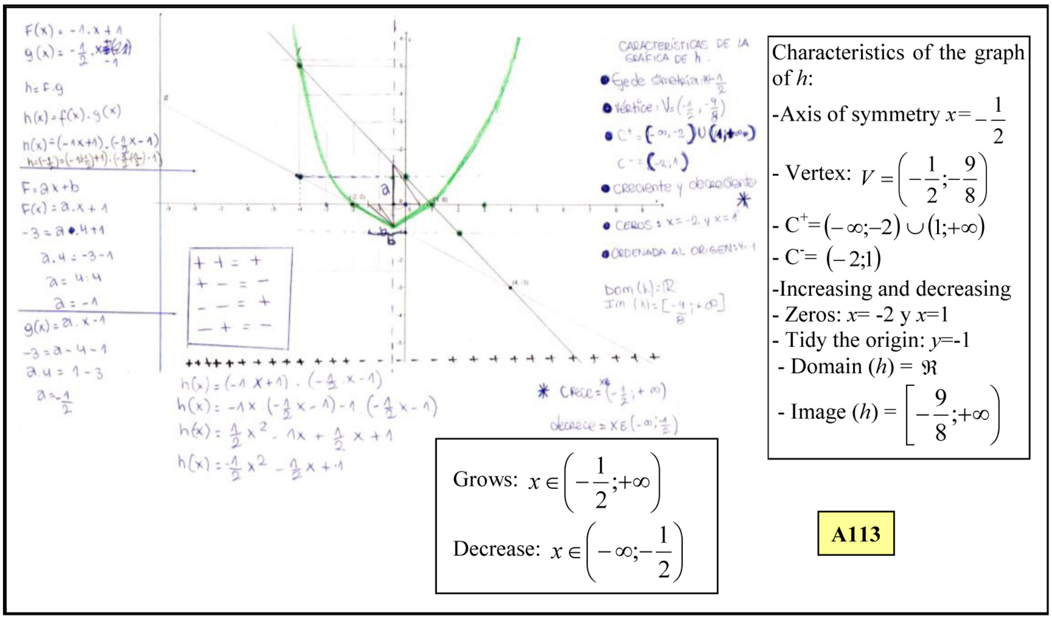

Figure 5: Protocol corresponding to student A153.

The MO of Polynomial Functions

The study also focuses on how the MO in the polynomial functions arises from the RSP (Llanos, Otero, Bilbao, 2011) resulting from the derivative question Q_2: *How to multiply more than two straight lines, or straight lines and parabolas, or parabolas; knowing only its graphic representation and the unit in the axes?* The curve p which the students can reconstruct becomes justified by the techniques obtained within the paths generated in Q_1. According to protocols in A77 and A128, the students have constructed a graphic for $p = f \cdot g \cdot j$ from the multiplication of three straight lines (Figure 6) and between a straight line and a parabola $p = f \cdot h$ (Figure 7).

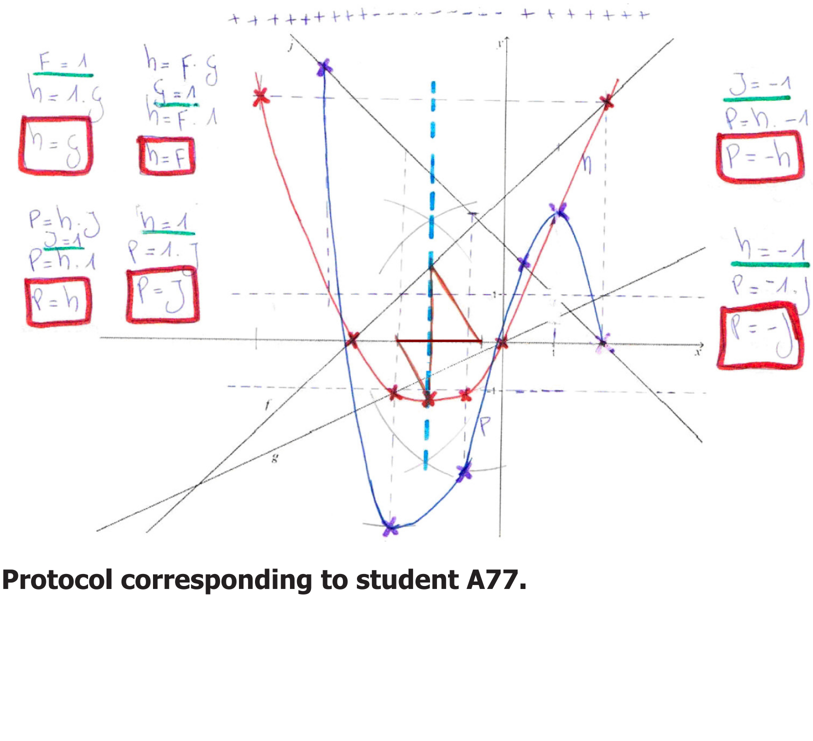

Figure 6: Protocol corresponding to student A77.

ISSN 1822-7864

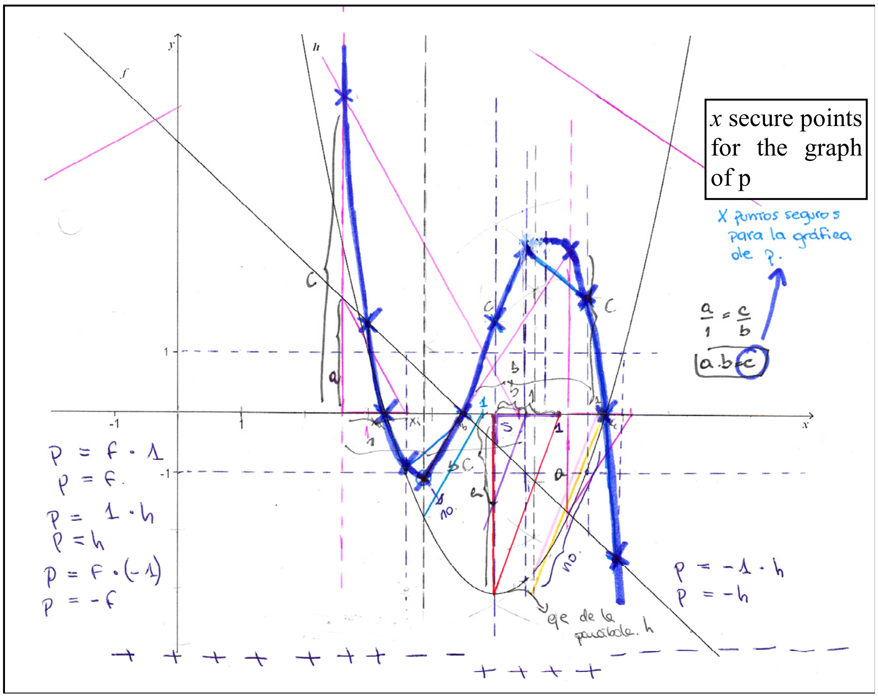

Figure 7: Protocol corresponding to student A128.

The MO of Rational Functions

The path generated by the quotient of polynomials is studied as well, facilitating the reconstruction of the MO of the rational functions q (Otero, Llanos, Gazzola, 2013), from the derivative question Q_3: *How to obtain the quotient between polynomial functions, knowing only its graphical representation and the unit in the axes?* By means of an adaptation in the techniques constructed to multiply the curves, students can advance in relation to the quotient problem. The protocols corresponding to A132 (Figure 8) and A79 (Figure 9) are selected. Then, the students construct $q = \dfrac{f}{g}$ and $q = \dfrac{f}{h}$ the graphic from the quotient of the two straight lines and between a straight line and a parabola respectively.

Protocols allow us to interpret that it is possible to adapt those techniques which have been constructed in one part of the path, into another part. Each derivative question Q_1, Q_2 y Q_3 made possible the construction of other characteristics apart from graphic representation of the functions due to the switching into an analytic framework. It is from a RSP as posed in the current investigation, that it is possible to recapture the *raisons d'être* of the MOs corresponding to the curriculum at secondary school in Argentina.

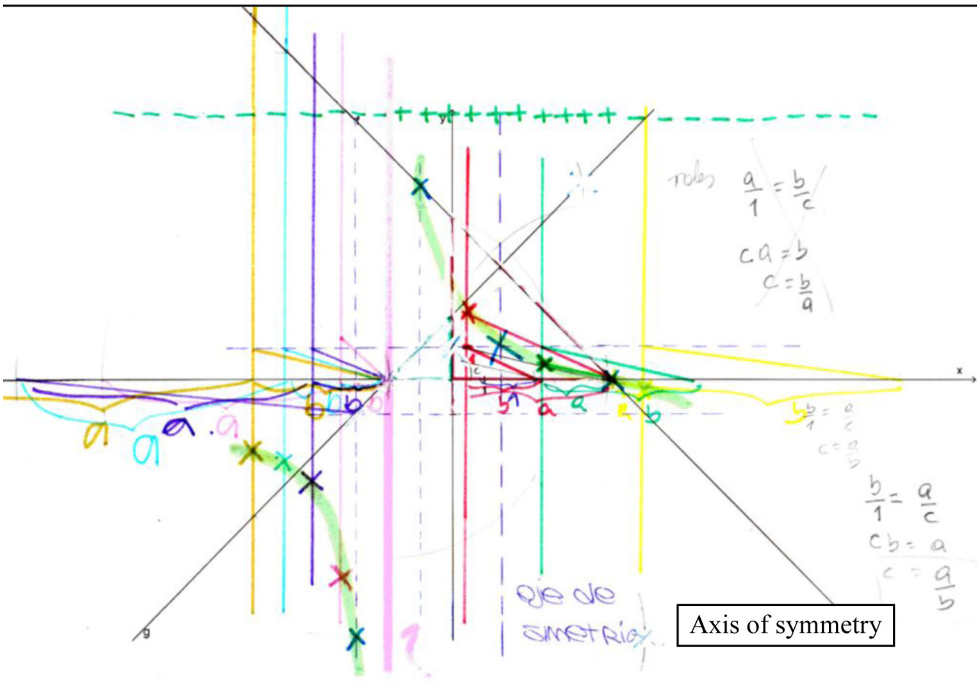

Figure 8: Protocol corresponding to student A132.

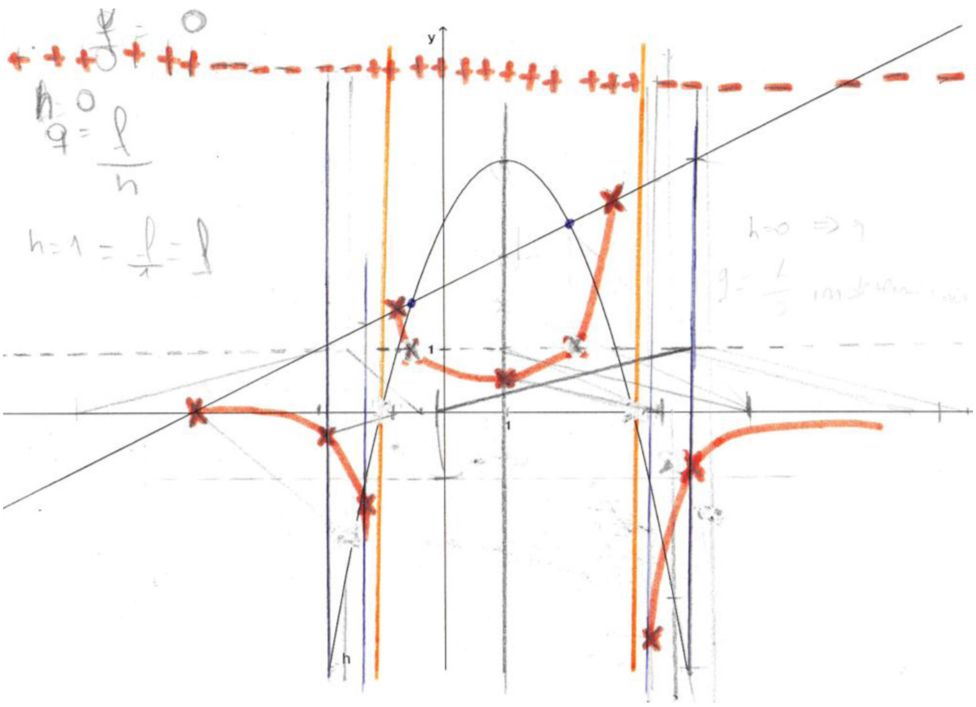

Figure 9: Protocol corresponding to student A79.

Conclusions

To proceed along the path of introducing the *pedagogy of research and questioning the world* based on the RSPs in the classrooms in secondary school, it is necessary to carry out important modifications. The change towards a teaching based on the study of questions rather than answers, has been one of the major difficulties.

The proposed RSP enables the study of algebraic functions at secondary school level. At the beginning of the path, the generation of geometric techniques has been a drawback. The study of the geometric notions has been part of the research in the RSP. The results achieved both in the construction of the curves and from the analytic representations are encouraging. Thus, knowledge has become the object of study and of construction by the course group. Protocols have shown the quality and relevance of the mathematical work carried out.

The RSPs have proved to be an important tool to face changes in the teaching at secondary school. Therefore, the current investigation seeks to:

- Promote paths based on the feasibility of the instruments, which not necessarily initiate by the multiplication of two straight lines, where the geometric techniques may not be essential;
- Attempt to find answers which produce different paths and constructions of other MOs in another order;
- Deepen the study of the difficulties and the ways in which those obstacles might be overcome, given the depth displayed by the devise to introduce a significant change in the secondary school.

Acknowledgments

National Council of Scientific and Technical Researches (CONICET). http://www.conicet.gov.ar/.

Nucleus of Research in Education in Science and Technology (NIECyT), Faculty of Exact Sciences, National University of the Center of the Province of Buenos Aires (UNCPBA).

References

Chevallard, Y. (1999). El análisis de las prácticas docentes en la teoría antropológica de lo didáctico. *Recherches en Didactique des Mathématiques, 19* (2), 221-266.

Chevallard, Y. (2004). *Vers une didactique de la codisciplinarité. Notes sur une nouvelle épistémologie scolaire*. Retrieved 08/04/2010 from http://yves.chevallard.free.fr/

Chevallard, Y. (2007). *Passé et présent de la théorie anthropologique du didactique*. Retrieved 14/06/2009 from http://yves.chevallard.free.fr/

Chevallard, Y. (2009). *La notion de PER: problèmes et avancées*. Retrieved 27/11/2009 from http://yves.chevallard.free.fr/

Chevallard, Y. (2011). Quel programme pour l´avenir de la recherche en TAD?. Trabajo publicado en *Actas del III International Conference on the Anthropological Theory of the Didactic (III CIT-AD)*, pp. 23-32. Cataluña, España. Available in http://www.crm.es/Publications/Documents/Documents_10.pdf.

Chevallard, Y. (2012) *Teaching mathematics in tomorrow's society: A case for an oncoming counter paradigm*. Retrieved 22/08/2012 from http://yves.chevallard.free.fr/

Ferrari, M., Farfán, R. M. (2008). Un estudio socioepistemológico de lo logarítmico: la construcción de una red de modelos. *Revista latinoamericana de investigación en matemática educativa (RELIME)*, *11* (3), 309-354.

Llanos, V. C., Otero, M. R. (2012). Las funciones polinómicas de segundo grado en el marco de un Recorrido de Estudio y de Investigación (REI): alcances y limitaciones. *Revista Iberoamericana de Educación Matemática UNIÓN*, *31*, 45-63.

Llanos, V. C., Otero, M. R. (2013a). La pédagogie de l'enquête et du questionnement du monde: une étude longitudinale dans l'école secondaire argentine. *Review of Science, Mathematics and ICT Education. Re SM TICE,* 7. En prensa desde el 12-11-2012. Fecha de publicación: junio de 2013. University of Patras.

Llanos, V. C., Otero, M. R. (2013b). Operaciones con curvas: las funciones polinómicas de grado dos y la generalización a otras funciones. *Revista SUMA+ para la enseñanza y el aprendizaje de la matemática*, *73*. Valencia. En prensa desde el 28-12-2012. Fecha de publicación: julio de 2013.

Llanos, V. C., Otero, M. R., Bilbao, M. P. (2011). Funciones Polinómicas en la Secundaria: primeros resultados de una Actividad de Estudio y de Investigación (AEI). *Revista Electrónica de Investigación en Educación en Ciencias (REIEC)*, *6* (1), 102-112. Available in http://www.exa.UNCPBA.edu.ar/reiec/.

Otero, M. R., Llanos, V. C. (2011). Enseñanza por REI en la Escuela Secundaria: desafíos, incertidumbres y pequeños logros al cabo de seis implementaciones. *Actas del I CIECyM y del II ENEM*. NIECyT, Facultad de Ciencias Exactas. UNCPBA. pp. 15-23. Available in http://iciecymiienem.sites.exa.unicen.edu.ar/actas.

Otero, M. R., Llanos, V. C., Gazzola, M. P. (2012). La pedagogía de la investigación en la escuela secundaria y la implementación de Recorridos de Estudio e Investigación en matemática. *Revista Ciencia Escolar: enseñanza y modelización*, *1* (2). Available in: http://www.revistacienciaescolar.cl/

Advised by Paolo Bussotti, University of West Bohemia, Pilsen, Czech Republic

Received: *January 18, 2013* Accepted: *March 21, 2013*

Viviana Carolina Llanos	Dr. in Sciences Education: Mathematical Mention, University of the Center of the Province of Buenos Aires (UNCPBA), Argentina. Nucleus of Research in Education in Science and Technology, Teachers' Formation Department, UNCPBA. Scholar of the National Council of Scientific and Technical Researches (CONICET). E-mail: vcllanos@exa.unicen.edu.ar
Maria Rita Otero	Dr. in Sciences Education, University of Burgos, Spain. Nucleus of Research in Education in Science and Technology. Titular Teachers' formation Department, Faculty of Exact Sciences, National University of the Center of the Province of Buenos Aires (UNCPBA). Independent Researcher of the National Council of Scientific and Technical Researches (CONICET). E-mail: rotero@exa.unicen.edu.ar Website: https://sites.google.com/site/mariaritaotero/