MENTAL ARITHMETIC SKILLS: COMPARATIVE RESEARCH BASED ON ESTONIAN AND FINNISH PUPILS

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Abstract

Estonian pupils achieved good results in international tests like TIMSS and PISA (IEA's TIMSS 2003 International Report on Achievement in the Mathematics Cognitive Domains, 2005; Assessing Scientific, Reading and Mathematical Literacy, 2006). Despite the high results of Estonian pupils, annual official tests showed that there were problems in subtraction, multiplication and time calculations at primary school level (The National Examinations and Qualifications Centre, 2009).

The key question this research poses is what kind of mental arithmetic skills do children in the first three forms of primary school possess? The aim is to analyze what kind of changes should be implemented in the national curriculum in order to improve the maths studies of Estonian pupils.

Mental arithmetic tests were carried out in late April and early May 2006 both in Estonia and in Finland. Two groups of pupils participated in the tests, one of which consisted of pupils from seven Estonian schools from different parts of Estonia, and the other consisted of the first, second and third form pupils at Turku University Practice School which is the training school for teacher trainees studying at the Turku University, Finland. In this research, mental arithmetic is studied among children attending the first, second and third forms of primary school. Differences between the sexes are studied and, in addition, differences between Finnish and Estonian pupils are examined by comparing the Estonian results with those of the Finnish group. On the basis of the test, Estonian pupils received higher results in mental arithmetic exercises. Both Estonian and Finnish boys performed better than girls in mental arithmetic exercises in the second and third forms.

The results confirm a good level of skills amongst the Estonian pupils, but also show in which tasks the most mistakes occurred. The exercises which acquired the use of working memory caused more mistakes. Further, the results indicate a difference between the mastery of boys and girls in mental arithmetic which highlights a need for the differentiation of teaching strategies according the sex of pupils.

Key words: *teaching mathematics, mental arithmetic, long-term memory, short-term memory, working memory.*

Introduction

Despite calculators and computers becoming more and more important in our daily lives, the importance of mental arithmetic in school teaching has not decreased. The application of such practical skills include, for example, understanding figures, knowing the time when looking at a clock, weighing and paying for shopping purchases, giving the correct amount of change, understanding timetables and performing the respective arithmetical tasks. According to several studies carried out, pupils consider mental arithmetic skills more important than physical arithmetic because mental arithmetic is used more in daily life (McIntosh & Reys, 1997, p.322- 329).

Mental arithmetic is a combination of thinking and acting. It is a process in which both creating an image and simultaneous calculating are required and it means counting in one's head without any tools such as a calculator, a computer or paper and a pen. During this process the thinking ability of a pupil develops, which itself is a foundation for different processes of cerebration (McIntosh & Reys, 1997, p.322-329). This very complicated process, during which it is easy to lose motivation and abandon the action of thinking if exertion is needed, presumes a great effort by a pupil. Although the results of research carried out in 2008 (Piht, Eisenschmidt) showed that the attitude towards mathematics among first to third forms pupils is generally positive, boys have a more positive attitude than girls.

Which topics should be handled in the school curriculum and in what quantity is one of the challenging issues in every country. In 2011 a new national curriculum is going to be implemented in Estonia. Studies which analyse the problems of acquiring knowledge in mathematics are valuable input to shape preparation of teaching facilities and teachers for implementation of the curriculum.

Theoretical Aspects of Mental Arithmetic

Vygotsky formulated the so-called interiorisation hypothesis about mental development: "Every function in the child's cultural development appears twice: first on the social level, between people (inter- psychological), and then inside the child (intra-psychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relations between human individuals" (Vygotsky, 1978, p. 57).

In mental arithmetic, a person discerns, interprets, analyses and memorises figures and information of their respective relations in their memory (Baddeley, 1986, 1996; Kyttälä, 2008, p.2). The processes which are primarily applied in mental arithmetic exercises are based on long-term memory (LM), where the respective absorbed information is saved and retrieved when necessary, and short-term memory (STM), which is used for processing and storing information for a short period of time (Logie, Gilhooly & Winn, 1994). The long term memory refers to general memorizing such as memorizing multiplication tables. Short-term memory is an operative memory which assists, for example, in memorizing cue numbers (Svensson, 1977, p.6; Virrankoski, 1983, p.2).

Some researchers make a distinction between short-term memory (STM) and working memory (WM). Others, for example Baddeley, whose model has been applied in most mental arithmetic studies, combine these functions under the same concept i.e. working memory (Baddeley, 1986, 1996; Kyttälä, 2008, p.2). Working memory is associated with both the storage and processing of short-term information when performing cognitive tasks such as learning, logical thinking and comprehension (Baddeley et al., 1999). Working memory is not only the short-term storage of the brain, but a necessary, yet a limited tool, essential to several cognitive functions.

It is evident that noting and borrowing are not only related to declarative information, such as only retrieving information, but also to procedural information (Ashcraft, 1992). On the basis of this, arithmetic performance is likely to be slower and less accurate in problems requiring noting or borrowing as compared to those exercises in which such tools are not needed. Therefore, the time required for mental arithmetic increases if taking notes or borrowing are needed (e.g. Ashcraft & Faust, 1994, Ashcraft & Kirk, 2001), because an extra step is required after the phase in which the figure is noted or borrowed when the information is being transferred to the working memory. In a later phase, this information needs to be retrieved again

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from the working memory and if the saved information has disappeared from the working memory, a mistake will occur. Faulty memory and borrowing methods are evidently among the most common reasons for mistakes in the mental arithmetic exercises carried out by both children and adults, as well as among people suffering from severe brain dysfunctions (e.g. Fürst & Hitch, 2000; Noel et al., 2001; Sandrini et al., 2003; Imbo et al., 2005, p.7). Thus possibilities and limitations of the working memory may influence the learning of mental arithmetic (e.g. Saariluoma, 1990, p.57-58; Vygotski, 1984, p.340; Van der Heijden, 2004, p.12-14).

The teaching and learning of mental arithmetic has been studied extensively. The results of a Canadian study, where participating children were aged four to five, focused on how information is reworked in the brain. The study showed that the younger the child, the more inefficient their short-term memory. The older the child gets, the better their working memory functions (Brainerd, 1983). Imbo, Rammelaere, Vandierendonck (2005) write about the role of the working memory in mental arithmetic, and about the mistakes made in carrying out mental arithmetic exercises as well as the reasons for such mistakes. Almost all studies concerning the noting operation were carried out on addition problems, though this operation also appears in multiplication problems. The role of the performing working memory component grew even larger when more carry or borrow operations had to be carried out. This role in noting and borrowing may be explained by the controlling function of the performing working memory component. When a carry or borrow operation has to be performed, a conflict occurs between this carry or borrow sequence and the no-carry or no-borrow sequence (Imbo et al., 2005). Almost no studies investigated the borrow operation in subtractions, although severe problems with the borrow procedure have been observed in children (e.g., Brown & Burton, 1978) and in brain damaged patients (e.g., Sandrini, Miozzo, Cotelli, & Cappa, 2003).

Van der Heijden (1993) studied pupils' approach behaviour, i.e. solving strategies, by applying the LDRT (Leiden Diagnostic Arthmetic Approach Test) in mental arithmetic tests. Leiden's test is a standardized process diagnostic tool that is used for measuring eight approach behaviours, i.e. angles on solving strategies, in summation and subtraction exercises with natural figures between 0 and 100. These angles are: lack of attention, relevant orientation, solving methods, automatization, and flexibility, understanding i.e. realization, awareness and control (Van der Heijden, 2004, p.7-8). According to Heijden's study, girls achieved slightly lower results in mental arithmetic exercises than boys; the level of automatization was also lower among girls than among boys. Also, girls' lower level of awareness and the differences between the sexes by the means of approach behaviour became evident (ibid: 14).

International studies provide an opportunity to compare pupils' achievements and learning motivation. The results of the PISA research in 2006 show a high level of skills in the results of both Estonian and Finnish pupils (Assessing Scientific, Reading and Mathematical Literacy, 2006).

Since 2003, national tests have been carried out in Estonia, which are aimed at evaluating pupils' mathematical skills and knowledge at the end of the third form based on the national curriculum. In the tests carried out in 2008, Estonian pupils received good results. The main problems arise in subtraction and multiplication exercises, as well as in the calculation of time (The National Examinations…, 2009).

Methodology of Research

The purpose of current research is to analyze what kind of mental arithmetic skills children in the first three forms of primary school possess, and to clarify the differences in mental arithmetic skills between the two sexes (boys and girls) and between the two countries (Estonia and Finland). The practical aim is to support the development of the national curriculum and the compiling of study materials. Analysis of the main calculation mistakes enables the shaping of emphases in the curriculum and the development of tasks to be included in teaching materials by adding exercises which help a pupil to achieve greater proficiency in performing mental arithmetic calculations.

Sample

The research sample consisted of first, second and third-form pupils (n=287) in Estonia (n=157) and in Finland (n=130) (Table 1). Estonian pupils were first, second and third-form pupils in Lääne County from randomly selected schools. In Finland, pupils were from the first, second and third forms at the Turku University Practice School. The respective number of pupils from each form and country is illustrated in Table 1.

Table 1. Pupils participating in the research.

	Estonians			Finnish		
	Total	Boys	Girls	Total	Boys	Girls
1st form	47	30	17	56	27	29
2nd form	39	19	20	37	16	21
3rd form	71	39	32	37	20	17
Total	157	88	69	130	63	67

Instrument and Procedures

Mental arithmetic tests were carried out in April and May 2006 both in Estonia and Finland. Exercises applied in the test were based on the Estonian mathematics curriculum (Põhikooli ja gümnaasiumi riiklik õppekava, 2002) and are used annually by The National Examinations and Qualifications Centre. The test consists of mental arithmetic tasks, writing down the strategies used in mental arithmetic tasks (the pupils wrote down how they performed the calculations) and questions about attitudes towards mathematics and mental arithmetic. In this paper we analyze the results of mental arithmetic tasks.

The test results were given points in order to create summary variables. The evaluation of the mental arithmetic test was based on the assessment criteria applied in Estonian schools. The pupils' achievements were evaluated based on Estonian pupils' assessment criteria (points and percentages) as follows: 100% to 90% Excellent; 89% to 70% Good; 69% to 45% Satisfactory; 44% to 20% Poor; 19% to 0% Unsatisfactory (Õpilaste hindamise ja …, 2006).

The results were analyzed by SPSS 16.0 programme; the average means, dispersion and chi-square test were found. In the first form it was possible to get 16 points for the mental arithmetic calculations and 4 points for solving the text exercises (20 points in all). In the second form, points were given as follows: 10 points for the mental arithmetic calculations and 4 points for solving verbal tasks (14 points altogether). In the third form a pupil received 20 points for solving a mental arithmetic task and 3 points for solving a verbal task (23 points altogether).

Research Results

Estonian and Finnish pupils who participated in this research performed well in the mental arithmetic exercises. The percentage of Estonian pupils who received excellent or good results in the mental arithmetic exercises was higher than among Finnish pupils (Table 2).

Pupils' results were classified according to the evaluation scale in order to enable a comparison of the results of the mental arithmetic exercises. A significant statistical difference was noted in the results of the mental arithmetic exercises between Estonian and Finnish pupils (χ^2 =18.24, df =4, p<0.001).

Table 2. Division of pupils' results (percentages).

Result of mental arithmetic exercise	Estonian (n=157)	Finnish (n=130)	Total (n=287)
Poor	1	2	1
Adequate	4	18	10
Good	34	37	36
Very good	36	26	31
Excellent	25	18	22

Differences between sexes were not noted in the results of Estonian pupils. On the basis of the mental arithmetic exercises, Finnish boys performed these exercises with better results than girls.

Table 3 portrays the results of the mental arithmetic exercises from the different forms. Minor dispersion shows that the results of the first and second-form pupils in Estonian schools are quite homogenized. The biggest dispersion is among third form pupils in both, Finnish and Estonian schools.

Table 3. Results of Estonian and Finnish pupils in mental arithmetic exercises.

Form/sex	Highest points	Estonian			Finnish		
		N	Average	SD	N	Average	SD
1st form	20	47	18.70	1.68	56	17.60	2.54
Girls		17	19.20	1.28	29	18.10	2.41
Boys		30	18.50	1.83	27	17.10	2.61
2nd form	14	39	11.80	1.31	37	11.50	2.09
Girls		20	11.75	1.65	21	10.85	2.37
Boys		19	11.90	0.87	16	12.40	1.25
3rd form	23	71	20.70	2.48	37	17.10	2.97
Girls		32	20.60	2.71	17	16.05	2.58
Boys		39	20.80	2.31	20	18.00	3.06

On the basis of the quality of the exercise, good results were noted among the first- form pupils in summation and subtraction exercises with figures between 0 and 10 (Table 4). Most mistakes occurred amongst both Estonian and Finnish pupils in mathematical exercises within the numerical range of 0 to 20 (e.g. 12+5, 19—4). Among the second-form pupils, both countries performed well in summation and subtraction exercises not below tens (e.g. 12+7, 24+12).

Table 4. Mental arithmetic results of Estonian and Finnish pupils on the basis of the quality of the exercise.

Form	Exercise quality	Estonian		Finnish	
		Average / highest	% highest	Average/ highest	% high-est
1st form	Summation on a numeric range from 0 to 10	7.9 / 8	98	7.3 / 8	91
	Subtraction on a numeric range from 0 to 10	5.8 / 6	97	5.5 / 6	91
	Summation on a numeric range from 0 to 20	1.7 / 2	87	1.6 / 2	78
	Subtraction on a numeric range from 0 to 20	3.3 / 4	82	3.3 / 4	83
2nd form	Summation and subtraction without exceeding tens	4.8 / 5	96	4.4 / 5	89
	Summation and subtraction up to tens (from tens)	1.8 / 3	59	2.4 / 3	80
	Summation and subtraction exceeding tens	5.3 / 6	88	4.7 / 6	78
3rd form	Summation and subtraction of two figures	8.3 / 9	93	8.0 / 9	89
	Multiplying and division of two figures	8.4 / 9	93	6.8 / 9	75
	Three figure calculations (including brackets in exercises)	4.0 / 5	80	2.3 / 5	45

Challenging exercises for both Estonian and Finnish pupils included summation and subtraction exercises to tens and / or from tens (e.g. 28+12, 50─14), as well as summation and subtraction exercises exceeding tens (e.g. 53+8, 81─9). Good results were achieved among third-form pupils with two figure summation and subtraction exercises (e.g. 47+35, 97─4). Most mistakes occurred in three figure exercises (e.g. 5+7·6) which Estonian pupils performed better.

Significant differences between the countries and the sexes appear in the written exercises (Table 5).

Table 5. Results of Estonian and Finnish pupils in written exercises.

Form/ sex	Estonian			Finnish		
	1st exercise	2nd exercise	3rd exercise	1st exercise	2nd exercise	3rd exercise
	Performance rate %	Performance rate %	Performance rate %	Performance rate %	Performance rate %	Performance rate %
1st form						
Girls	94	100		70	76	
Boys	80	97		67	74	

2nd form						
Girls	95	100	35	91	62	10
Boys	100	90	16	100	75	0
3rd form						
Girls	100	59	66	65	18	35
Boys	97	51	80	85	40	100

Estonian first-form pupils received significantly better results in the second written exercise (χ^2=12.491, df=2, p<0.002). The second-form Estonian pupils received better results in the second (χ^2=9.419, df=2, p<0.002) and third exercise (χ^2=25.551, df=2, p=0.000). The difference between the two countries is notable. Among the third-form pupils, the difference between the countries is significant in the first exercise (χ^2=15.203, df=2, p=0.000) as well as in the second exercise (χ^2=6.213, df=2, p<0.013), where Estonian pupils received higher marks. No differences between the sexes among the first and second-form pupils were noted. However, differences between the sexes in the results of the third exercise were noted, as Finnish boys received better results than Finnish girls (χ^2=13.104, df=1, p=0.000).

Discussion

Working memory has an influence on the learning of mental arithmetic and is a foundation for the thinking processes of a pupil in general, irrespective of the subject. Therefore mental arithmetic cannot be handled only in a context of developing mathematical skills. This highlights the importance of arithmetic exercises. The more chance the pupils have to practice, the better are their results in mental arithmetic, and the more the working memory develops and forms a basis for learning in general. The methods which the teachers apply in arithmetic exercises are important, because it is a mental activity which helps to develop the working memory.

Differences in the results of mental arithmetic tests

In this research, the first, second and third-form pupils received good results in mental arithmetic exercises of addition and subtraction of up to tens. On the other hand, transferring to memory of noting and borrowing are needed in more complex arithmetic exercises. It is expected that arithmetic performance will be slower and less accurate in exercises that require noting or borrowing compared to problems that do not require the application of such methods. In these instances a pupil needs the long term memory, where knowledge has been saved, and the working memory for maintaining and using information in the short term. With reference to earlier studies (e.g. Fürst & Hitch, 2000, Noel et al., 2001, Sandrini et al., 2003, Imbo et al., 2005, p.7), mistakes in the results of mental calculations can be caused by the incomplete noting and borrowing methods. From the other hand this is essentially related with development psychology and how the child's working memory functions develop. This study didn't give us information on pupils' approach behaviours which have strong influences on the results (Van der Heijden, 2004).

Differences between the sexes

In the third form Estonian girls received better results in written exercises, and the first-form girls received better results in mental arithmetic. When comparing the results of the mental arithmetic exercises, it became evident that boys in general received slightly better results.

The earlier studies have confirmed better results in mathematics among girls (Linnanmäki, 1997, p.283–300; Hyde, Fennema & Lamon, 1990, p.139–155). According to Van der Heijden (2004, p.14), the differences between the results of boys and girls may be explained by their approach behaviour i.e. the solution strategy they apply in mental arithmetic exercises.

We have some evidence on distinctions between the sexes in cognitive development of the child. However, the social and cultural environment as an important factor is more highlighted.

Differences between the countries

We noticed that generally dispersion among the results of Finnish pupils is larger. It could be explained by the differences in national curricula, but it could depend on teaching approach. An earlier survey points out that Estonian pupils feel pressure at school (Sarv, 2008). For example, the results of the PISA research in 2006 showed that Estonian pupils had quite low learning motivation (Assessing Scientific ..., 2006). The TALIS survey (2009) showed that Estonian teachers value the classroom climate highly (especially discipline in the classroom). It could be that individual needs and development of the pupils' are not taken into consideration. This may give good results in marks but reduces the learning motivation and doesn't support cognitive development of pupils.

Conclusions

The current research provides a comparative view of the Estonian national curriculum, especially in math studies. Based on our research we can make the following suggestions for curriculum development.

Since the first form pupils acquire summation and subtraction on the numeric range from 0 to 10 relatively fast, subtraction on a numeric range from 0 to 20 causes more mistakes. In the second form, summation and subtraction up to tens (from tens) and summation and subtraction with figures exceeding tens needs more practice, because noting or borrowing is needed here for performing a mental arithmetic task. Teachers need to acknowledge that in shaping a mental arithmetic skill it is essential to introduce, teach and use different strategies. The teaching literature should also contain different strategies for performing mental arithmetic.

In implementing the national curriculum, the use of pupils' mental arithmetic work as a means of developing memory should be emphasized. Although there remains the option of using calculators and computers, more emphasis should be placed on the skills of calculation in relation to managing everyday life, as well as in terms of an activity which supports mental development. Teaching materials have to contain different problem exercises, which need different calculation strategies in order to solve them. To avoid calculation becoming tedious, it is important that learning games are used in the learning process, because through games pupils make the first steps in the science world. Further, these learning games should be a natural part of the lesson. To differentiate learning it is important to take different levels into consideration, and offer the possibility of choice for pupils. In relation to sex differences, boys have to be offered short-term, but exertion taking and challenging exercises, where they can focus on a solution of a problem. To motivate girls, calculation should be combined with different role-plays, descriptions and explanations.

In further studies it is important to analyse the factors which influence pupils' results according to the demands of the national curriculum and how the experts working with curriculum development consider the cognitive development of the child.

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