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Analysis of Dispersion and Attenuation Characteristics of the Bragg Fiber

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Abstract

In the article, a model of analysis and an algorithm for calculation of the group velocity dispersion and the losses of light, spreading in a Bragg optical fiber, are proposed. A mathematical model, based on the method of the geometrics optics is included. The influence of the fiber clad dielectric properties on the light propagation is taken into account. A comparison of the dispersion and the bandwidth of fibers with different number layers for low modes are made. Examples of Bragg fibers with different optical properties and attenuation are shown. Finally, advantages of the Bragg fiber, observed on the diagrams of attenuation, dispersion and bandwidth, are listed.

Keywords

Bragg Fiber; Bandwidth; Dispersion; Losses

Introduction

In the conventional optical fibers, propagation of the light only in the core is caused by the total internal reflection from the clad. This is possible when the effective refractive index of the core is greater than the cladding. The Bragg fiber is one dimensional fiber and is constructed as uniaxial cylindrical layers. The fiber cladding is a dielectric mirror, realized as a multilayer dielectric coverage. This kind of fibers is a part of so called Photonic Crystal Fibers (PCF) or microstructured fibers. The term marks the most of the optical fibers with a complicated clad structure; which often includes one- or two-dimensional periodic structures and has substantial influence on the optical properties of the fiber PCF decrease significantly the restrictions, caused by the materials and the construction of the conventional optical fibers and allow the characteristics of the fibers as bandwidth, dispersion, nonlinearity, modal area to be controlled more freely.

The attenuation of the light in the fiber during transmission to long distances is critical. In the usual fibers, the attenuation is limited by the Rayleigh scattering of the energy of spreading modes. Because of that, when manufacturing, the fibers should use pureness materials with low dielectric losses.

All PCF allow easy changing of dispersion and can compensate the dispersion in the telecommunication fibers. The high attenuation can be overcome by a new design.

Examples of different kinds of optical cylindrical PCF are shown in FIG. 1. These fibers have different mechanism to keep the light inside the core.

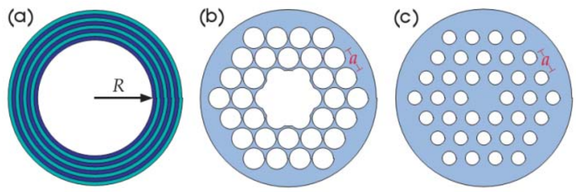

FIG.1 PCF FIBERS. (A) BRAGG FIBER, WITH A ONE-DIMENSIONAL PERIODIC CLADDING OF CONCENTRIC LAYERS. (B) TWO-DIMENSIONAL PERIODIC STRUCTURE (A TRIANGULAR LATTICE OF AIR HOLES – "HOLEY FIBER"), CONFINING LIGHT IN A HOLLOW CORE BY A BAND GAP. (C) HOLEY FIBER THAT CONFINES LIGHT IN A SOLID CORE BY INDEX GUIDING.

Nowadays there is no universal and precise enough method of evaluation of the fiber losses, which allows an optimizing its construction properties (diameter, layers thickness, refractive index of the material) in a given frequency band. Numerical solution of the Maxwell equations is realized in many software systems, but it needs significant computation power. Solution of the system of partial differential equations takes tens of minutes or hours. Despite of some accessible software modules, creating such a completed software tool is not an easy task. The software systems, ready to use, are specialized to analyse a group of similar problems and they are expensive.

Next we will propose an engineering accuracy method for analysing the light propagation in a multilayer Bragg fiber with a hollow core as a result of tunnelling and taking account the dielectric losses of the fiber clad.

Analysis of Bragg Fiber by the Geometric Optics Method

Suppose we have a Bragg optical fiber with *M* cylindrical layers of cladding. The core refractive index is n_a. A cross-section view of the fiber is presented in FIG. 2.

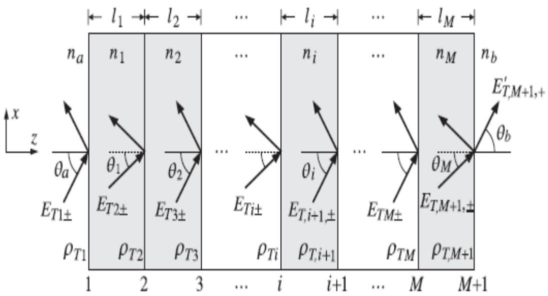

FIG. 2 OBLIQUE INCIDENCE IN MULTILAYER DIELECTRIC STRUCTURE

Because of central symmetry of the fiber, only half of a longitudinal section is shown. We suppose that the fiber is like a flat multilayer dielectric structure. From the left to the right in the figure is located the core, layer 1, layer 2, layer 3,…, layer *M* of the cladding. The symbol n_a marks the core refractive index. n_i is the refractive index of the layer number i. n_b is refractive index of the uppermost layer. The thickness of the layer i is signed as, *i=1, 2, 3...M*. Here $E_{T,k\pm}$, *k=1, 2,…, M+1* marks falling (+) or refracted (-) parts of electrical field at the border of layer k and the layer on the left before it. The leaking field from waveguide is signed as $E'_{T,M+1,+}$. The Snell's law describes the relationship between the angles of incidence and refraction and the refractive indexes of the material at the border surface between every two of all *M+1* boundary layers:

$$n_a sin\theta_a = n_i sin\theta_i = n_b sin\theta_b, \tag{1}$$

when *i=1,2,3,…,M.*

We suppose there is no incident field on the right of the structure. Here ρ_{Ti} is a transverse refractive index of surface number *i*, defined as:

$$\rho_{Ti} = \frac{n_{Ti-1} - n_{Ti}}{n_{Ti-1} + n_{Ti}},\ i=1,2,3,\dots,M+1, \tag{2}$$

where $n_{T_0} = n_{T_a}$ and $n_{T_{M+1}} = n_b$. The phase shifting of incident beams, passing through the layer number i is given by $\delta_i = k_0 n_i cos\theta_i$, where k_0 is the wave number in the free space, $k_0 = \frac{2\pi}{\lambda}$.

In every layer, the transverse refraction factors for *TM* and *TE* polarization are described as:

$$n_{Ti} = \begin{cases} \dfrac{n_i}{\cos\theta_i}, & TM\ polarization \\ n_i, & TE\ polarization \end{cases}, \tag{3}$$

$$i = a, 1, 2, \dots, M, b.$$

The spreading matrix associates the electric field density of the incident with refracted beam recursively:

$$\begin{bmatrix} E_{Ti,+} \\ E_{Ti,-} \end{bmatrix} = \frac{1}{\Gamma_{Ti}} \begin{bmatrix} \exp(j\delta i) & \rho_{Ti}\exp(-j\delta i) \\ \rho_{Ti}\exp(j\delta i) & \exp(-j\delta i) \end{bmatrix} \begin{bmatrix} E_{Ti+1,+} \\ E_{Ti+1,-} \end{bmatrix}, \tag{4}$$

i=M, M-1,…, 1.

Here the refractive factor is $\Gamma_{Ti} = E_{Ti,-}/E_{Ti,+}$ and associates Γ_{Ti+1} with recursion:

$$\Gamma_{Ti} = \frac{\rho_{Ti} + \Gamma_{Ti+1}\exp(-2j\delta i)}{1 + \rho_{Ti}\Gamma_{Ti+1}\exp(-2j\delta i)},\ i=M, M-1,\dots, 1, \tag{5}$$

where $\Gamma_{T,M+1} = \rho_{T,M+1}$. A similar association exists between electric and magnetic field on the boundary between 2 layers:

$$\begin{bmatrix} E_{Ti} \\ H_{Ti} \end{bmatrix} = \begin{bmatrix} \cos\delta i & j\eta_{Ti}\sin\delta i) \\ j\eta_{Ti}^{-1}\sin\delta i & \cos\delta i \end{bmatrix} \begin{bmatrix} E_{Ti+1} \\ H_{Ti+1} \end{bmatrix}, \tag{6}$$

where *i=M, M-1,…, 1.*

In (6) the transverse characteristic impedances are defined as:

$$\eta_T = \begin{cases} \eta\cos\theta & TM\ polarization \\ \dfrac{\eta}{\cos\theta} & TE\ polarization \end{cases}, \tag{7}$$

where $\eta_{Ti} = \eta_0 / n_{Ti}$. The total power of incident beam P_{in} towards the wave vector direction, its z component and the input power into first layer P_1 for *TE* and *TM* polarizations are given by:

$$P_{in} = \frac{1}{2\eta_a}\left|E_{in}\right|^2,\ P_{in,z} = P_{in}\cos\theta_a,$$

$$\eta_a = \eta_0 / n_a,\ P_1 = P_{in,z}(1 - \left|\Gamma_1\right|^2). \tag{8}$$

The losses of reflection are:

$$\frac{P_1(\lambda)}{P_{in,z}(\lambda)} = 1 - \left|\Gamma(\lambda)\right|^2, \qquad (9)$$

The normalized attenuation is:

$$B(\lambda) = \frac{P_{in,z}(\lambda) - P_1(\lambda)}{P_{in,z}(\lambda)} = \left|\Gamma(\lambda)\right|^2. \qquad (10)$$

We discuss a fiber with radius a, core refractive index n_a, cladding refractive index n_1. The light is spreading when $sin\theta_a > n_1/n_a$. FIG. 3 shows the trajectory of beams.

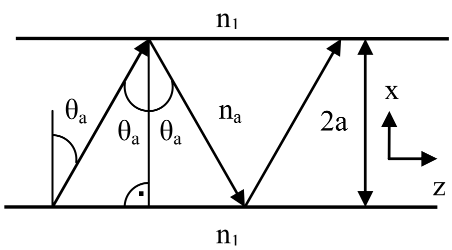

FIG. 3 MERIDIONAL BEAMS IN THE CORE

The spreading modes of index m have discrete values of the angle θ_a. The transverse resonance condition expresses the phase change for one zigzag of the beam, multiple by 2π:

The spreading modes of index m have discrete values of the angle θ_a. The transverse resonance condition expresses the phase change for one zigzag of the beam, multiple by 2π:

$$4ak_0 n_a cos\theta_a + \varphi = 2m\pi \qquad (11)$$

where φ is a phase changed by reflection from the boundary core - cladding. Between two sequential refraction, the path of light beam is $lz = 4a.tg\theta_a$. If the length of the fiber is equal to L, there are N reflections inside the core, where $= L/lz$. The losses $B(\lambda)$ in a fiber with length L for *TE* and *TM* polarization are proportional to N:

$$B(\lambda) = N.\left|\Gamma(\lambda)\right|^2 = \frac{L.\left|\Gamma(\lambda)\right|^2}{lz}. \qquad (12)$$

The reflection factor is $\Gamma(\lambda) = \left|\Gamma(\lambda)\right| \exp(\arg(\Gamma(\lambda))$, and the phase change when the beam passes through the entire fiber is $N.arg(\Gamma(\lambda))$. The delays of beams with *TE* and *TM* polarization are respectively τ_e and τ_m. They are calculated by an expression:

$$\tau(\lambda) = \frac{\lambda^2}{2.\pi.c}\frac{d(\arg(\Gamma(\lambda))}{d\lambda}, \qquad (13)$$

where $\Gamma(\lambda)$ is reflection factor for *TE* and *TM* modes.

The chromatic dispersion of the fiber d_{cr} is given by the expressions:

$$d_{cr}(\lambda) = \frac{d\tau(\lambda)}{d\lambda} = \frac{2\tau(\lambda)}{\lambda} - \frac{\lambda^2}{2\pi.c}.\frac{d^2(\arg(\Gamma(\lambda))}{d\lambda^2} \qquad (14)$$

The total attenuation $BN(\lambda)$ along the whole length of the fiber is expressed as a sum of the particular area attenuations $Bi(\lambda)$ in which the reflected beam is signed with integer number *1,2,3,…N*, as follows:

$$B_N(\lambda) = B_1(\lambda).B_2(\lambda).B_3(\lambda)...B_N(\lambda) = B_i^N(\lambda) = (\Gamma^2(\lambda))^N. \qquad (15)$$

In expression (15) $\Gamma(\lambda)$ are complex refractive coefficients in areas with incident angle θ_a :

$$\Gamma(\lambda) = \alpha + j\beta\ . \qquad (16)$$

The real part α accounts the propagation losses, and the imaginary part β is a phase coefficient. When in the clad, there are dielectrical losses, the refractive index n and wave vector k are complex numbers too:

$$n = n' - jn'',\ k = k' - jk''. \qquad (17)$$

The propagating electrical field trough z axis is represented as:

$$\begin{aligned} E(z) &= E_0\, exp(-\Gamma z) = E_0\, exp(-\alpha z)\, exp(j\beta z) = \\ &= E_0\, exp(-jkz) = E_0\, exp(-k''z)\, exp(-jk'z) = \\ &= E_0\, exp(-k''z)\, exp(-j\frac{2\pi n'}{\lambda}z)\ . \end{aligned} \qquad (18)$$

The phase coefficient β can be expressed by the real part of the refractive coefficient α and the wave length in free space λ as $= \frac{2\pi n'}{\lambda}$. Usually

$$\alpha[dB/km] = 10\, lg(\alpha z)\ . \qquad (19)$$

From (17, 18, 19) follows:

$$\alpha = k'' = \frac{2\pi n''}{\lambda}\ , \qquad (20)$$

$$n'' = \frac{\lambda\alpha}{2\pi}\ . \qquad (21)$$

When dielectrical losses are low, $n'' \approx tg\ \delta$.

Example

Let's have a *M*-layers Bragg fiber with an air core, a core radius *a=62 μm; na=1.0, nb=1.5*. The cladding consists of three different materials, each of them having different losses. (*H, L, b* layers). The thickness of the layers is chosen so as to make a phase difference of 90⁰ for the central wave length λ_0*=1500nm*.

The user enters loss coefficients of the layers α_H, α_L, α_b. After solving the characteristic equation (11) for *m=1* and doing some calculations according to the above proposed mathematical model, we obtain as a result the refraction coefficients of the multilayer cladding.

Then we calculate the normalized attenuations $B_{te}(\lambda)$ for *TE* polarization in [dB] for a fiber without deformation.

After that, we obtain the group delay and dispersion. The influence of the losses α[dB/km] of the clad on the attenuation for *TE* wave is shown in FIG. 4.

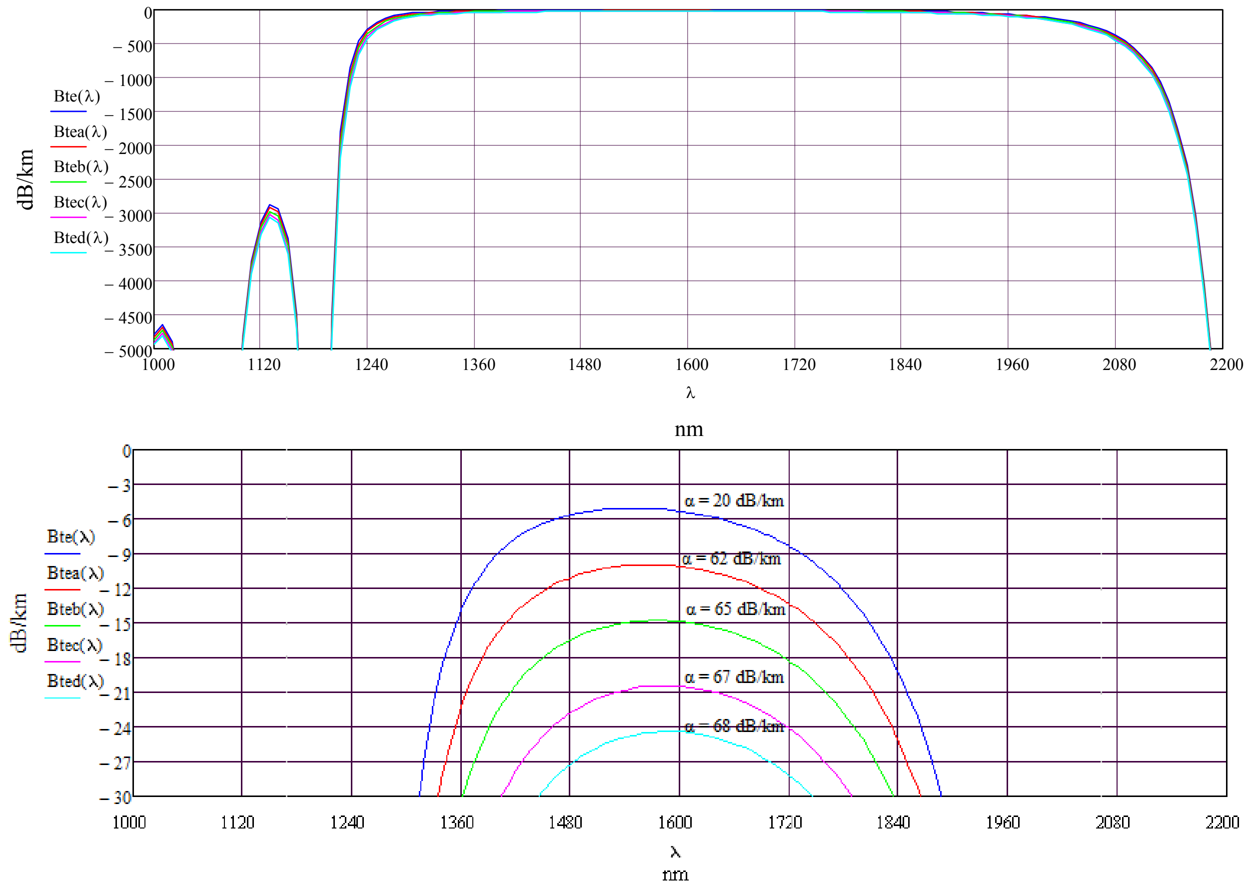

FIG. 4 ATTENUATION BTE(Λ) WHEN THE MATERIAL HAS DIFFERENT LOSSES. THE UPPER DIAGRAM RANGE IS 30 DB, THE LOWER DIAGRAM RANGE IS 5000 DB

FIG. 5 shows a comparison between attenuations Bte of the first *TE* mode in the Bragg fiber with *M=9* layers of the cladding and conventional SMF, depending on losses of material α[dB/km], when the wavelength is fixed to λ*=1550nm*.

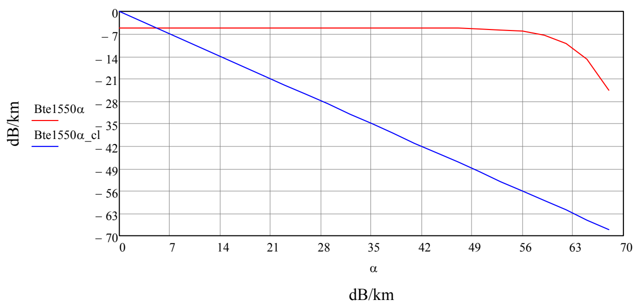

FIG 5 ATTENUATION OF BRAGG FIBER AND SMF WHEN LOSSES OF THE CLAD ARE DIFFERENT

The comparison of the attenuation of the Bragg fiber and conventional SMF shows, that in case of α*<7dB/km* the conventional SMF have lower attenuation per unit length. When losses α rise above *7dB/km*, the Bragg fiber can have lower attenuation than SMF.

FIG. 6 shows changing of the attenuation of the fundamental *TE* mode (*m=1*), in a Bragg fiber with core radius $a = 62\mu m$, λ*=1550nm*, as function of losses in material α. Here are some diagrams of the attenuation *Bte(*α*)* in a fiber with different number of clad layers (*M=9-21*). The diagram shows, that with increasing of the number of the dielectric clad layers the attenuation of the fibers decreases, this decreasing changing more intensively in fibers containing fewer numbers of layers in the clad, than in fibers containing over 13 layers in the clad. The increasing of α above *50dB/km* abruptly increases total attenuation of the fiber. However, even in fibers with 11 layers and losses α*<68dB/km*, total attenuation is less than *-20dB/km*.

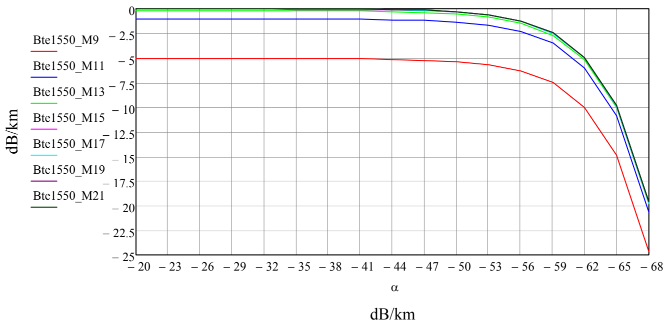

FIG. 6 ATTENUATION BTE(A) IN A BRAGG FIBER WITH M=9-21 CLADDING LAYERS

FIG. 7 shows shifting of the maximum of the wave characteristic of the Bragg fiber with different numbers of layers *M=9, 11, 13, 15*, when losses of material in cladding *α* increase. It is shown that this shifting depends more strongly on losses of the material cladding *α* in fiber with larger numbers of layers in the clad.

FIG. 8 shows the changing of the bandwidth *Δλ* [nm] of the wave characteristic *Bte(λ)* on level *-3dB*, depending on the losses of the fiber cladding material *α*[dB/km] when *M=9, 11, 13, 15* cladding layers.

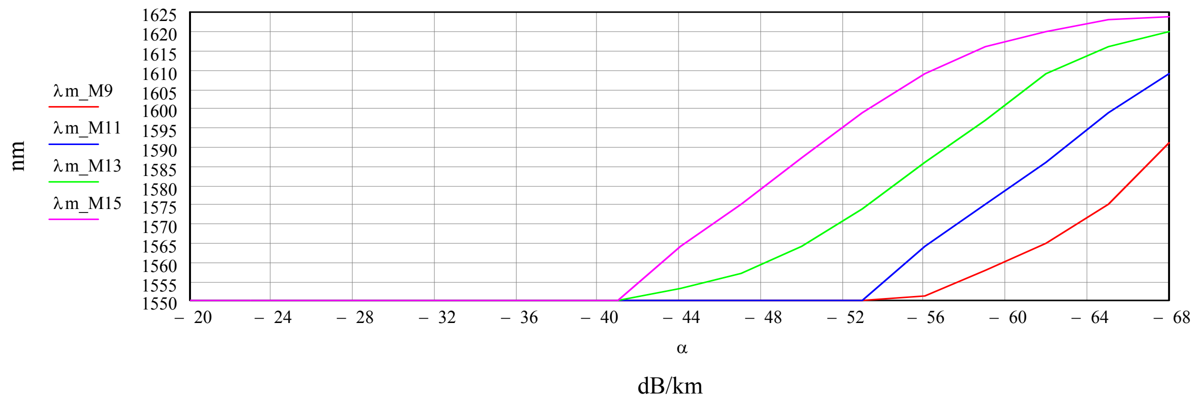

FIG.7 MAXIMUM OF THE WAVE CHARACTERISTIC, DEPENDING ON THE LOSSES IN A FIBER WITH DIFFERENT NUMBER CLAD LAYERS

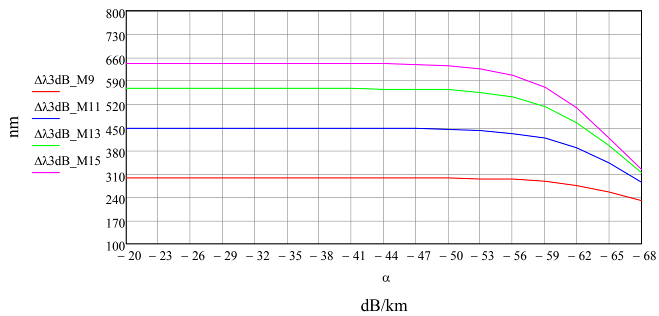

FIG. 8 CHANGING THE BANDWIDTH, DEPENDING ON LOSSES IN A FIBER WITH DIFFERENT NUMBER CLAD LAYERS

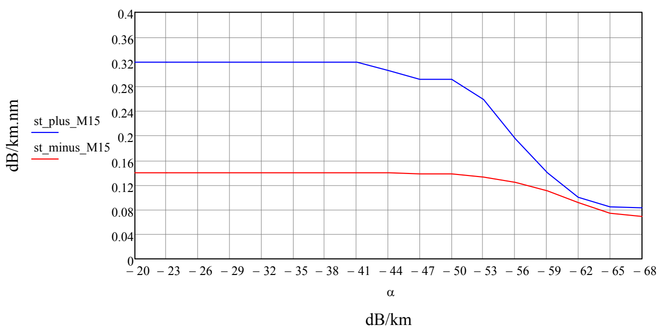

FIG. 9 SLOPE OF THE WAVE CHARACTERISTIC AS A FUNCTION OF LOSSES

The bandwidth *Δλ* is narrower in fiber with fewer numbers of layers and decreases when *α* rises. For example, when *M=9* layers, and *α=0dB/km* the bandwidth is *Δλ=300nm* and when *α=68dB/km* the bandwidth decreases to *Δλ=230nm*.

FIG. 9 shows the changing of the steepness st_plus and st_minus of short wave and long wave slopes of the wave characteristic *Bte(λ)*, depending on the losses *α*[dB/km], the number of clad layers being *M=15*.

The studies show that steepness of the slopes of wave characteristics rises when numbers of fiber layers rise due to the decreasing of the attenuation and increasing the flatness of the wave characteristic *Bte(λ)* in the bandtwidth.

FIG. 10 shows the attenuation *Bte(λ=1550nm)* in the fiber as a function of clad material losses *α=20-68dB/km*. On a common diagram are represented attenuations in fibers with *M=9* layers and different core radius (*a=10-100 μm*).

The chart demonstrates the decreasing of attenuation of the Bragg fiber, when the radius of the hollow core increases.

FIG. 11 presents the attenuation of the same fiber as a function of the clad material losses, presented by tg δ and with different fiber core radius (*a=10-100μm*).

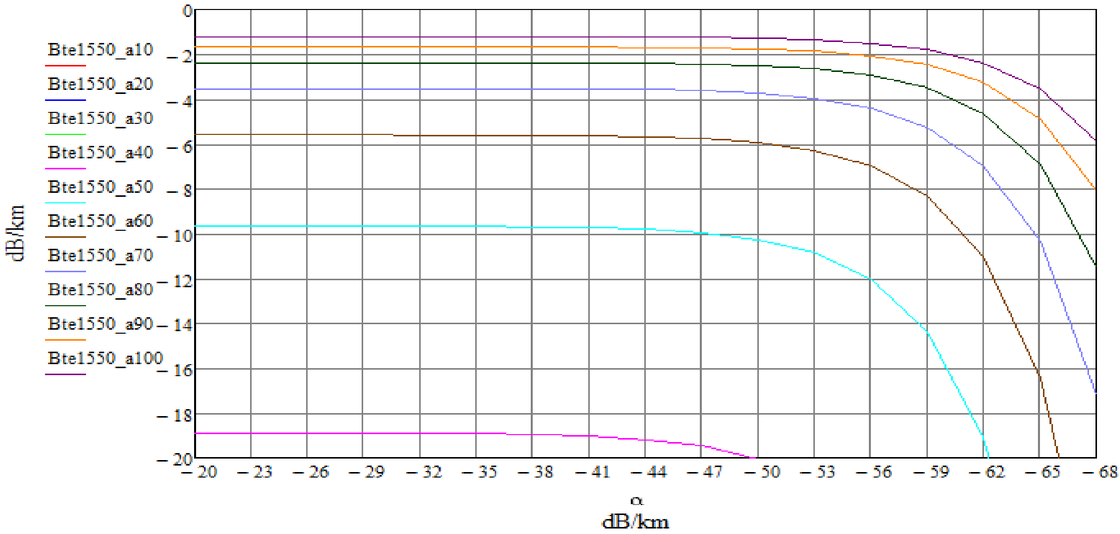

FIG. 10 ATTENUATION, DEPENDING ON THE LOSSES IN A FIBER WITH DIFFERENT CORE RADIUS

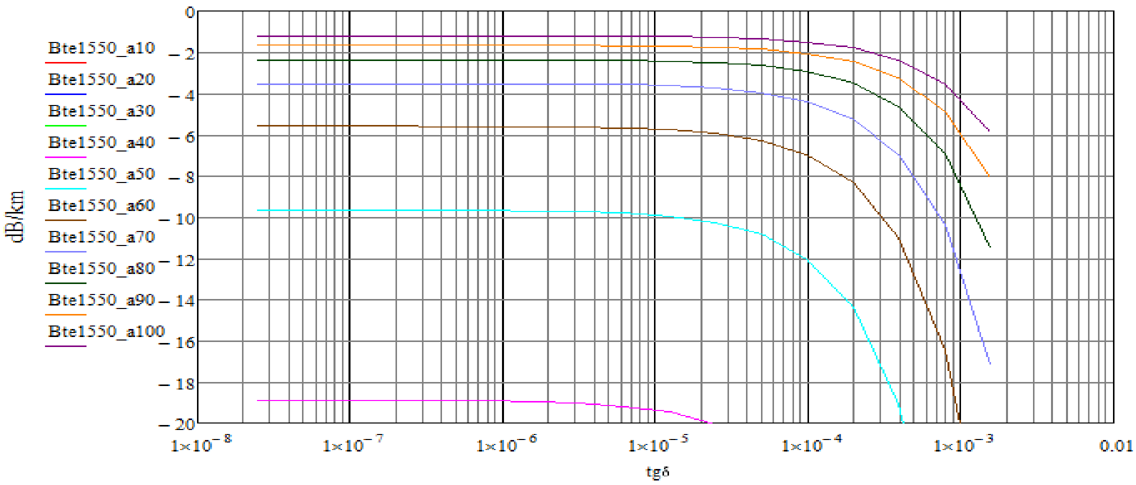

FIG 11 ATTENUATION OF THE FIBER AS A FUNCTION OF CLAD MATERIAL LOSSES AND CORE RADIUS

Conclusions

The proposed mathematical model and the calculation algorithm allow an approximate analysis of the basic characteristics of Bragg fiber. This method makes it possible for optical fibers with *M* (*M*≤*21*) clad layers to be analyzed. The obtained numerical results show the influence of losses of real dielectric materials over the attenuation of the wave propagation in the Bragg fibers.

The comparison between the attenuation of Bragg fiber with small numbers of layers (*M*=*9*) and conventional SMF with the same dielectrical losses. *α* of the material clearly shows that the Bragg fiber has an attenuation greater than the usual SMF, when the losses in the dielectric clad are small (*α*<*7dB*/*km*) due to limited transparency of the cladding. The increasing the losses in the dielectrical cladding (*α*>*7dB*/*km*) makes the Bragg fiber attenuation less than the attenuation in the SMF. In case when *α*<*20dB*/*km*, losses of the dielectric material slightly influence over the attenuation in the fiber. This difference becomes very small when losses exceed *60dB*/*km* ($tg\,\delta > 10^{-4}$). The attenuation increases very fast (by more than *20dB*/*km*) when *α*>*70dB*/*km*. This means that to manufacture Bragg fibers, it is not necessary to use the complicated technology, to get pure materials, usually used for conventional optical fibers.

Increasing the fiber air core radius decreases the attenuation in the fiber.

The bandwidth decreases relatively slightly when the losses of the material increase. The wave characteristic is asymmetric, its short wave slope being shorter and steeper and the long wave one more gradual and longer. The steepness of the slopes depends on value of the losses. When the losses of clad material increase, the steepness of short wave slope decreases and the steepness of the long wave one increases.

When the dielectric losses increase, the dispersion and the group delay time decrease.

The method of geometrical optics is less accurate than numerical methods of calculations of the field, used in some software packets, based on a large group of general and specific methods of solving Maxwell equations. The obtained quantitative and qualitative dependences for PCF Bragg fiber are similar to the other ones. This analytical method of calculation is a lot faster than the numerical ones, because of its simplicity.

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