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Abstract

The focus of this article is to present the modelling tehcniques used on international practice in the evaluation of right life premiums based. The knowledge and models obtained have a common element of mortality risk indicators but these are varied in different parts of the world. The common elements of these studies and models are generally based on a series of indicators which mainly point out their probability of survival and they are named the mortality indicators. These indicators represent the basis for the calculation of the premiums quotes and for the elaboration by the insurers of premium tables. The benefit for the policyholder is to obtain insurance at a fair and competitive price and for the insurer, to maintain the experience of its portfolio in line with mortality assumptions.

Keywords: Insurance Premiums Quota, Actuarially Selection Criteria, Experience Studies.

JEL Classification Codes: C81, O50, G22

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1. Introduction

In order to set the right premiums for the consumers, insurers use the experience studies which are based on information about the past and the future. Usually, an experience study refers to mortality (for life insurance) or morbidity (for health insurance) experience. Experience studies help insurers minimize anti-selection and focus risk selection towards better and more profitable risk categories.

An experience study compares the actual experience on a block of business with a model of how the insurer anticipated that experience to look. This is referred to as an A/E ratio, where *A stands for actual*, and *E stands for expected* risks (JJ lane Caroll, 2007:31).

A detailed understanding of historical experience is important in order to estimate how changes in underwriting selection might plausibly impact future results. By evaluating its past experience, the company can make more confident decisions in the development of future products and the underwriting of its business. By tracking the right data elements, insurers can closely monitor developing experience driven by changes in underwriting and/or business practice.

The very basic requirements to complete any type of mortality study include: policy issue date, issue age (or date of birth), gender, smoker status (dependent on the expected basis), policy face amount in force policy status (active, death, lapse, etc), termination date, claim settlement amount (if different from policy face amount), rating information about substandard risks, cause of death, and history of underwriting guidelines and preferred criteria throughout the study period (published and internal to the insurance company).

Among these requirements, the most common *risk selection* factors that demonstrate sufficient credibility to form the basis of a study include gender, age at policy issue, smoker status, amount assured (as an indicator for socioeconomic status), and preferred class or underwriting rating. The latter two categories evaluate the combination of all underwriting that is performed on the group of lives.

It is also important to evaluate policy duration since issue. This is because risk factors tend to manifest themselves over time, resulting in a mortality curve with a selection period that eventually grades into ultimate mortality (ie the mortality expected at a given age in the absence of underwriting). Evaluation of results by

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duration can yield information about the effectiveness of the underwriting selection.

Specific underwriting criteria whose effects can be studied include the depth and scope of a medical or paramedical exam, The usage of these tests can be included in the study by incorporating historical underwriting guidelines. In the United States, the top five medical tests used to evaluate preferred lives business are blood pressure, cholesterol, cholesterol ratio, build and family history. In each case it is the actual result of the test, and not just the existence of the test, that is important. Currently, however, few companies store this level of historical underwriting data electronically.

2. The factors of determining of life premiums

When speaking of life insurance, one should know that the insured person's rights and obligations are generally based on a series of indicators which mainly point out their probability of survival. These indicators are calculated by the National Commission for Statistics in each country and determine the insurance premiums quota. In the Annex no. 1 are presented mortality indicators in the Romania. The most important role in determining the premium quota is played by the *actuary*, also named the life insurance mathematician.

The figure 1 shows the factors according to which life premiums are determined: the indicators of the mortality tables; the frequency of payment of the premiums; the types of life insurance policies differing in terms of the covered risk (survival insurance, death insurance or mixed life insurance) and the way of paying the indemnity by the insurer.

From the insurer's point of view, the life premium owed by the insured person is designed as the gross premium and consists of two elements: the net premium and the supplement or the extra premium.

The net premium serves to create the necessary fund for covering the indemnities or the insurance indemnities. The *determination of the net premium* takes into account the probability of risk occurrence and the intensity or the frequency of its manifestation. The *probability of risk occurrence* is given by the indicators of the mortality tables determined by the age of the insured person, in case of survival as well as in case of death. The *intensity of risk manifestation* is also given by the premium level, for risks of high intensity, the premium is also high, and for risks of low intensity, the premium is also low. If the risk has a variable manifestation during the contract, the premium will be modified in proportion to its intensity. The *supplement or the extra premium* covers the insurer's purchase and management

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overhead, as well as the ways of creating benefits. The value of these costs varies in terms of different types of insurance products and of different ways of dealing with them.

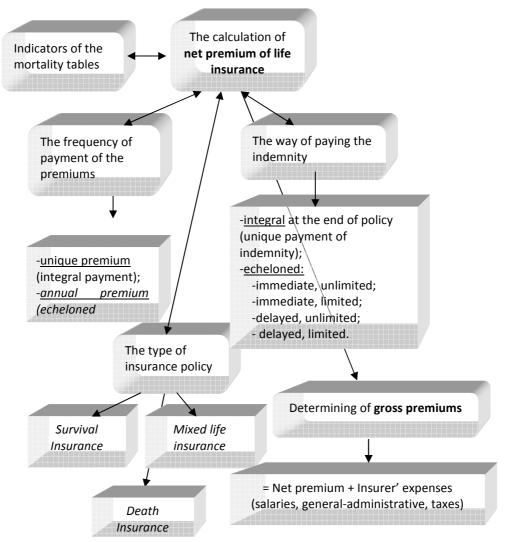


Figure 1. The factors which influence the determining of life premiums

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2.1. Indicators of the mortality tables

For an easy understanding of the mortality table as well as for the planning of scientific rules in order to determine different calculation elements of the premium, the *mortality tables indicators* are based on an international system of symbols as it follows:

• x – age of the insured persons;

• l(x) - survival function indicates how many persons belonging to an assumed generation of 100,000 living persons are still alive when attaining the age of x years;

• p(x) – *survival probability* expresses the chances of a person attaining the age of x years to continue to live till the age of x+1 years;

• q(x) - death probability expresses the risk undertaken by a person who has already turned x years, that of dying before attaining the age of x+1 years;

• d(x) - number of persons supposed to pass away within x and (x+1) years indicates how many persons of x years old passed away before the age of x+1 years, being determined by the difference between the number of survivors aged x years (I_x) and the number of survivors aged x+1 years (I_{x+1}) ;

• E(x) – hope of life at the age of x or life expectancy represents the average number of years left to be lived for a person surviving the age of x;

• *Hope of life at birth* also named the span of life indicates the average number of years supposed to be lived by a newborn baby.

A common measure is the difference in the life expectancy between male and female. Life expectancy can be measured from any age, and is often measured from birth, but for assessing the impact of the mortality differential between males and females is appropriate to consider life expectancy in middle age.

For example, the researchers studies show the difference between life expectancy at age 45 for females and males in the EU Member States. Female life expectancy at age 45 significantly exceeds that of males in all EU countries. The difference ranges from 3.4 years (in Denmark) to 6.5 years (in France). Looking forward, in the UK Government's official national population projections (produced by the Government Actuary's Department), the sex differential in life expectancy at age 50 is expected to continue for many years in the future. From 3.6 years in 2002, it is expected to narrow only to 3.1 years by 2041 (Memorandum by Swiss Re, http://www.publications.parliament.uk).

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For the life insurances, *these indicators represent the basis for the calculation of the net premiums* and for the elaboration of premium tables. In order to estimate the net premiums, the actuarial science provides general formulas for the estimation of these indicators, made up of symbols. A real determination of the premiums involves the subrogation (commutation) of one formula made of symbols with the figures corresponding to the insured person's age and to the interest which increase the premium. These figures meant to achieve the commutation of the formula are already calculated within the mortality tables for each age and they are called numbers of commutation. For the actualization factor (discount) marked with vⁿ, the percentage used by the insurance company in order to calculate the interest for the premiums, is considered and it accumulates in time.

2.2. The frequency of payment of the premiums

For the estimation of the premium, the payment possibilities are taken into consideration. For the life insurance, the premium to be paid is cashed once as a unique premium or echeloned premiums.

The *unique premium* is estimated in order to cover the risk during the whole insured period as the insurer cashes the total amount afferent to the insurance duration at the beginning of the contract. The cashed unique premium and the afferent interest resulting from its investment, will be used for the payment of the indemnity. This modality of payment is less used in practice, being applied for long term life insurances.

Insurances involving the *payments echeloned premiums* are often requested by the insured persons, because the amount representing the unique premium constitutes an important financial effort.

2.3. The way of paying the indemnity and the type of insurance policy

Another factor influencing the amount of the net premium is determined by the *payment possibility used for the insurance indemnity and the type of the insurance policy* (survival insurance, death insurance or mixed life insurance) and it is considered separately for each insured risk. Thus, in practice, there are the following situations considering the modality of paying the indemnity:

1. The indemnity is completely paid in a certain number of years starting from the contract moment of the insurance policy (the unique payment of the indemnity).

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2. The indemnity is paid in installments as it follows:

- unlimited immediate annuities – the insured person pays the net premium at the conclusion of the insurance policy in order to receive the indemnity in installments (annuities), shortly after the conclusion of the contract (immediate annuities), during his whole life period, at the beginning or at the end of the year;

- limited immediate annuities - the insured person pays the net premium at the conclusion of the insurance policy in order to receive the indemnity in installments, shortly after the conclusion of the contract, for a limited period of time (limited immediate annuities);

- unlimited delayed annuities - the insured person pays the net premium at the conclusion of the insurance policy in order to receive the indemnity in installments (annuities), after a certain period of time from the conclusion of the insurance policy (delayed annuities), during his whole life period (delayed life annuities);

- limited delayed annuities - the insurer pays the indemnity in a certain number of years from the conclusion of the insurance policy (delayed annuities), but for a limited period of time (limited).

The *death insurance* relies on the premise that the insurer will pay the beneficiary of the insurance a certain amount of money at the date of death of the insured person. The determination of the unique net premium, in the case of death insurances, takes into consideration the contractual duration of the insurance which may be: undetermined (for life), a period of several years or a short period of time.

In practice, the *life insurance policy reflects more often a mixed nature*, thus, it covers the survival risk, as well as the death risk. Therefore, the insurer will pay the indemnity to the insured person, at the termination of the contract if the last one is alive, or the sum will be paid to the successors, at the date of death of the insured person. So, the unique net premium owed by the insured person is calculated by summing up the net shares of the premiums afferent to the two insured risks.

3. The net and gross premiums for all types of life insurances

As we mentioned before, the net premium differs in accordance with the type on insurance policy, the insurance period and way of indemnity payment by the insurers. In the Table 1, we synthesize the formulas for determining the unitary (at 1 c.u. indemnity) unique (integral payment of premium by the insured person) net premium for the three basic life insurance types.

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S	urvival Insurance	Death Insurance				
The payment of	The unitary unique net premium	Insurance period	The unitary unique net premium			
indemnity The unique payment of the indemnity	$_{n}E_{x}=\frac{D_{x+n}}{D_{x}}$	Death insurance one year term	$A_x = \frac{d_x}{l_x}$			
Unlimited immediate annuities	$a_x = \frac{N_x}{D_x}$	Undetermine d term	$A_x = \frac{M_x}{D_x}$			
Limited immediate annuities	$a_x = \frac{N_x - N_{x+n}}{D_x}$	Limited immediate insurance	$A_x = \frac{M_x - M_{x+n}}{D_x}$			
Unlimited delayed annuities	$a_x = \frac{N_{x+n}}{D_x}$	Unlimited delayed insurance	$_{n/}A_{x}=\frac{M_{x+n}}{D_{x}}$			
Limited delayed annuities	$\int_{r/n} a_x = \frac{N_{x+r} - N_{x+r+n}}{D_x}$	Limited delayed insurance	$_{r/n}A_{x} = \frac{M_{x+r} - M_{x+r+n}}{D_{x}}$			
Mixed Life Insurance						
	Insurance period	The unitary unique net premium				
case of s	payment of the indemnity in survival risk and limited alability for death risk	$AM_x = \frac{D}{L}$	$\frac{D_{x+n}}{D_x} + \frac{M_x - M_{x+n}}{D_x}$			

Table 1: The unitary unique net premium for survival, death and mixed life insurances

Notations used in the table have the following meaning:

• $_{n}E_{x}$ indicates the unitary unique net premium (the indemnity for a currency unit), due to be paid by the insured person aged x in order to receive the sum of 1 currency unit (c.u.) at the date of attaining the age of (x+n);

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• a_x = unitary unique net premium paid by the insured person aged x for the insurance which provides him 1 c.u. during his whole life as indemnity. It results from reporting the total number of survivors aged over x years (N_x) to the number of living persons attaining the age of x years (D_x);

• $a_x = a_x$ unitary unique net premium paid by the insured person aged x for the insurance which provides him 1 c.u. in the next *n* years as indemnity. The difference from the numerator indicates the total number of survivors within the period of *x* and *x*+*n* years.

• $a_x = a_x$ unitary unique net premium paid by the insured person aged x for the insurance which provides him (if he is alive) 1 c.u. as indemnity, in n years, during his whole life;

• $r/n a_x$ = unitary unique net premium paid by the insured person aged x for the insurance which provides him (if he is alive) 1 c.u. as indemnity, in r years for n years;

• $/_n AM_x$ = unitary unique net premium of the mixed insurance for a period of *n* years, by means of which the insurer pays the sum of 1 c.u. if the insured person is still alive in *n* years or the sum of 1 c.u. at the date of his death if it occurred before the moment *n*;

• Dx, N_x and M_x are commutation numbers.

Number of commutation $D_x = v^x \cdot l_x$, where: v^n – life actualization factor or life discount factor ($v = \frac{1}{1+i}$, *i* indicates the actualization rate); *x* – age of the insured person; *n* – duration of the insurance policy.

Number of commutation $N_x = \sum_{k=x}^{\omega} D_k$ and it indicates the total number of the survivors after attaining the age of x years, for which the cashed premiums are capitalized using an actualization factor v^X . ω indicates the age of death of the last survivor.

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Number of commutation $M_x = \sum_{k=x}^{\omega} C_k$, where $C_x = d_x \cdot v^x$. M_x expresses

the total number of persons who will die after attaining the age of x years, considering a factor of capitalization $v^{\mathcal{X}}$.

The values of $D_{x_i} N_x$ and M_x for each age become constant and they are listed in the tables of commutation numbers, elaborated considering the mortality table and

the values assumed by the rate of actualization $v^{\mathcal{X}}$ (see Annex 2.).

For an indemnity, *S*, the total unique net premium (P) is obtained by multiplying the unitary unique net premium specific to each type of insurance by *S* value.

Most frequently in practice, the insured person chooses the *premium payment in annual, semestrial, quarterly or monthly installments.*

The *annual net premium* is determined by reporting the unique net premium to the annuity specific to the premium payment modality. Considering r years the duration of the premium payment, the annuity specific to the payment modality (a limited immediate annuity is considered), is given by the relation:

$$a_{x}^{(A)} = \frac{N_{x} - N_{x+r}}{D_{x}}.$$

For example, for *survival insurance with unique payment of the indemnity*, annual net premium (p_x) is given by:

$$p_{x} = \frac{\frac{D_{x+n}}{D_{x}}}{\frac{N_{x} - N_{x+r}}{D_{x}}} = \frac{D_{x+n}}{N_{x} - N_{x+r}}.$$

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4. Case studies concerning premium fees for different types of insurance policies

4.1. Survival Insurance

One family made up of two persons aged 30, respectively 40 conclude a survival insurances for a period of 10 years and an insurance indemnity of 10,000 c.u., each one completely payable at the conclusion of the contract, if the insured persons are alive. The net premium is integrally paid at the conclusion of the insurance and the interest used by the insurer is of 30%.

Referring to the estimation report the unitary unique net premium specific to the survival insurance, for a unique payment of the insurance indemnity (Table no. 1) it results a total unique net premium, for the insured person aged 30, using the commutation numbers for 30%, which is determined as it follows:

$$P_{30} = S_{10}E_{30} = S \cdot \frac{D_{x+n}}{D_x} = S \cdot \frac{D_{40}}{D_{30}} = 10,000 \cdot \frac{2.523003}{35.986319} = 701.1$$
c.u

So, for the insured person aged 30, in order to receive after 10 years the indemnity of 10,000 c.u., he has to pay as net premium, in the moment of contract policy 701.1 c.u.

If insured person is older, of 40 years, total net premium becomes:

$$P_{40} = S_{10}E_{40} = S \cdot \frac{D_{50}}{D_{40}} = 10,000 \cdot \frac{0.168916}{2.523003} = 669.5 \text{ c.u}$$

We notice that, for an older age, net premium insurance is decreasing, because the insured person is older, the probability as insured person to be alive at the end of policy is reduced.

We presume that the insured person of 30 years chooses for the payment of net premium during o period of 5 years. Thus, annual net premium becomes:

$$P_x = S \cdot \frac{D_{x+n}}{N_x - N_{x+r}} = 10,000 \cdot \frac{D_{40}}{N_{30} - N_{35}} = 10,000 \cdot \frac{2.523003}{154.345 - 40.791} = 222c.u$$

So, for the insured person aged 30, in order to receive after 10 years the indemnity of 10,000 c.u., he has to pay as annual net premium, of 222 c.u., time of 5 years. Thus, he pays a total net premium of 1,110 c.u. (222 c.u. x 5 years), confronted by 701.1 c.u, in the case on integral payment of premium.

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4.2. Death Insurance

The main issue discussed at the conclusion of the death insurance policy consists in the *determination of the need for insurance*. There is no need for this type of insurance when considering persons with small debts, who gain important revenues from investments or persons who do not depend on the insured person's income.

One person, mentioned as the contractor, wishes to conclude a death insurance for his son, aged 35, mentioned as the insured person, for an insured amount of 10,000 c.u., for a period of 12 years. In Romania, the minimum age for concluding an insurance policy is 16 years and the maximum age is 55 ani. At the conclusion of the insurance policy, another important factor is that the insured persons must not pass by the age of 67 years at the termination of the contract. The duration of the contract is optionally determined by the insured person or by the insurance contractor and it may vary from 12, 15, 20, 25 to 30 years.

The indemnity is determined by the insured person having the insurer's agreement and in accordance to the terms imposed by the insurance company.

Personal data belonging to contracting party of the policy as well as to the insured party are provided in the insurance application. The contract stipulates several questions concerning the health condition of the insured persons, such as: whether he is exposed to serious risks in practicing his profession or apart from his work, whether he has suffered from a certain disease - Lung troubles, heart diseases, stomach complaints, pancreas, intestine, kidney or genital disorders, neuropsychical disorders, epilepsy, diabetes, arterial hypertension, tuberculosis, cancer or any other form of tumour, blood diseases, articular, osseous or skin diseases - or a surgical intervention, whether he was retired due to different health matters, if there were other life insurance petitions addressed to other insurance companies in the past and, moreover, if they were rejected or if there were penalties applied, questions regarding the medication frequency, drug and alcohol consumption, the existing of other life insurance policies.

It is also mentioned here the medical service and the family doctor. When the insured person dies, his wife becomes the beneficiary of the insurance indemnity.

The premium is completely paid at the date of concluding the insurance policy and it is determined according to the tables of commutation numbers for an average interest of 20% (see Annex 2).

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Dealing with a death insurance with a limited validity, the net premium is determined according to the table of commutation numbers, considering the following the estimation report (see Table 1):

$$P = S \cdot_{/n} A_x = S \cdot \frac{M_x - M_{x+n}}{D_x}$$

Knowing S = 10,000 c.u., n = 12 years and x = 35 years, we obtain:

$$P = 10,000 \cdot \frac{M_{35} - M_{35+12}}{D_{35}} = 10,000 \cdot \frac{4.46794 - 1.10180}{157.56222} = 214c.u.$$

All contributions corresponding to the costs made by the insurance company are added to the value of the unique net premium, obtaining thus the gross premium. Considering these costs totalizing 80 c.u., the unique gross premium achieves the value of 294 c.u.

Even though, the insurance policy is handed to the insured person, the obligation of paying the premiums rests upon the contractor.

4.3. Mixed Life Insurance

An insured person aged 40 wishes to conclude a mixed life insurance on a period of 10 years, so that, at the age of 50 he may be returned an indemnity meant to provide him a certain standard of living. The insured person decides to pay maximum 1,000 c.u. annually, considering the value of the indemnity determined for both case, according to the table of commutation numbers: unique and annual payment of the premium, for a period of 5 years.

In order to determine the indemnity, one should take into account the estimation report between the annual net premium and the numbers of commutation from the table of commutation numbers, which corresponds to the age of 40 and for an annual interest of 20%.

$$p_x = S \times \frac{D_{x+n} + M_x - M_{x+n}}{N_x - N_{x+r}} \Longrightarrow S = \frac{p_x (N_x - N_{x+r})}{D_{x+n} + M_x - M_{x+n}} = \frac{1,000(N_{40} - N_{45})}{D_{50} + M_{40} - M_{50}}$$

From number of commutation table for the 20% discount rate (see Annex 2), there are selected the values of number of commutation, thus: N_{40} =356.61024, N_{45} = 136.27742, D_{50} =9.24219, M_{40} =2.56643 and M_{50} =0.75306.

Replacing in formula, we obtain:

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$$S = \frac{1,000(356.61024 - 136.27742)}{9.24219 + 2.56643 - 0.75306} = \frac{1,000 \times 305.67545}{11.05556} = 19,930c.u.$$

The insured person has the possibility of an unique payment of premium, determined as follows:

$$P = S \frac{D_{50} + M_{40} - M_{50}}{D_{40}} = 19,930 \frac{9.24219 + 2.56643 - 0.75306}{62.00147} = 3,554$$
c.u

If insured person will choose for integral payment of premium in the moment of contracting the policy, he will pay 3,554 c.u. for an indemnity of 19,930 c.u., and if he will choose for echeloned payments, he will pay, for the period of 5 years a total amount by 5,000 c.u. The supplement or the extra premium which represents insurer's expenses will be added at the net premium value and it will be obtained the gross premium (total premium).

5. Modeling techniques for premium ratings

The insurance industry has developed some models for an accurate calculation of premium ratings. The most acknowledged models are two types of them (Chessman Wekwete, 2007: 43):

- *multivariable models*, most important being *the Cox model*;
- generalized linear models.

In the absence of the previous experience of the risks (statistical data), needful to transform these into premium ratings, there is used other tools, known as *multiple state models*, Markov model being one significant.

"The Cox model is a multivariable model used to analyze the effect of different risk factors on the time to an event" (Chessman Wekwete, 2007:45). The risk factors (named covariates) used by Cox model are age, cholesterol level as continuous variables or gender and smoker status, as categorical variables. Various statistical software packages are available to calculate relative risks using different data which can produce an output that includes the estimates of the relative risks and their standard errors.

Generalised linear models – GLMs – allow for the analysis not only of survival times, but also other risk classification measures such as hazard rates, insurance claims exacted over a specified duration, and proportions of diseased lives in a specified group of lives.

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A significant constraint for Cox model and GLMs is that only risk factors for which values are available for all observations can be included in the model.

Markov models are defined by the specified states of insured lives and the hazard rates between the states. The *Markov property* is that the hazard rate coming out of any state should depend only on the information that defines the state and not on the history of the life prior to entering state.

Many-sided of Markov models is proved by the fact that premiums can be calculated easily for any life insurance policy, on the condition that certain grounds are met: the states must be defined; all the hazard rates between the states must be defined, and must meet the Markov property criteria; the insurance payments related to these states and any other required information, such us inflation values or interest rates, must be provided.

6. Conclusion

Practical experience of the around the world life insurers demonstrates that, they have to quantify and group similar risks together, based on actuarially selection criteria. This criteria may not produce *equal* prices between insured persons, but it is *equitable* and guarantees that the consumer pays fair and competitive premiums based on the risk everyone brings to the group. Underwriting enables insurers to group together those with the same level of expected risk and to charge them the same premium for the protection they choose. The benefit for the policyholder is to obtain insurance at a fair and competitive price. For the insurer, the benefit is to maintain the experience of its portfolio in line with mortality assumptions.

The key finding of researchers' analysis is that, in all developed countries in the world, males have higher overall rates of mortality than females, even after allowing for other factors including whether or not a person smokes, their age and marital status, where they live and their lifestyle in general. Moreover, not only does female life expectancy exceed that of males today, but this has been the case in these countries for many years.

By working with a diverse group of professionals such as statisticians, actuaries, doctors, underwriters, geneticists and other research professionals, it is possible to take a holistic approach to reviewing experience studies. The accumulated knowledge gained as pricing and underwriting environments change over time, in different parts of the world, can be employed to help insurers minimize antiselection and focus risk selection towards better and more profitable risk categories. This ultimately leads to fairer and more competitive, and therefore more affordable, premiums for the end consumer.

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Popul	ation Mor	tality Tabel	from Roma	ania – both	sexes - period	1990-1992
Age (years) x	Survival function I_x	Number of deceased people d(x)	Death probability q(x)	Survival probability p(x)	Average number of survivals from x to x+1 years Im(x)	Life expectancy E(x)
0	10000	2328	0,02328	0,97417	98836	69,78
1	97672	267	0,00273	0,99706	97538	70,43
2	97405	168	0,00172	0,99810	97321	69,63
3	97237	120	0,00123	0,99868	97177	68,75
4	97117	83	0,00085	0,99903	97075	67,83
5	97034	64	0,00066	0,99924	97002	66,89
6	97970	56	0,00058	0,99929	96942	65,93
7	96914	58	0,00060	0,99928	96885	64,97
8	96856	51	0,00053	0,99935	96830	64,01
9	96805	41	0,00042	0,99948	96784	63,04
10	96764	41	0,00042	0,99949	96743	62,07
11	96723	42	0,00043	0,99953	96702	61,09
12	96681	42	0,00043	0,99945	96660	60,12
13	96693	44	0,00046	0,99943	96617	59,15
14	96595	47	0,00049	0,99940	96571	58,21
15	96548	54	0,00056	0,99926	96521	57,20
16	96494	62	0,00064	0,99918	96463	56,23
17	96432	59	0,00061	0,99916	96402	55,27
18	96373	73	0,00076	0,99899	96336	54,30
19	96300	85	0,00087	0,99879	96258	53,34
20	96216	84	0,00087	0,99878	96174	52,39
21	96132	87	0,00090	0,99874	96098	51,43
22	96045	91	0,00095	0,99869	95999	50,48
23	95954	91	0,00095	0,99865	95908	49,53
24	95863	96	0,00100	0,99859	95815	48,57
25	95767	102	0,00106	0,99857	95716	47,62
26	95665	113	0,00118	0,99832	95608	46,67
27	95552	113	0,00118	0,99845	95495	45,73
28	95439	124	0,00130	0,99819	95377	44,78
29	95315	133	0,00140	0,99808	95248	43,84
30	95182	153	0,00161	0,99770	95105	42,90
31	95029	162	0,00171	0,99759	94948	41,96
32	94867	169	0,00178	0,99751	94782	41,04
33	94698	179	0,00189	0,99731	94608	40,11

ANNEX 1 Population Mortality Tabel from Romania – both sexes - period 1990-1992

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					Average	
Age	с ·	Number of	Death	Survival	number of	Life
(years)	Survival	deceased	probability	probability	survivals from x	expectancy
х	function $\mathbf{I}_{\mathbf{x}}$	people	q(x)	p(x)	to x+1 years	E(x)
		d(x)			lm(x)	
34	94519	195	0,00206	0,99711	94421	39,19
35	94324	220	0,00233	0,99683	94214	38,27
36	94104	224	0,00238	0,99664	93992	37,35
37	93880	248	0,00264	0,99622	93756	36,44
38	93632	268	0,00286	0,99596	93498	35,54
39	93364	288	0,00309	0,99564	93220	34,64
40	93076	303	0,00326	0,99541	92924	33,74
41	92773	338	0,00364	0,99492	92604	32,85
42	92435	373	0,00404	0,99440	92248	31,97
43	92062	418	0,00454	0,99375	91853	31,10
44	91644	430	0,00469	0,99350	91429	30,24
45	91214	447	0,00490	0,99319	90990	29,38
46	90767	488	0,00538	0,99246	90523	28,52
47	90279	538	0,00596	0,99158	90010	27,67
48	89741	579	0,00645	0,99094	89451	26,83
49	89162	638	0,00715	0,98979	88843	26,00
50	88524	672	0,00759	0,98915	88188	25,19
51	87852	724	0,00824	0,98837	87490	24,38
52	87128	772	0,00886	0,98719	86742	23,58
53	86356	833	0,00965	0,98645	85939	22,78
54	85523	906	0,01059	0,98493	85070	22,00
55	84617	951	0,01124	0,98433	84141	21,23
56	83666	1012	0,01210	0,98275	83160	20,47
57	82654	1055	0,01277	0,98190	82126	19,71
58	81599	1134	0,01390	0,98079	81032	18,96
59	80465	1190	0,01479	0,97957	79870	18,22
60	79275	1302	0,01643	0,97746	78624	17,48
61	77973	1360	0,01744	0,97623	77293	16,77
62	76613	1476	0,01926	0,97442	75875	16,06
63	75137	1566	0,02084	0,97204	74354	15,36
64	73571	1640	0,02229	0,97058	72751	14,68
65	71931	1777	0,02471	0,96723	71042	14,00
66	70154	1833	0,02613	0,96577	69237	13,34
67	68321	1976	0,02892	0,96219	67333	12,69

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Experience Studies on	Determining Life Premium	Insurance Ratings: Practical Approaches	5.

		Number of			Average	
Age	Survival	deceased	Death	Survival	number of	Life
(years)	function I.	people	probability	probability	survivals from x	expectancy
x	· · · · ^	d(x)	q(x)	p(x)	to x+1 years	E(x)
					lm(x)	
68	66345	2061	0,03106	0,95976	65314	12,05
69	64284	2201	0,03424	0,95662	63183	11,42
70	62083	2366	0,03811	0,95268	60900	10,81
71	59717	2519	0,04218	0,94769	58457	10,22
72	57198	2716	0,04748	0,94117	55840	9,65
73	54482	2700	0,04956	0,94077	53132	9,10
74	51782	2788	0,05384	0,93496	50388	8,55
75	48994	2961	0,06043	0,92823	47513	8,01
76	46033	3115	0,06767	0,92030	44475	7,49
77	42918	3188	0,07429	0,91506	41324	7,00
78	39730	3281	0,08258	0,90639	38089	6,52
79	36449	3377	0,09265	0,89700	34760	6,06
80	33072	3369	0,10187	0,88753	31387	5,63
81	29703	3383	0,11389	0,87677	28011	5,21
82	26320	3442	0,13078	0,85925	24599	4,82
83	22878	3306	0,14452	0,84938	21225	4,47
84	19572	3108	0,15878	0,83331	18018	4,14
85	16464	2913	0,17694	0,81628	15007	3,82
86	13551	2642	0,19497	0,79918	12230	3,54
87	10909	2336	0,21415	0,78102	9741	3,27
88	8573	2010	0,23447	0,76181	7568	3,03
89	6563	1680	0,25594	0,74154	5723	2,80
90	4883	1360	0,27855	0,72022	4203	2,59
91	3523	1065	0,30231	0,69784	2990	2,40
92	2458	804	0,32722	0,67441	2056	2,22
93	1654	584	0,35327	0,64993	1362	2,05
94	1070	407	0,38047	0,62439	866	1,90
95	663	271	0,40881	0,59780	527	1,76
96	392	172	0,43830	0,57016	306	1,63
97	220	103	0,46893	0,54146	168	1,51
98	117	59	0,50071	0,51170	87	1,40
99	58	31	0,53364	0,48089	42	1,31
100	27	15	0,56771	0,44903	19	1,24

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Age (year) x	Survival Function I _x	$v^{x} = (1+i)^{-x}$	$D_X = l_X \cdot v^X$	$N_x = \sum_{k=x}^{\omega} D_k$	$C_x = v^{x+1} \cdot (l_x - l_{x+1})$	$M_x = \sum_{k=x}^{\omega} C_k$
10	96458	0,1615056	15578,50551	93089,11518	6,86399	63,65298
11	96407	0,1345880	12975,22394	77510,60967	5,15921	56,78900
12	96361	0,1121567	10807,52741	64535,38572	4,95359	51,62979
13	96308	0,0934639	9001,32926	53727,85831	3,89433	46,67621
14	96258	0,0778866	7497,20505	44726,53905	4,34867	42,78188
15	96191	0,0649055	6243,32221	37229,33400	4,00250	38,43321
16	96117	0,0540879	5198,76600	30986,01179	3,24527	34,43071
17	96045	0,0450732	4329,05973	25787,24579	2,96732	31,18543
18	95966	0,0375610	3604,58245	21458,18606	2,78578	28,21811
19	95877	0,0313009	3001,03293	17853,60360	2,71274	25,43233
20	95773	0,0260841	2498,14804	14852,57067	2,43451	22,71959
21	95661	0,0217367	2079,35552	12354,42263	2,49972	20,28508
22	95523	0,0181139	1730,29654	10275,06711	2,18877	17,78536
23	95378	0,0150949	1439,72502	8544,77057	1,72334	15,59659
24	95241	0,0125791	1198,04751	7105,04555	1,44660	13,87325
25	95103	0,0104826	996,92633	5906,99803	1,26665	12,42666
26	94958	0,0087355	829,50529	4910,07171	1,04826	11,16001
27	94814	0,0072796	690,20615	4080,56641	1,03734	10,11175
28	94643	0,0060663	574,13445	3390,36026	0,84928	9,07441
29	94475	0,0050553	477,59609	2816,22581	0,80463	8,22512
30	94284	0,0042127	397,19211	2338,62971	0,69510	7,42050
31	94086	0,0035106	330,29833	1941,43760	0,61728	6,72540
32	93875	0,0029255	274,63133	1611,13927	0,59729	6,10812
33	93630	0,0024379	228,26215	1336,50794	0,53837	5,51083
34	93365	0,0020316	189,68009	1108,24579	0,50451	4,97245
35	93067	0,0016930	157,56222	918,56571	0,45711	4,46794
36	92743	0,0014108	130,84474	761,00348	0,40797	4,01083
37	92396	0,0011757	108,62932	630,15874	0,38014	3,60286
38	92008	0,0009797	90,14429	521,52942	0,34536	3,22272
39	91585	0,0008165	74,77489	431,38512	0,31093	2,87736

ANNEX 2 Commutation Numbers Table, for a discount rate of 20% - Case of Romania -

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Age	Survival	r _r	r	Ø	r+1	Ø
(year)	Function	$v^{x} = (1+i)^{-x}$	$D_{\chi} = l_{\chi} \cdot v^{\chi}$	$N_x = \sum_{k=1}^{\omega} D_k$	$C_{\chi} = v^{\chi + 1} \cdot$	$M_{\chi} = \sum_{k=1}^{\omega} C_k$
x	I _x				$\cdot (l_x - l_{x+1})$	
			co. oo	k=x		k=x
40	91128	0,0006804	62,00147	356,61024	0,28689	2,56643
41	90622	0,0005670	51,38100	294,60877	0,26412	2,27954
42	90063	0,0004725	42,55338	243,22777	0,22797	2,01542
43	89484	0,0003937	35,23318	200,67439	0,19720	1,78745
44	88883	0,0003281	29,16378	165,44121	0,18730	1,59025
45	88198	0,0002734	24,11586	136,27742	0,16178	1,40295
46	87488	0,0002279	19,93477	112,16157	0,13937	1,24117
47	86754	0,0001899	16,47293	92,22680	0,12532	1,10180
48	85962	0,0001582	13,60212	75,75387	0,11815	0,97648
49	85066	0,0001319	11,21695	62,15175	0,10527	0,85833
50	84108	0,0001099	9,24219	50,93479	0,09441	0,75306
51	83077	0,0000916	7,60742	41,69260	0,08432	0,65865
52	81972	0,0000763	6,25519	34,08518	0,07472	0,57433
53	80797	0,0000636	5,13794	27,82999	0,06523	0,49961
54	79566	0,0000530	4,21638	22,69205	0,05984	0,43438
55	78211	0,0000442	3,45382	18,47566	0,05067	0,37454
56	76834	0,0000368	2,82751	15,02184	0,04324	0,32387
57	75424	0,0000307	2,31302	12,19434	0,03874	0,28063
58	73908	0,0000256	1,88877	9,88132	0,03341	0,24188
59	72339	0,0000213	1,54056	7,99255	0,02852	0,20847
60	70732	0,0000177	1,25528	6,45199	0,02504	0,17995
61	69039	0,0000148	1,02103	5,19671	0,02170	0,15491
62	67278	0,0000123	0,82916	4,17568	0,01871	0,13321
63	65456	0,0000103	0,67225	3,34652	0,01657	0,11450
64	63520	0,000086	0,54364	2,67427	0,01421	0,09793
65	61527	0,0000071	0,43882	2,13063	0,01245	0,08371
66	59433	0,0000059	0,35324	1,69182	0,01062	0,07127
67	57288	0,0000050	0,28374	1,33858	0,00925	0,06064
68	55046	0,0000041	0,22720	1,05484	0,01125	0,05139
69	51775	0,0000034	0,17808	0,82764	0,00401	0,04014
70	50377	0,0000029	0,14439	0,64956	0,00596	0,03613
71	47882	0,0000024	0,11437	0,50517	0,00505	0,03017
72	45346	0,0000020	0,09026	0,39080	0,00422	0,02512
73	42801	0,0000017	0,07099	0,30054	0,00371	0,02090
74	40119	0,0000014	0,05545	0,22955	0,00322	0,01720
75	37321	0,0000012	0,04299	0,17410	0,00261	0,01397
76	34604	0,0000010	0,03322	0,13111	0,00220	0,01137
77	31857	0,000008	0,02548	0,09789	0,00186	0,00917

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Age (year)	Survival Function	$v^{x} = (1+i)^{-x}$	$D_{\mathcal{X}} = l_{\mathcal{X}} \cdot v^{\mathcal{X}}$	$N_x = \sum_{k=1}^{\omega} D_k$	$C_x = v^{x+1}$.	$M_{\chi} = \sum_{k=r}^{\omega} C_k$
x	I _x			k = x	$\cdot (l_x - l_{x+1})$	k = x
78	29063	0,0000007	0,01937	0,07241	0,00155	0,00731
79	26266	0,000006	0,01459	0,05303	0,00125	0,00575
80	23575	0,0000005	0,01091	0,03844	0,00102	0,00451
81	20935	0,000004	0,00808	0,02753	0,00084	0,00349
82	18312	0,000003	0,00589	0,01945	0,00067	0,00264
83	15799	0,000003	0,00423	0,01357	0,00051	0,00197
84	13519	0,0000002	0,00302	0,00933	0,00040	0,00146
85	11352	0,000002	0,00211	0,00632	0,00030	0,00106
86	9387	0,000002	0,00146	0,00420	0,00023	0,00075
87	7628	0,0000001	0,00099	0,00275	0,00017	0,00053
88	6085	0,0000001	0,00066	0,00176	0,00012	0,00036
89	4758	0,0000001	0,00043	0,00111	0,00008	0,00024
90	3642	0,0000001	0,00027	0,00068	0,00006	0,00016
91	2725	0,000001	0,00017	0,00041	0,00004	0,00010
92	1990	0,0000001	0,00010	0,00024	0,00002	0,00006
93	1415	0,0000000	0,00006	0,00014	0,00002	0,00004
94	979	0,0000000	0,00004	0,00008	0,00001	0,00002
95	657	0,0000000	0,00002	0,00004	0,00001	0,00001
96	428	0,0000000	0,00001	0,00002	0,00000	0,00001
97	269	0,0000000	0,00001	0,00001	0,00000	0,00000
98	163	0,0000000	0,00000	0,00000	0,00000	0,00000
99	95	0,0000000	0,00000	0,00000	0,00000	0,00000
100	0	0,0000000	0,00000	0,00000	0,00000	0,00000

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