International Journal of Mathematics And Its Applications
Vol. 2 No. 3 (2014), pp.17-27.
ISSN: 2347-1557(online)

# Cordial Labeling of Snakes 

S. K. Vaidya ${ }^{11}$ and N. H. Shah ${ }^{\ddagger}$<br>${ }^{\dagger}$ Saurashtra University, Rajkot-360 005, Gujarat, India.<br>samirkvaidya@yahoo.co.in<br>${ }^{\ddagger}$ Government Polytechnic, Rajkot-360 003, Gujarat, India.<br>nirav.hs@gmail.com


#### Abstract

We prove that the graphs alternate triangular snake, alternate quadrilateral snake, double alternate triangular snake and double alternate quadrilateral snake admit cordial labeling.


Keywords : Cordial graph, Cordial labeling, Snake.

## 1 Introduction

In this paper, we consider only finite, connected, undirected and simple graph $G=(V(G), E(G))$ with $p$ vertices and $q$ edges. For standard terminology and notation we refer to Balakrishnan and Ranganathan [4. A brief summary of definitions and information related to the present work is given in order to maintain the compactness.

Definition 1.1. A graph labeling is an assignment of numbers to the vertices or edges or both subject to certain condition(s). If the domain of the mapping is the set of vertices (edges) then the labeling is called a vertex labeling (edge labeling).

[^0]Various labeling schemes have been introduced so far and explored as well by many researchers. Several diversified applications of graph labeling have been reported by Yegnanaryanan and Vaidhyanathan [13. A dynamic survey on different graph labeling problems along with an extensive bibliography can be found in Gallian 6].

Definition 1.2. A mapping $f: V(G) \rightarrow\{0,1\}$ is called binary vertex labeling of $G$ and $f(v)$ is called the label of the vertex $v$ of $G$ under $f$.

If for an edge $e=u v$, the induced edge labeling $f^{*}: E(G) \rightarrow\{0,1\}$ is given by $f^{*}(e)=|f(u)-f(v)|$ then we introduce following notations,

$$
\left.\begin{array}{l}
v_{f}(i)=\text { number of vertices of } G \text { having label } i \text { under } f \\
e_{f}(i)=\text { number of edges of } G \text { having label } i \text { under } f^{*}
\end{array}\right\} \text { where } i=0 \text { or } 1
$$

Definition 1.3. A binary vertex labeling $f$ of a graph $G$ is called a cordial labeling if $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$. A graph $G$ is cordial if it admits cordial labeling.

The concept of cordial labeling was introduced by Cahit [5] as a weaker version of graceful and harmonious labeling. In the same paper Cahit investigated many classes of cordial graphs and a necessary condition for an Eulerian graph to be cordial graph. Andar et al. [1, 2] and Ho et al. 88 have contributed many results on cordial labeling. Vaidya and Dani 9, 10] as well as Vaidya and Vihol [11] have investigated many results on cordial labeling for the graphs arising from different graph operations. Vaidya and Shah [12] have discussed cordial labeling for some bistar related graphs. Lawrence and Koilraj [7] have discussed cordial labeling for the splitting graph of some standard graphs. Motivated through the concept of cordial labeling Babujee and Shobana [3] have introduced the concepts of cordial languages and cordial numbers. Some labeling schemes are also introduced with minor variations in cordial theme.

Definition 1.4. An alternate triangular snake $A\left(T_{n}\right)$ is obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i}$ and $u_{i+1}$ (alternately) to a new vertex $v_{i}$. That is every alternate edge of path is replaced by $C_{3}$.

Definition 1.5. An alternate quadrilateral snake $A\left(Q S_{n}\right)$ is obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i}, u_{i+1}$ to new vertices $v_{i}, w_{i}$ respectively and then joining $v_{i}$ and $w_{i}$. That is every alternate edge of path is replaced by $C_{4}$.

Definition 1.6. A double alternate triangular snake $D A\left(T_{n}\right)$ consists of two alternate triangular snakes that have a common path. That is, double alternate triangular snake is obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i}$ and $u_{i+1}$ (alternately) to new vertices $v_{i}$ and $w_{i}$.

Definition 1.7. A double alternate quadrilateral snake $D A\left(Q S_{n}\right)$ consists of two alternate quadrilateral snakes that have a common path. That is, it is obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i}$ and $u_{i+1}$ (alternately) to new vertices $v_{i}, v_{i}^{\prime}$ and $w_{i}, w_{i}^{\prime}$ respectively and adding the edges $v_{i} w_{i}$ and $v_{i}^{\prime} w_{i}^{\prime}$.

## 2 Main Results

Theorem 2.1. $A\left(T_{n}\right)$ admits cordial labeling.

Proof. Let $A\left(T_{n}\right)$ be alternate triangular snake obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i}$ and $u_{i+1}$ (alternately) to new vertex $v_{i}$ where $1 \leq i \leq n-1$ for even $n$ and $1 \leq i \leq n-2$ for odd $n$. Therefore $V\left(A\left(T_{n}\right)\right)=\left\{u_{i}, v_{j} / 1 \leq i \leq n, 1 \leq j \leq\left\lfloor\frac{n}{2}\right\rfloor\right\}$. We note that
$\left|V\left(A\left(T_{n}\right)\right)\right|=\left\{\begin{array}{cl}\frac{3 n}{2} & , n \equiv 0(\bmod 2) \\ \frac{3 n-1}{2} & , n \equiv 1(\bmod 2)\end{array}\right.$ and $\left|E\left(A\left(T_{n}\right)\right)\right|= \begin{cases}2 n-1 & , n \equiv 0(\bmod 2) \\ 2 n-2 & , n \equiv 1(\bmod 2) .\end{cases}$
To define vertex labeling $f: V\left(A\left(T_{n}\right)\right) \rightarrow\{0,1\}$ we consider following five cases.
Case 1: $n=2,3$.
For $n=2, A\left(T_{2}\right)=C_{3}$, which is a cordial graph as proved by Ho et al. 8 .
For $n=3, f\left(u_{1}\right)=0, f\left(u_{2}\right)=1, f\left(u_{3}\right)=1$ and $f\left(v_{1}\right)=0$. Then $v_{f}(0)=2, v_{f}(1)=2$ and $e_{f}(0)=2=$ $e_{f}(1)$. Hence, $A\left(T_{3}\right)$ admits cordial labeling.
Case 2: $n \equiv 0(\bmod 4)$.
Let $n=4 k$,

$$
\begin{array}{ll}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(v_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n}{4}-1 \\
f\left(v_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n}{4}-1
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=\frac{3 n}{4}=v_{f}(1)$ and $e_{f}(0)=n-1, e_{f}(1)=n$.
Case 3: $n \equiv 1(\bmod 4)$.
Let $n=4 k+1$,

$$
\begin{array}{cl}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{n}\right)=0 ; & \\
f\left(v_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n-1}{4}-1 \\
f\left(v_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-1}{4}-1
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=\left\lceil\frac{3 n-1}{4}\right\rceil, v_{f}(1)=\left\lfloor\frac{3 n-1}{4}\right\rfloor$ and $e_{f}(0)=n-1=$ $e_{f}(1)$.
Case 4: $n \equiv 2(\bmod 4)$.

Let $n=4 k+2$,

$$
\begin{array}{cl}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{n-1}\right)=0 ; & \\
f\left(u_{n}\right)=1 ; & \\
f\left(v_{1+2 i}\right)=1 ; \quad 0 \leq i \leq \frac{n-2}{4}-1 \\
f\left(v_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-2}{4}-1 \\
f\left(v_{\frac{n}{2}}\right)=1 ; &
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=\left\lfloor\frac{3 n}{4}\right\rfloor, v_{f}(1)=\left\lceil\frac{3 n}{4}\right\rceil$ and $e_{f}(0)=n-1, e_{f}(1)=n$. Case 5: $n \equiv 3(\bmod 4)$.
Let $n=4 k+3$,

$$
\begin{array}{ll}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{n-2}\right)=0 ; & \\
f\left(u_{n-1}\right)=1 ; & \\
f\left(u_{n}\right)=1 ; & \\
f\left(v_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n-3}{4}-1 \\
f\left(v_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-3}{4}-1 \\
f\left(v_{\frac{n-1}{2}}\right)=0 ; &
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=\frac{3 n-1}{4}=v_{f}(1)$ and $e_{f}(0)=n-1=e_{f}(1)$.
Thus, in each case we have $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
Hence, $A\left(T_{n}\right)$ admits cordial labeling.

Example 2.2. A cordial labeling of $A\left(T_{11}\right)$ is shown in Figure 1.


Fig. $1 A\left(T_{11}\right)$ and its cordial labeling.

Theorem 2.3. $A\left(Q S_{n}\right)$ is admits cordial labeling.

Proof. Let $A\left(Q S_{n}\right)$ be an alternate quadrilateral snake obtained from a path $u_{1}, u_{2}, \ldots, u_{n}$ by joining $u_{i}, u_{i+1}$ (alternately) to new vertices $v_{i}, w_{i}$ respectively and then joining $v_{i}$ and $w_{i}$ where $1 \leq i \leq n-1$ for even $n$ and $1 \leq i \leq n-2$ for odd $n$. Therefore $V\left(A\left(T_{n}\right)\right)=\left\{u_{i}, v_{j}, w_{j} / 1 \leq i \leq n, 1 \leq j \leq\left\lfloor\frac{n}{2}\right\rfloor\right\}$. We note that
$\left|V\left(A\left(Q S_{n}\right)\right)\right|=\left\{\begin{array}{cl}2 n & , n \equiv 0(\bmod 2) ; \\ 2 n-1 & , n \equiv 1(\bmod 2) .\end{array}\right.$ and $\left|E\left(A\left(Q S_{n}\right)\right)\right|=\left\{\begin{array}{cl}\frac{5 n-2}{2} & , n \equiv 0(\bmod 2) ; \\ \frac{5 n-5}{2} & , n \equiv 1(\bmod 2) .\end{array}\right.$
To define vertex labeling $f: V\left(A\left(Q S_{n}\right)\right) \rightarrow\{0,1\}$ we consider following five cases.
Case 1: $n=2,3$.
For $n=2, A\left(Q S_{2}\right)=C_{4}$, which is a cordial graph as proved by Ho et al. 88 .
For $n=3, f\left(u_{1}\right)=1, f\left(u_{2}\right)=0, f\left(u_{3}\right)=1$ and $f\left(v_{1}\right)=1, f\left(w_{1}\right)=0$. Then $v_{f}(0)=2, v_{f}(1)=3$ and $e_{f}(0)=2, e_{f}(1)=3$. Hence, $A\left(Q S_{3}\right)$ admits cordial labeling.
Case 2: $n \equiv 0(\bmod 4)$.
Let $n=4 k$,

$$
\begin{array}{cc}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(v_{i}\right)=1 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(w_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n}{4}-1 \\
f\left(w_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n}{4}-1
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n=v_{f}(1)$ and $e_{f}(0)=n+\frac{n}{4}-1$ and $e_{f}(1)=n+\frac{n}{4}$.
Case 3: $n \equiv 1(\bmod 4)$.
Let $n=4 k+1$,

$$
\begin{aligned}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{n}\right)=0 ; & \\
f\left(v_{i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(w_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n-1}{4}-1 \\
f\left(w_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-1}{4}-1
\end{aligned}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n, v_{f}(1)=n-1$ and $e_{f}(0)=n-1+\frac{n-1}{4}=$ $\frac{5 n-5}{4}=e_{f}(1)$.
Case 4: $n \equiv 2(\bmod 4)$.

Let $n=4 k+2$,

$$
\begin{array}{cc}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k \\
f\left(u_{2+4 i}\right)=0 ; & 0 \leq i \leq k \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(v_{i}\right)=1 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(w_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n-2}{4} \\
f\left(w_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-2}{4}-1
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n=v_{f}(1)$ and $e_{f}(0)=\frac{5 n-2}{4}=e_{f}(1)$.
Case 5: $n \equiv 3(\bmod 4)$.
Let $n=4 k+3$,

$$
\begin{array}{cc}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k \\
f\left(u_{2+4 i}\right)=0 ; & 0 \leq i \leq k \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
& \\
f\left(u_{n}\right)=1 ; & \\
f\left(v_{i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(w_{1+2 i}\right)=1 ; & 0 \leq i \leq \frac{n-3}{4} \\
f\left(w_{2+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-3}{4}-1
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n-1, v_{f}(1)=n$ and $e_{f}(0)=\frac{5 n-7}{4}, e_{f}(1)=$ $\frac{5 n-3}{4}$.
Thus in each case we have $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
Hence, $A\left(Q S_{n}\right)$ admits cordial labeling.

Example 2.4. A cordial labeling of $A\left(Q S_{8}\right)$ is shown in Figure 2.


Fig. $2 A\left(Q S_{8}\right)$ and its cordial labeling.

Theorem 2.5. $D A\left(T_{n}\right)$ admits cordial labeling.

Proof. Let $G$ be a double alternate triangular snake $D A\left(T_{n}\right)$ then $V(G)=\left\{u_{i}, v_{j}, w_{j} / 1 \leq i \leq n, 1 \leq j \leq\right.$ $\left.\left\lfloor\frac{n}{2}\right\rfloor\right\}$. We note that
$|V(G)|=\left\{\begin{array}{cc}2 n, & n \equiv 0(\bmod 2) \\ 2 n-1, & n \equiv 1(\bmod 2)\end{array}\right.$ and $|E(G)|=\left\{\begin{array}{cc}3 n-1, & n \equiv 0(\bmod 2) \\ 3 n-3, & n \equiv 1(\bmod 2)\end{array}\right.$
To define vertex labeling $f: V\left(D A\left(T_{n}\right)\right) \rightarrow\{0,1\}$ we consider following five cases.
Case 1: $n=2,3$.
For $n=2, f\left(u_{1}\right)=0, f\left(u_{2}\right)=1$ and $f\left(v_{1}\right)=0, f\left(w_{1}\right)=1$. Then $v_{f}(0)=2=v_{f}(1)$ and $e_{f}(0)=$ $2, e_{f}(1)=3$. Hence, $D A\left(T_{2}\right)$ admits cordial labeling.
For $n=3, f\left(u_{1}\right)=0, f\left(u_{2}\right)=1, f\left(u_{3}\right)=1$ and $f\left(v_{1}\right)=0, f\left(w_{1}\right)=1$. Then $v_{f}(0)=2, v_{f}(1)=3$ and $e_{f}(0)=3=e_{f}(1)$. Hence, $D A\left(T_{3}\right)$ admits cordial labeling.
Case 2: $n \equiv 0(\bmod 4)$.
Let $n=4 k$,

$$
\begin{array}{cc}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(v_{i}\right)=0 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(w_{i}\right)=1 ; & 1 \leq i \leq \frac{n}{2}
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n=v_{f}(1)$ and $e_{f}(0)=\frac{3 n-2}{2}$ and $e_{f}(1)=\frac{3 n}{2}$.
Case 3: $n \equiv 1(\bmod 4)$.
Let $n=4 k+1$,

$$
\begin{array}{cl}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{n}\right)=0 ; & \\
f\left(v_{i}\right)=0 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(w_{i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2}
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n, v_{f}(1)=n-1$ and $e_{f}(0)=\frac{3 n-3}{2}, e_{f}(1)=$ $\frac{3 n-3}{2}$.
Case 4: $n \equiv 2(\bmod 4)$.

Let $n=4 k+2$,

$$
\begin{array}{cc}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(v_{i}\right)=0 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(w_{i}\right)=1 ; & 1 \leq i \leq \frac{n}{2}
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n=v_{f}(1)$ and $e_{f}(0)=\frac{3 n}{2}-1, e_{f}(1)=\frac{3 n}{2}$.
Case 5: $n \equiv 3(\bmod 4)$.
Let $n=4 k+3$,

$$
\begin{array}{cc}
f\left(u_{1+4 i}\right)=0 ; & 0 \leq i \leq k \\
f\left(u_{2+4 i}\right)=1 ; & 0 \leq i \leq k \\
f\left(u_{3+4 i}\right)=1 ; & 0 \leq i \leq k-1 \\
f\left(u_{4+4 i}\right)=0 ; & 0 \leq i \leq k-1 \\
f\left(u_{n}\right)=1 ; & \\
f\left(v_{i}\right)=0 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(w_{i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2}
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=n-1, v_{f}(1)=n$ and $e_{f}(0)=\frac{3 n-3}{2}=e_{f}(1)$. Thus, in each case we have $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.

Hence, $D A\left(T_{n}\right)$ admits cordial labeling.

Example 2.6. A cordial labeling of $D A\left(T_{10}\right)$ is shown in Figure 3.


Fig. $3 D A\left(T_{10}\right)$ and its cordial labeling.

Theorem 2.7. $D A\left(Q S_{n}\right)$ admits cordial labeling.
Proof. Let $G$ be a double alternate quadrilateral snake $D A\left(T_{n}\right)$ then $V(G)=\left\{u_{i}, v_{j}, w_{j}, v_{j}^{\prime}, w_{j}^{\prime} / 1 \leq i \leq\right.$ $\left.n, 1 \leq j \leq\left\lfloor\frac{n}{2}\right\rfloor\right\}$. We note that
$|V(G)|=\left\{\begin{array}{cc}3 n, & n \equiv 0(\bmod 2) \\ 3 n-2, & n \equiv 1(\bmod 2)\end{array}\right.$ and $|E(G)|=\left\{\begin{array}{cl}4 n-1, & , n \equiv 0(\bmod 2) \\ 4 n-4, & n \equiv 1(\bmod 2)\end{array}\right.$

To define vertex labeling $f: V(G) \rightarrow\{0,1\}$ we consider following two cases.
Case 1: $n \equiv 0(\bmod 2)$.
Let $n=2 k$,

$$
\begin{array}{cc}
f\left(u_{1+2 i}\right)=0 ; & 0 \leq i \leq \frac{n}{2}-1 \\
f\left(u_{2 i}\right)=1 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(v_{i}\right)=1 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(w_{i}\right)=1 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(v_{i}^{\prime}\right)=0 ; & 1 \leq i \leq \frac{n}{2} \\
f\left(w_{i}^{\prime}\right)=0 ; & 1 \leq i \leq \frac{n}{2}
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=\frac{3 n}{2}=v_{f}(1)$ and $e_{f}(0)=2 n, e_{f}(1)=2 n-1$.
Case 2: $n \equiv 1(\bmod 2)$.
Let $n=2 k+1$,

$$
\begin{array}{cl}
f\left(u_{1+2 i}\right)=0 ; & 0 \leq i \leq \frac{n-1}{2} \\
f\left(u_{2 i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(v_{i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(w_{i}\right)=1 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(v_{i}^{\prime}\right)=0 ; & 1 \leq i \leq \frac{n-1}{2} \\
f\left(w_{i}^{\prime}\right)=0 ; & 1 \leq i \leq \frac{n-1}{2}
\end{array}
$$

In view of the above defined labeling pattern, $v_{f}(0)=\frac{3 n-1}{2}, v_{f}(1)=\frac{3 n-3}{2}$ and $e_{f}(0)=2 n-2=e_{f}(1)$.
Thus, in both the cases we have $\left|v_{f}(0)-v_{f}(1)\right| \leq 1$ and $\left|e_{f}(0)-e_{f}(1)\right| \leq 1$.
Hence, $D A\left(Q S_{n}\right)$ admits cordial labeling.

Example 2.8. A cordial labeling of $D A\left(Q S_{9}\right)$ is shown in Figure 4.


Fig. $4 D A\left(Q S_{9}\right)$ and its cordial labeling.

## 3 Concluding Remarks

The snakes are the graphs obtained from paths by attaching some graphs in various fashion. We have investigated cordial labelings for the same.

## 4 Acknowledgement

Our thanks are due to the anonymous referees for constructive comments and useful suggestions on the first draft of this paper.

## References

[1] M. Andar, S. Boxwala and N. B. Limaye, Cordial labelings of some wheel related graphs, J. Combin. Math. Combin. Comput., 41(2002), 203-208.
[2] M. Andar, S. Boxwala and N. B. Limaye, A note on cordial labeling of multiple shells, Trends Math., (2002), $77-80$.
[3] J. B. Babujee and L. Shobana, Cordial Languages and Cordial Numbers, Journal of Applied Computer Science \& Mathematics, 13(6)(2012), 9-12.
[4] R. Balakrishnan and K. Ranganathan, A text book of Graph Theory, 2/e, Springer, New York, (2012).
[5] I. Cahit, Cordial Graphs: A weaker version of graceful and harmonious Graphs, Ars Combinatoria, 23 (1987), 201-207.
[6] J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics, 16 (2013), \#DS6.

Available online: Available:http://www.combinatorics.org
[7] P. Lawrence Rozario Raj and S. Koilraj, Cordial labeling for the splitting graph of some standard graphs, International Journal of Mathematics and Soft Computing, 1(1) (2011), 105-114.
[8] Y. S. Ho, S. M. Lee and S. C. Shee, Cordial labelings of unicyclic graphs and generalized petersen graphs, Congr. Numer., 68 (1989), 109-122.
[9] S. K. Vaidya and N. A. Dani, Some new star related graphs and their cordial as well as 3-equitable labeling, Journal of Sciences, 1(1)(2010), 111-114.
[10] S. K. Vaidya and N. A. Dani, Cordial labeling and arbitrary super subdivision of some graphs, International Journal of Information Science and Computer Mathematics, 2 (1)(2010), 51-60.
[11] S. K. Vaidya and P. L. Vihol, Cordial labeling for middle graph of some graphs, Elixir Dis. Math., 34C (2011), 2468-2476.
[12] S. K. Vaidya and N. H. Shah, Cordial labeling for some bistar related graphs, International Journal of Mathematics and Soft Computing, (Accepted for publication).
[13] V. Yegnanaryanan and P. Vaidhyanathan, Some Interesting Applications of Graph Labellings, J. Math. Comput. Sci., 2(5)(2012), 1522-1531.


[^0]:    ${ }^{1}$ Corresponding author E-Mail:samirkvaidya@yahoo.co.in (S. K. Vaidya) AMS Subject Classification: 05C78.

