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Computational Experiment for the Analysis of Functioning of Technological Process of Filtering of a Suspension

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Abstract. For the Analysis of Functioning of Technological Process of Filtering of a Suspension the adequate mathematical model is developed, numerical algorithm and the computing experiments on the computer are carried out. The results of computing experiments are illustrated as the diagrams.

Keywords: mathematical model, numerical algorithm, porous media, filtration, gas, oil, water.

Introduction. In a large cycle of production of commodities, which have an important social status, the role of filtering to obtain final product is huge. Therefore to achieve maximum result in refining of final product from undesirable admixtures in the process of filtering, the latter should be organized as a technological cycle with optimal parameters, including characteristics of the filter itself and the mode of operation. Since the process of filtering of severely polluted liquid ionized solutions represents a complex non-stationary process, that depends on many factors, the solution of the problem of optimization of the managing of the process of filtering is a very difficult task. One of the methods of solution of above mentioned problem, discussed in this paper, is based on a realization of computational experiment aimed to study the parameters of the process of filtering of suspension. This method applies the concept «model – algorithm – program».

General equation of the problem. Consider general non-linear differential (integral-differential) equation of the form

$$L\Theta = F[t, X, u, f(t, x, \Theta, \Theta_\Gamma)], \tag{1}$$

here L is a differential operator; F – arbitrary function of parameters t, X, Θ, f; $f(t, X, \Theta, \Theta_\Gamma)$ – non-linear function of variable Θ, which includes derivatives and/or integrals of Θ, or a certain function of its boundary values Θ_Γ ; $X = \{x_1,..., x_n\}$ – vector of coordinates. Integral operators are present in right side part of the equation (1). Function f (t, X, Θ, Θ_Γ) also may be present in one of boundary conditions of the problem. The solution of the equation (1) is connected with a number of difficulties due to non-linear dependence F on Θ. The equation of such type with given boundary conditions are typical for the problems of physical-chemical hydrodynamics. There are methods of solution of general equation (1), based on analytical or approximate solutions with high accuracy in the statement of a linear problem (with constant value of a parameter) on the basis of interpolation procedures [1]. In this case the function $f(t, X, \Theta, \Theta_\Gamma)$ is substituted by constant value $\langle f \rangle$ on the basis of averaging of $f(t, X, \Theta, \Theta_\Gamma)$ in a certain region of t and X. Here, (1) is simplified and acquires the form

$$L\Theta = F[t, X, u, f(t, x, \langle f \rangle)]. \quad (2)$$

In case when one of boundary conditions depends on mentioned function, it is here substituted by $\langle f \rangle$ and is calculated at the moment of time t on the border of Ω on the basis of solution of integral equation

$$\langle f \rangle = \frac{1}{t\Omega} \int_0^t \int_\Omega f(t, X, \Theta(t, X, \langle f \rangle)) d\Omega d\tau \quad (3)$$

with iterative algorithm:

$$\langle f \rangle^{(i+1)} = \frac{1}{t\Omega} \int_0^t \int_\Omega f(t, X, \Theta(t, X, \langle f \rangle^i)) d\Omega d\tau . \quad (3a)$$

Physical interpretation of (3), (3a) consists in the aspect that approximate curve calculated on the basis of function Θ from the expression (2), is maximally approaching numeric solutions for f on the end of the interval of averaging.

The problem of analysis of technological process of filtering of suspensions. A large majority of parameters entering the equations (1), (2), differ in their specific weights. In mathematical statement there appears a somewhat incorrect problem: inconsiderable deviations of parameters lead to severe changes of the function sought for, that is to qualitative and quantitative changes of the process on the whole. Such behavior of the function and its consequences may be studies on the basis of realization of computational experiment.

The presentation of the model of non-stationary technological process of filtering of severely polluted ionized liquid mixtures based on (1) is written down in the form of the system of differential equations in dimensionless coordinates, that include the law of conservation of mass, quantity of motion and kinetics of the process [2–4]:

$$\frac{\partial w}{\partial t} + \mathrm{Re}\, w \frac{\partial w}{\partial x} - \frac{w}{1-\Theta_3}\frac{d\Theta_3}{dt} = -Eu \cdot \mathrm{Re}\frac{\partial P}{\partial x} + \frac{HK_0}{H_0^3}\frac{\partial^2 w}{\partial x^2} - \frac{w}{(1-\Theta_3)(1-\delta)^2}; \quad (4)$$

$$\frac{\partial P}{\partial x} = \frac{1}{Eu}\left[\frac{m}{\mathrm{Re}(1-\delta)^2} - \frac{d\Theta_3}{dt}\right]; \quad (5)$$

$$\frac{\partial \Theta}{\partial t} = -\frac{\partial(\Theta w)}{\partial x} - \frac{\partial \alpha}{\partial t} - (1-m_0)\frac{\partial \delta}{\partial t} + \frac{\mu_0 \alpha_\tau}{H_0^2}\frac{\partial^2 \Theta}{\partial x^2}; \quad (6)$$

$$\frac{\partial \delta}{\partial t} = \lambda(\Theta - \gamma\delta)$$

$$\Theta - \Theta_3 = \frac{\alpha}{1-\delta} \quad (7)$$

$$\frac{d\Theta_3}{dt} = \frac{1-\overline{\Theta}}{2-\delta}\frac{d\overline{\Theta}}{dt} + \frac{1}{1-\delta}\left[(1-\overline{\Theta}\frac{d\overline{\delta}}{dt} - \frac{\Theta_1 w_0}{mH_0(1-\delta_1)}\right] + \Theta_3\left[\frac{d\overline{\delta}}{dt} + \frac{w_0}{mH_0(1-\delta_1)}\right]\frac{1}{2-\delta}; \quad (8)$$

$$\frac{\partial v}{\partial t} = Sw \quad (9)$$

$$\frac{\partial n_i}{\partial t} = -\frac{w_0 \alpha_\tau}{H_0 m}\frac{\partial n_i}{\partial x} - \frac{N_0}{n_i m} + \frac{\partial N_i}{\partial t} + \frac{D_a \alpha_\tau}{H^2 m}\frac{\partial^2 n_i}{\partial x^2} + \frac{\alpha_\tau D_b}{PH_0}\frac{\partial P}{\partial x}; \quad (10)$$

$$\frac{\partial N_i}{\partial t} = \frac{\alpha_\tau \beta}{1}\left[\frac{n_0 n_i}{N_0} - \frac{a}{a - bN_i N_0}\right] \quad (11)$$

with initial conditions

$$\left.\begin{array}{ll} \Theta = e^{-\lambda H_0 Bx}; & \Theta_3 = 0; \\ \delta = 0; & w = 0; \\ n_1 = 0; & n_2 = \varphi_2; \\ N_2 = 1; & N_1 = \varphi_3 \end{array}\right\} at\, t = 0 \qquad (12)$$

and boundary conditions

$$\left.\begin{array}{l} \Theta = 1; \quad P = 1; \\ n_1 = 1/n_0; \; n_2 = 0; \\ w = 1; \end{array}\right\} at\ x = 0; \qquad (13)$$

$$\left.\begin{array}{l} \Theta(x,t) = \Theta_0 e^{-\lambda BH_0}\left[e^{-\lambda Bt} I_0\left(2\sqrt{\lambda^2 \gamma H_0 t}\right) + \dfrac{1}{\lambda BH_0}\displaystyle\int_0^{\lambda^2 BH_0 \gamma t} e^{-\tau/\lambda BH_0} I_0\left(2\sqrt{\tau}\right) d\tau \right]; \\ \dfrac{\partial n_1}{\partial x} = 0, \dfrac{\partial w}{\partial x} = \dfrac{a_0 H_0}{w_0}; \; n_2 = \dfrac{n_0}{N_0} \end{array}\right\} npu\, x = 1, \qquad (14)$$

here

$\bar{\Theta}(t) = \int_0^1 \Theta(x,t)dx, \bar{\delta}(t) = \int_0^1 \delta(x,t)dx; \; B = \dfrac{m_0(1-m)}{w}, Eu = \dfrac{\rho w^2}{P_0 m}$ Euler number;

$\mathrm{Re} = \dfrac{\rho k_0 w}{\mu H_0}$ - Reynolds number .

In (14) I_0- is Bessel's function of the first order. Then, N_i and n_i unbalanced concentrations of exchanging ions in the mixture and sorbent; P drop in pressure, Θ concentration of suspension, settled on the surface of a filter, Θ_1 initial concentration of suspension, Θ_3 concentration of the particles, passing through filtering partition, δ - concentration of suspension settled in the pores of a filter, m porosity of settled mass in the pores of a filter, m_0 initial porosity, H_0 the thickness of a filter; □, μ viscosity and density of a filtrate; λ kinematic coefficient; γ parameter of filtering; w velocity of filtering; β effective constant of exchanging ions; a, b constant isotherms; D_a,D_b coefficients of longitudinal diffusion and bar-diffusion; μ_0 coefficients of artificial viscosity, v volume of a filtrate, passing through filtering column; S- the area of a filter.

To close the system (4) - (14) the equations of equivalence of the exchange are added

$$n_1 + n_2 = n_0\,; \qquad (15)$$

$$N_1 + N_2 = N_0, \qquad (16)$$

here n_0 – is an initial concentration in the mixture of ions introduced into a column, N_0 exchanging capacity of sorbent absorption.

The solution of the system (4)-(16) was carried out with application of vector-difference scheme with accuracy O ($\Delta t+\Delta h^2$) [5]. The step of integration in time was taken as irregular one, that allowed to considerably reduce the amount of calculations and as a consequence – to decrease the errors of rounding off. For non-linear terms Bellman method of quasi-linearization [6] was applied. The number of iterations is defined by the condition:

$$\max\left|N_i^{(s)} - N_i^{(s-1)}\right| < \varepsilon, \qquad \frac{\left|W_i^{(s)} - W_i^{(s-1)}\right|}{W_i^{(s)}} < \varepsilon,$$

here $N_i^{(s)}.W_i^{(s)}, N_i^{(s-1)}, W_i^{(s-1)} -$ the values of the function, calculated on s time step.

To study and to analyze this process a program complex, realizing computational experiment, was developed on the basis of worked out algorithm. Numeric calculations were carried out according to the data of distribution of concentration of salts at water desalting by industrial device of chemical shop of complex purification in Tashkent Hydro-electric Plant for *H*-zeolite softening of the first stage of water purification from the ions of calcium, magnesium and sodium. The calculations have been carried out for the values:

$$\alpha = \theta(1-\delta);\ \mathrm{H} = 0.2;\ w_0 = 0.0025;\ \ \theta_0 = 0.00001\ .$$

Results of numeric experiments are given in Fig.1-2.

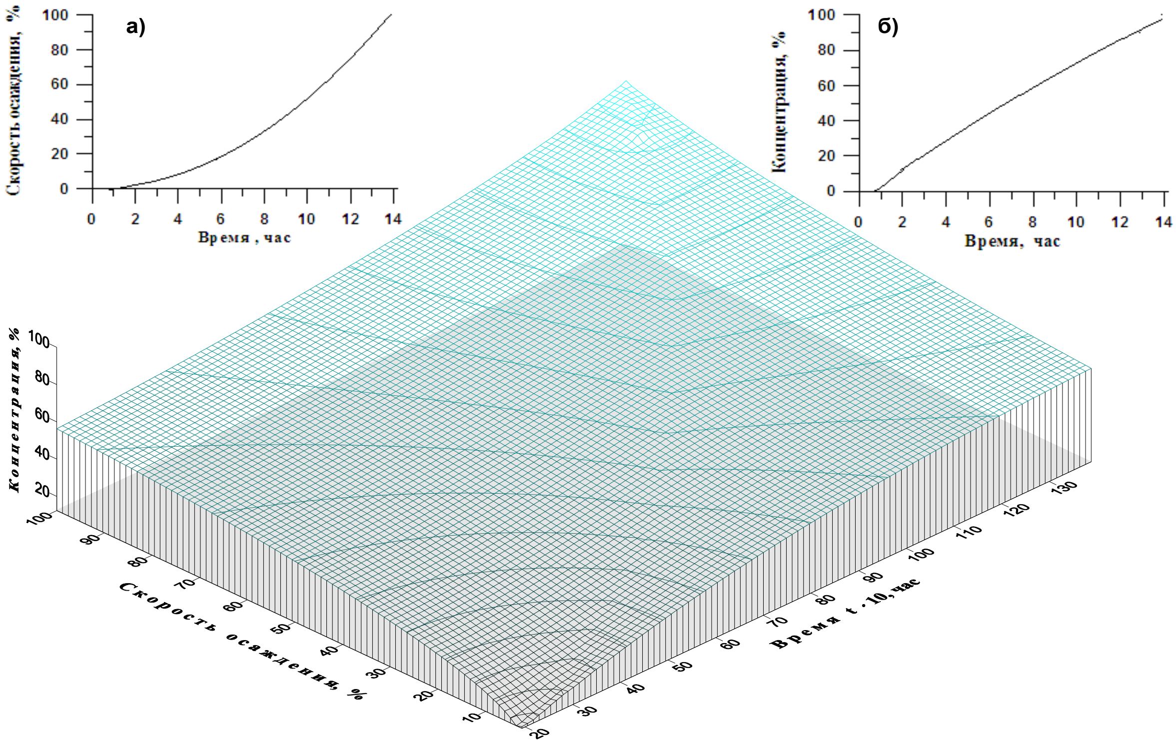

Fig. 1. The surface of changes of concentration (%) inside the filter, as a function of time (hour) and velocity of settlement (%) of gel-particles

Foot-note: a) – velocity of settlement (%) and b) – change in concentration (%), as a function of time (hour). On the graph of the surface time axis is multiplied by 10.

Fig. 1(a) shows that during discussed period of time with the rise in squeeze coefficient of a filter, the velocity of settlement of gel-particles in a porous media is parabolically increasing with the convexity directed downward. Concentration inside the column is changing in a similar way, but with convexity directed upward (fig.1b). The change of concentration with time as a function of velocity of settlement of gel-particles may be plainly seen in fig. 1 in the form of built surface. Such change in concentration leads to the situation when the basic mass of gel-particles is settled on the upper layer of a filter and with time gal-particles are settled along the thickness of the layer of a filter. It is evident that the concentration of suspension in upper layers of a filter is increasing with time. Because of the clogging of the pores of a filter by gel-particles, the velocity of filtration becomes less (fig.2a), creating an additional pressure inside the column of an aggregate. With the formation of the layer of settlement above the surface of a filter, the velocity of settlement and penetration of gel-particles into filtering partition becomes less. The process of permeation of the pores of a filter by suspended particles (with sufficient for the practice purposes accuracy) may be considered as done according to linear law.

Fig. 2б shows the changes of exchanging ions in a mixture. As seen from curves in fig. 2б the concentration of exchanging ions in a mixture is exponentially decreasing. Due to decrease of their concentration in a mixture, the concentration of exchanging ions in sorbent becomes higher.

The change in the volume of a filtrate, passing through filtering column of an aggregate, is shown in fig.2b. According to carried out numeric experiment the volume of a filtrate is parabolically increasing with time, with inconsiderable curvature directed upward. On the initial stages of the process of filtering the velocity of passage of a mixture through the columns of a filter is greater than in a final stage of the process.

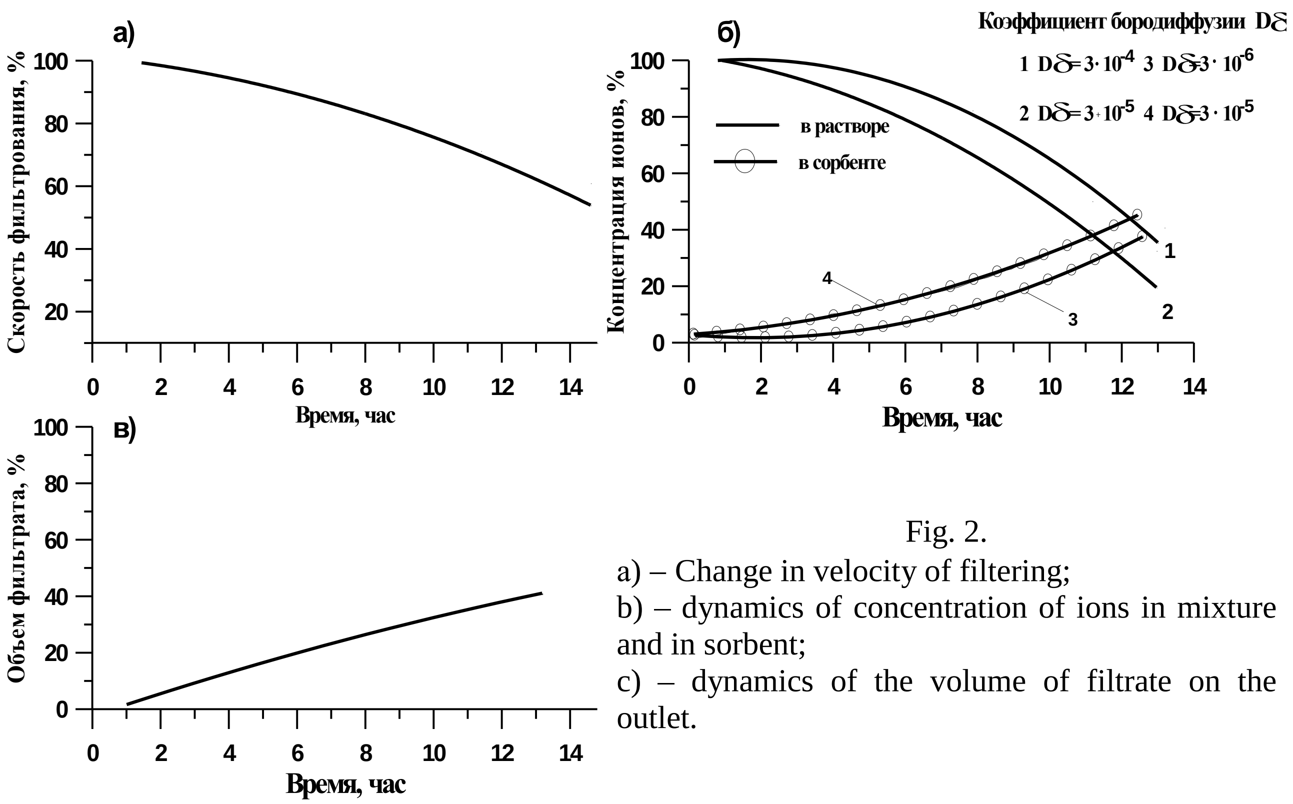

Fig. 2.
a) – Change in velocity of filtering;
b) – dynamics of concentration of ions in mixture and in sorbent;
c) – dynamics of the volume of filtrate on the outlet.

Conclusions. From carried out numeric experiment it follows that as a result of colmatation of suspended particles in the pores of ionite filter, the velocity of ion exchange and the time of duration of filter operation become less. Theoretical (design) time of switching with the absence of concentration of suspension in water is 16,6 hour, when adding concentration $\theta = 0{,}0001$, time of switching is 13,7 hour, with concentration $\theta = 0{,}0003$ - 15,1 hour. When the concentration is present average design time of switching is 16,2 hour.

Calculations carried out for the distribution of concentration of salts of citric acid with time show that theoretical time of slippage without the concentration of a suspension is 1 minute, with addition of concentration of a suspension $\theta = 0{,}0001$ 17 minutes, and practical time of a slippage 25 minutes.

Theoretical and experimental time of permeation without concentration of suspension is 110 minutes, and with concentration of suspension – 93 minutes. It should be noted that when solving the problem of mass-exchange with colmatation during a given time adsorption process ceased to go on, but the concentration of ionite was not saturated completely. This effect is explained by the fact that gel-particles, when colmatating ionite filter, somehow «isolate» the grains of ionite; and by this present the contact with liquid phase.

Carried out computational experiment has shown that at filtering of a filtrate with multi-layered filter: firstly, the degree of purification of mixtures from the admixtures and technological waste is greater, secondly, the time of operation of a filter becomes 1,2 – 1,3 times longer; thirdly, bearing capacity of a filter becomes greater. Average specific weight of the admixture at passing through the first layer is 0,0405, and after the passage through the second layer – 0,0285. Hence, when filtering the mixtures with two-layered filter, the volume of attached admixture and gel-particles is almost 2 times greater than with one-layered filter, that, in a sense, is evident and from the point of view of reliability of a worked out model shows its complete validity. With increase in velocity of settlement of gel-particles in the pores of a filter its bearing capacity is decreasing first in the inner and then in the upper layers. On the basis of results obtained, concerning the velocity of settlement of gel-particles in the layers of a filter, it becomes possible to assess the decrease in bearing capacity of a filtering aggregate. Carried out results at different modes of filtering have shown that with the change in the value of a coefficient of bar-diffusion, the velocity of exchange of ions in a sorbent and a mixture is changed as well.

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Компьютерная модель и вычислительный эксперимент для анализа функционирования технологического процесса фильтрования суспензии

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Аннотация. Для анализа функционирования технологического процесса фильтрования суспензии разработана адекватная математическая модель, численный алгоритм и проведены вычислительные эксперименты на ЭВМ. Результаты вычислительных экспериментов проиллюстрированы в виде графиков.

Ключевые слова: Компьютерная модель; вычислительный эксперимент; технологический процесс; фильтрования.