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Locally Bounded Function Spaces as the External Environment for Nonlinear Systems

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Abstract. In this paper we consider locally bounded function spaces that act as the external environment for nonlinear dynamic systems. We give non-traditional examples of above spaces in which the basis of the selected function Orlicz space.

Keywords: external environment of the system; locally bounded function spaces; nonlinear dynamical systems.

Introduction. It is known (see [1] - [2]) that the study of nonlinear dynamical systems in a number of situations is greatly simplified when viewed from the position of operator approach. A characteristic example is the non-linear multi-dimensional data processing systems containing Hammerstein and Urysohn integral operators.

Qualitative methods for solving analysis and synthesis problems of the systems of this type lead to non-local solvability of the corresponding operator equations in the original function spaces [3] as the external environment.

The objective of the analysis [4] is finding the exact (analytical) or approximate solution of the corresponding operator equation describing the dynamical system under consideration, and its further study in a given domain of the state space.

In turn, the purpose of synthesis is to form the optimal program and stabilizing controls, to construct algorithms processing the input observations and block diagrams of the optimal receivers, linear and non-linear filters, signal detectors, and so on within the specified quality criteria in the presence of disturbing factors of different nature.

Materials and Methods. Methodological and theoretical basis of the work are the fundamental works of national and international scientists in the study of nonlinear dynamical systems, the construction of their structure, identification, adaptation, and also new methods for the synthesis of the above systems.

Used special topics of functional analysis, operator theory, and publications related to the Orlicz spaces and amalgam spaces.

Discussion. In this paper we discuss the possibility of using locally bounded (generally nonnormable) Orlicz function spaces as an external environment for the system. Their topological structure, as well as various properties of operators and equations in them are of independent interest.

We give a number of non-traditional locally bounded function spaces (LBFS) (for details, see our monograf [5], which contains the necessary terminology and a great number of references.) As a model we have chosen Orlicz function space with sufficiently flexible structure covering a large family of LBFS.

Let (Ω, Σ, μ) be a measure space, where Ω is a compact set in R^n, $\sum - \sigma$-algebra of its measurable subsets, $\mu - \sigma -$ additive measure (for simplicity μ can be considered as Lebesgue measure).

Orlicz space $L^{*\Phi}$ consists of Lebesgue measurable functions defined on (Ω, Σ, μ),and is generated by a non-negative non-decreasing Young function Φ, that has several properties. In particular, if we consider measurable vector functions $\vec{\varphi}: T \to X$, where *X* is a normalized *B*-space, it is natural to assume that the generating LBFS Young function Φ is defined on *X*.

If the Young function Φ is defined on the interval $[0, \infty)$, then Orlicz space $L^{*\Phi}$ is defined not through the values $\vec{\varphi}(t)$ of vector functions $\vec{\varphi}$ themselves, but directly through their norms in the original space *X*. Obviously, this approach comes to the above if we replace the Young function Φ by $\Phi_1 = \Phi(\|\cdot\|)$, (for example, if $\Phi(u) = u^p$, $u \geq 0$, then $\Phi_1(u) = \|u\|^p$ and $L^{*\Phi} = L^p$.

Let $\Phi(x, y)$ be an arbitrary saddle Young function, $\Gamma_\Phi(x, y)$ – an integrated modular defined by Young function Φ, $L^{*\Phi}(\Omega, \Sigma, \mu)$ – locally bounded *F* - quasinormed Orlicz space, where $\left\|x; L^{*\Phi}\right\| = \inf\left\{\varepsilon > 0 : \Gamma_\Phi\left(\frac{|(x,\cdot)|}{\varepsilon}\right) \leq \varepsilon\right\}$, (similar for an element *y*) (for details see [6]). In particular, if a concave-convex saddle function $\Phi(x, y) = |x(\tau)|^{y(\tau)}$, where $0 < y(\tau) < 1$, then we obtain the corresponding partially ordered locally nonconvex topological space (defined by the integral modular $\Gamma(x, y) = \int\limits_\Omega |x(\tau)|^{y(\tau)} d\mu(\tau)$), whose structure is so far very little studied, although the solution of the relevant applied problems is of particular interest for the estimations of dual gap in the theory of nonconvex programming.

Let φ be an exact normal semifinite trace on von Neumann algebra $M, K(M, \varphi) - (*)$ – algebra of all measurable operators affiliated to *M*. *E* is a linear subspace in $K(M, \varphi)$ with *F*-quasinorm $\|\cdot; E\|$. Then *E* is a non-associative locally bounded space. The nature of *E* space is also poorly understood, as evidenced by few publications in the world (for details see [5]).

Let L^Φ be a class of all μ - measurable almost periodic functions $x(\tau)$, the ones that at each interval of 2*T* length the upper limit is finite $\Gamma_\Phi(x) = \overline{\lim\limits_{T \to \infty}} \frac{1}{2T} \cdot \int\limits_{-T}^{T} \Phi(|x(\tau)|) d\mu(\tau)$. Then the corresponding LBFS, denoted as M_Φ with *F*- quasinorm $\|x; M_\Phi\| = \inf\{\varepsilon > 0 : \Gamma_\Phi(|x| / \varepsilon) \leq \varepsilon\} < +\infty$, is a locally nonconvex Marcinkiewicz-Orlicz space that is a basic one for the synthesis of almost periodic systems.

Wiener amalgam spaces are a class of function spaces where the function's local and global behavior can be easily distinguished. These spaces are extensively used in Harmonic analysis which originated in the work of Wiener.

For each integer *k*, let $I_k = [k, k+1)$ be the interval of $\mathbb{R}$ for a function $f : \mathbb{R} \to X$, $f|_{I_k}$ will stand for the restriction of *f* on I_k.

Let $1 \leq p, q \leq \infty$. Define

$$(L^p(\mathbb{R}, X), l^q(Z)) = \{f : \mathbb{R} \to X / f|_{I_k} \in L^p(I_k, X)\}$$

for all $k \in Z$ and the bi-infinite sequence $\{\|f|_{I_k}\|_{L^p(I_k, X)}\}_{k \in Z} \in l^q(Z)\}$.

It easy to see that this set is a vector space over the complex field under pointwise operations. This space will be called a vector-valued amalgam space.

The amalgam space $(L^p(\mathbb{R}, X), l^q(Z))$ is a Banach space with respect to the norm $\|\cdot\|_{p,q}$, *for* $1 \le p, q \le \infty$.

It should be noted that the synthesis of a nonlinear system is usually an ill-posed problem (in the corresponding pair of linear topological Orlicz spaces) due to the fact that small discrepancies in initial data can result in the enormous discrepancy in solutions.

Results and conclusions. In this work we propose the possibility of using as an external environment for the nonlinear dynamic system of locally bounded Orlicz function spaces. We give some of non-traditional examples of the above-mentioned spaces.

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Локально ограниченные функциональные пространства как внешняя среда для нелинейных систем

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Аннотация. В статье рассматриваются локально ограниченные функциональные пространства, выступающие в роли внешней среды для нелинейных динамических систем. Приводятся нетрадиционные примеры вышеуказанных пространств, в которых за основу выбрано функциональное пространство Орлича.

Ключевые слова: внешняя среда системы; локально ограниченные функциональные пространства; нелинейные динамические системы.