SECTION 2. Applied mathematics. Mathematical modeling.

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TRANSPARENT SURFACE KHAN SHATYR AREA CALCULATION

In this article solves the problem of finding the square of construction of the Khan-Shatyr, like the surface of the oblique cone, on Maple.
Keywords: oblique cone, surface area.

Consider the process of constructing a mathematical model and calculation of the square surface of the construction Khan Shatyr.

Khan Shatyr (Fig.1) is hyperboloid design [1], and it is based on a hyperboloid.

The hyperboloid is kind of a surface of the second order three-dimensional space, specified in Cartesian coordinates by the equation

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$ (one-way hyperboloid),

where a and b are real axis, and c is the imaginary axis; or

$$-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$ (two-way hyperboloid),

where a and b are imaginary axis, and c is the real axis.

Figure 1 - Khan Shatyr.

By design, Khan Shatyr can be described by the equation one-way hyperboloid. Symmetry axis is directed along the spire. Known height of the main spire (Fig.2) [2]. All further calculations will bind to this size.

Practically, in mathematical terms, we have the following picture Fig.3..

Figure 2 - The Height Of The Khan Shatyr.

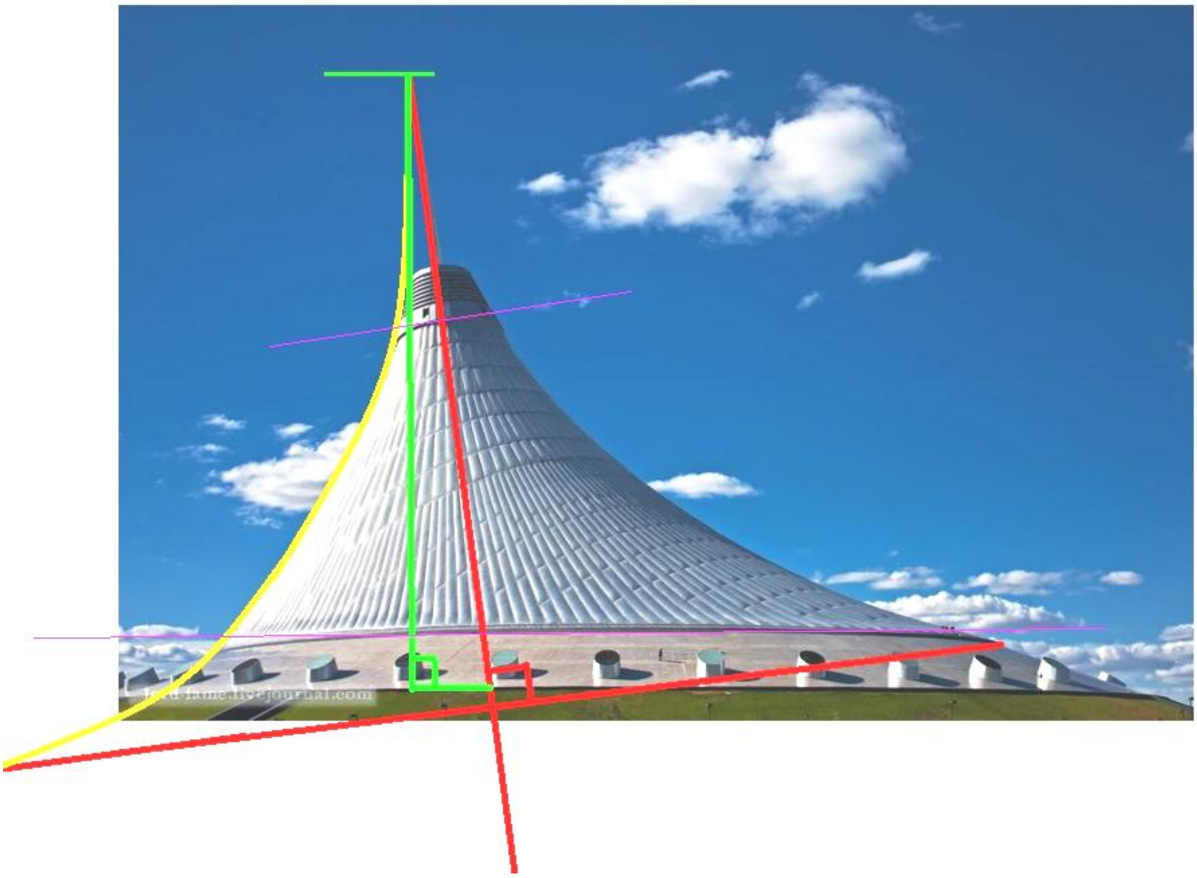


Figure 3 - Basic laws.

Do the analysis, and find out the main dimensions, for this we will make the program on Delphi (Annex 1). Program allows you to open the image, enter the basic data coordinates, move the origin to the specified point, rotate the coordinate system at a given angle, and translate all the coordinates in the SI metric system.

To find the coordinates of all points in the SI system (Fig.4), and shift slightly the origin at point «0» to the Ox axis was directed along the line of «03» (Fig.5).

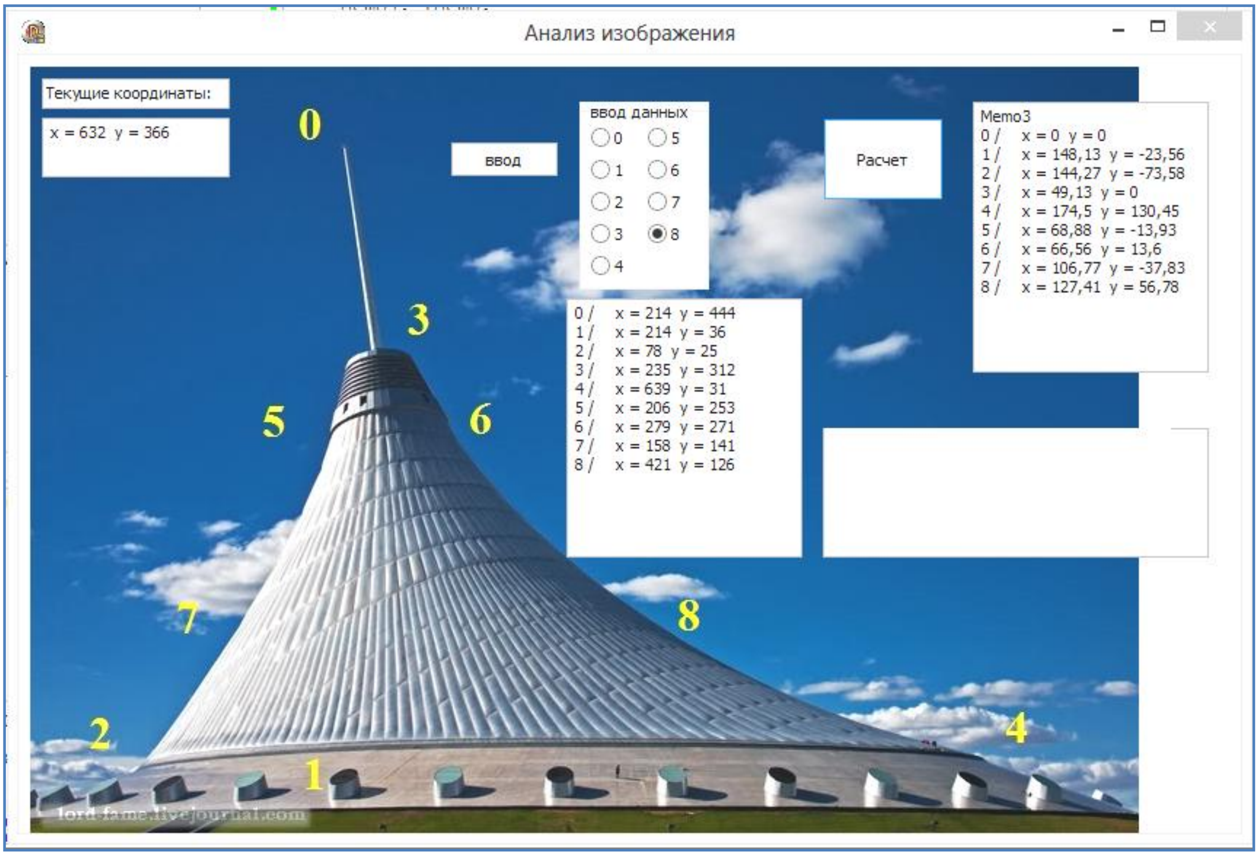


Figure 4 - Determination of coordinates and sizes.

It is obvious that the hyperbolic surface is formed by the rotation curve $y = f(x)$ around the ox axis. And the surface area can be found under [3-5]and is found by the formula:

$$S = S_1 + S_2 .$$

Moreover

$$S_1 = 2\pi \int_a^b f(x)\sqrt{1 + f'^2(x)}dx$$

Knowing the coordinates of the pointsт «6», «8», «4», via Lagrange interpolation polynomials [6-7] find coefficients of the curve $y = f(x)$.

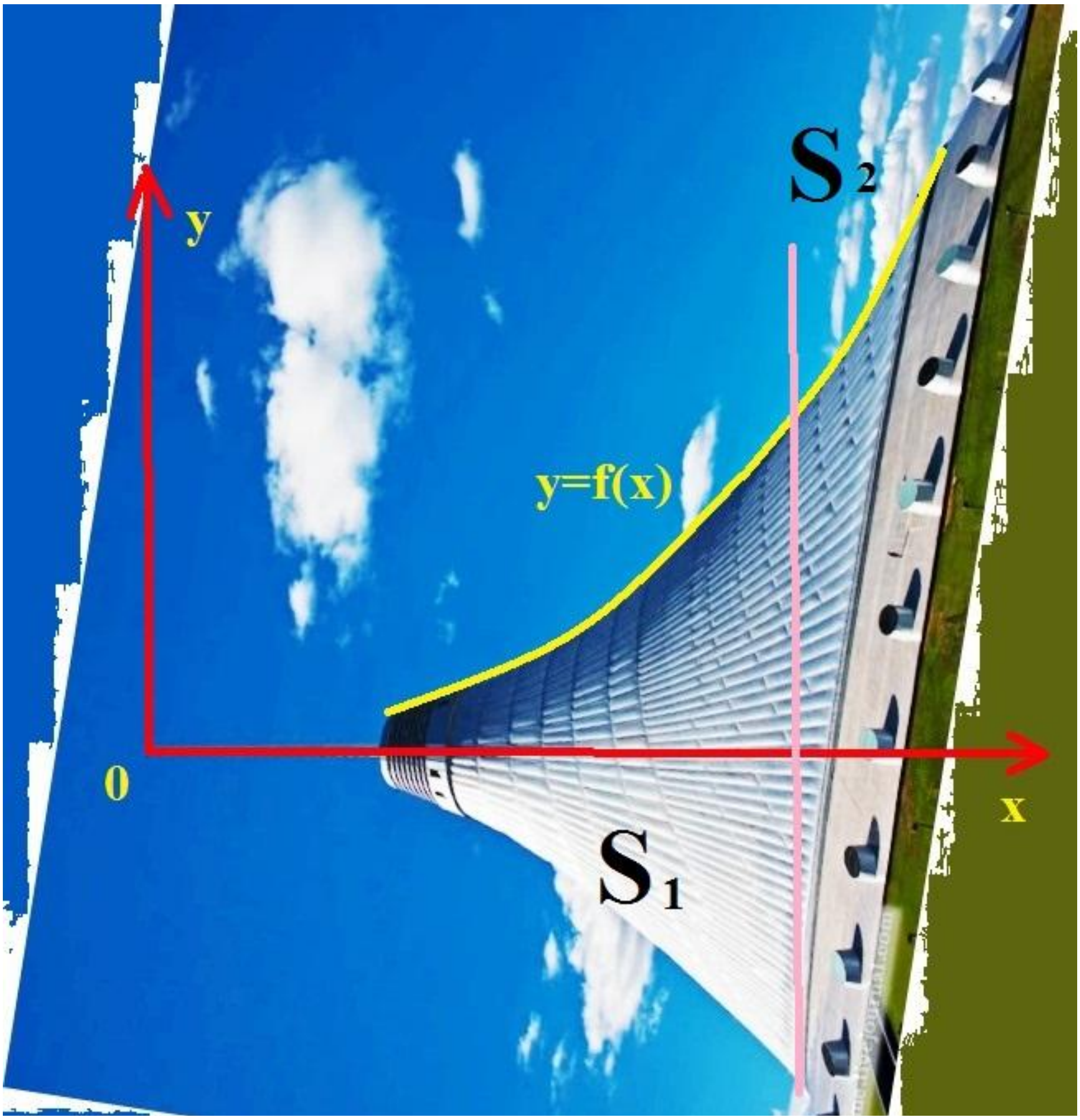


Figure 5 - New coordinate system.

$$y=\frac{(x-x_8)(x-x_4)}{(x_6-x_8)(x_6-x_8)}y_6+\frac{(x-x_6)(x-x_4)}{(x_8-x_6)(x_8-x_4)}y_8+\frac{(x-x_6)(x-x_8)}{(x_4-x_6)(x_4-x_8)}y_4$$

Derive the coefficients of the function, we get:

$$y=\left(\frac{y_6}{(x_6-x_8)(x_6-x_8)}+\frac{y_8}{(x_8-x_6)(x_8-x_4)}+\frac{y_4}{(x_4-x_6)(x_4-x_8)}\right)x^2-$$
$$-\left(\frac{y_6(x_8+x_4)}{(x_6-x_8)(x_6-x_8)}+\frac{y_8(x_6+x_4)}{(x_8-x_6)(x_8-x_4)}+\frac{y_4(x_8+x_6)}{(x_4-x_6)(x_4-x_8)}\right)x+$$
$$+\left(\frac{y_6(x_8x_4)}{(x_6-x_8)(x_6-x_8)}+\frac{y_8(x_6x_4)}{(x_8-x_6)(x_8-x_4)}+\frac{y_4(x_8x_6)}{(x_4-x_6)(x_4-x_8)}\right)$$

Let's substitute it found coordinates (Fig.6), and we obtain:

$$y = 0.009x^2 - 1.21x + 51.139$$

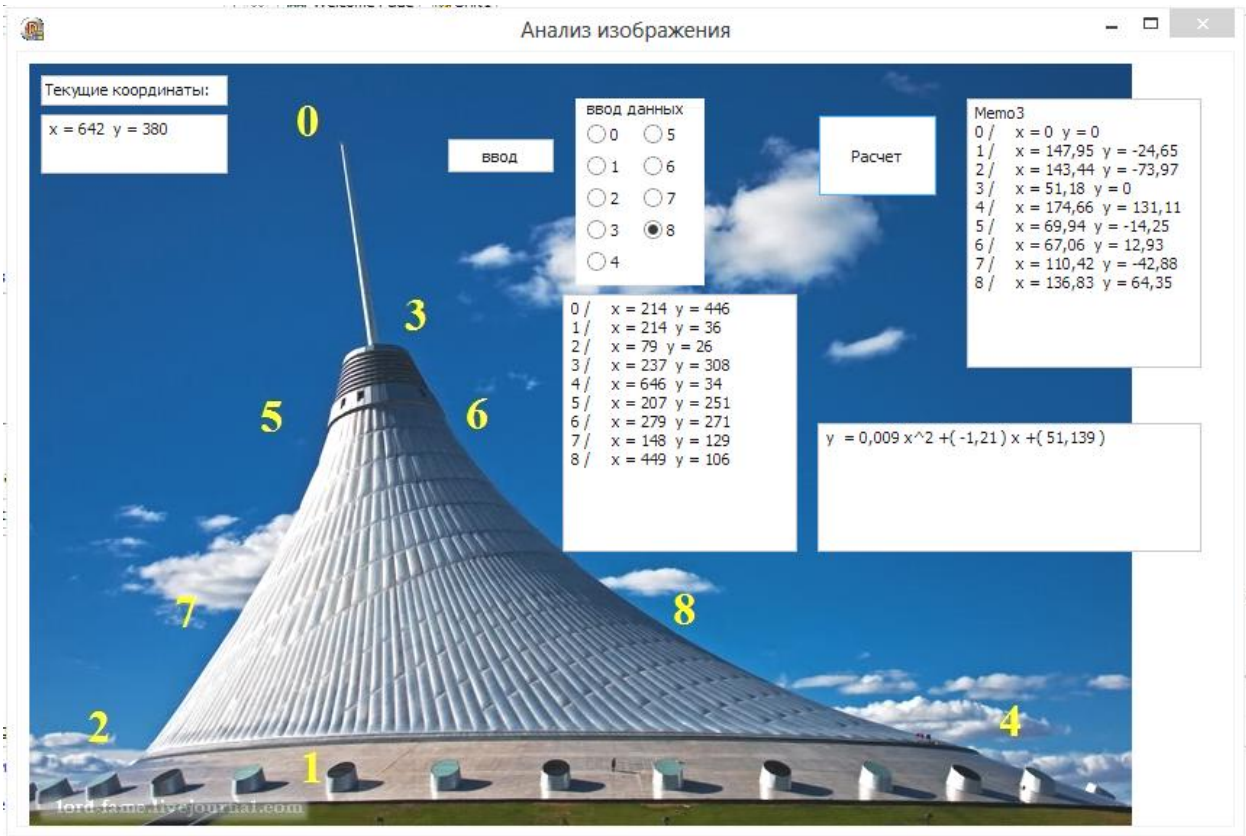


Figure 6 - Calculation functions : $y = f(x)$.

Find the square analytically, on Maple:

```
> restart;
  f:=0.009*x*x-1.21*x+51.139;
  ff:=diff(f,x$1);
  SS1:=(2*Pi*Int(f*sqrt(1+ff*ff),x));
```

$$f := .009\,x^2 - 1.21\,x + 51.139$$

$$ff := .018\,x - 1.21$$

$$SS1 := 2\,\pi \int (.009\,x^2 - 1.21\,x + 51.139)\sqrt{1 + (.018\,x - 1.21)^2}\,dx$$

For S_1 the limits of integration are equal

$$a = 67$$

$$b = 144.27$$

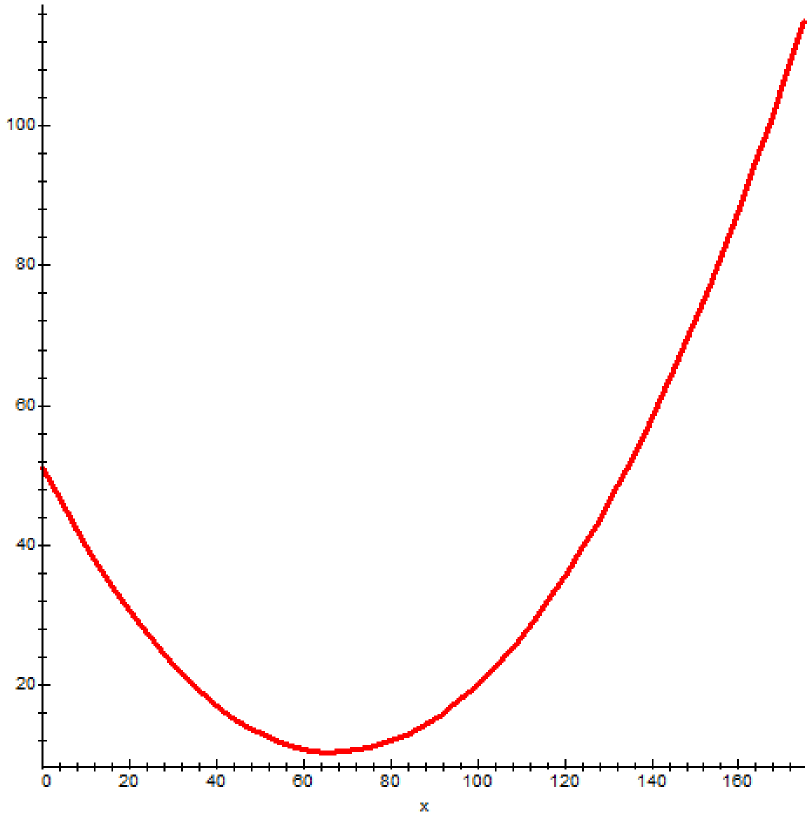
```
> restart;
  f:=0.009*x*x-1.21*x+51.139;
  ff:=diff(f,x$1);
  SS1:=abs(evalf(2*Pi*int(f*sqrt(1+ff*ff),x=67..144.27)));

  plot(f,x=0..175,thickness=3);
>
```

$$f := .009\,x^2 - 1.21\,x + 51.139$$

$$ff := .018\,x - 1.21$$

$$SS1 := 18972.67023$$



Find S_1

$$S_1 = 18972.67\ м^2$$

Find now S_2, it is obvious that

$$S_2 = k * 2\pi \int_b^c f(x)\sqrt{1 + f'^2(x)}dx$$

where k is a coefficient of proportionality,

$$b = 144.27$$
$$c = 174.5$$

Considering square figures (Fig. 7), we can find close to k coefficient of proportionality k^*, here $k \approx k^* = \frac{S_3}{S_4}$.

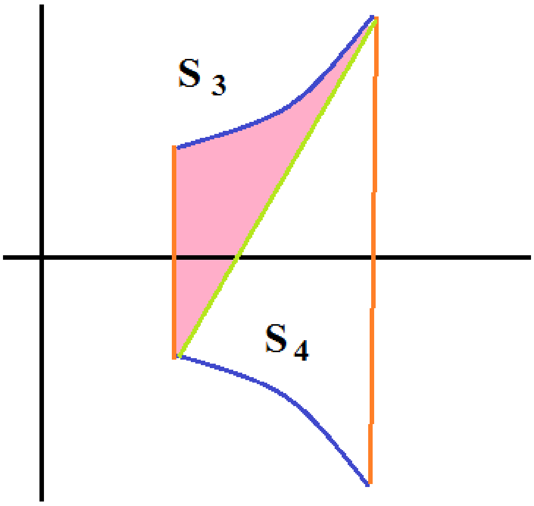


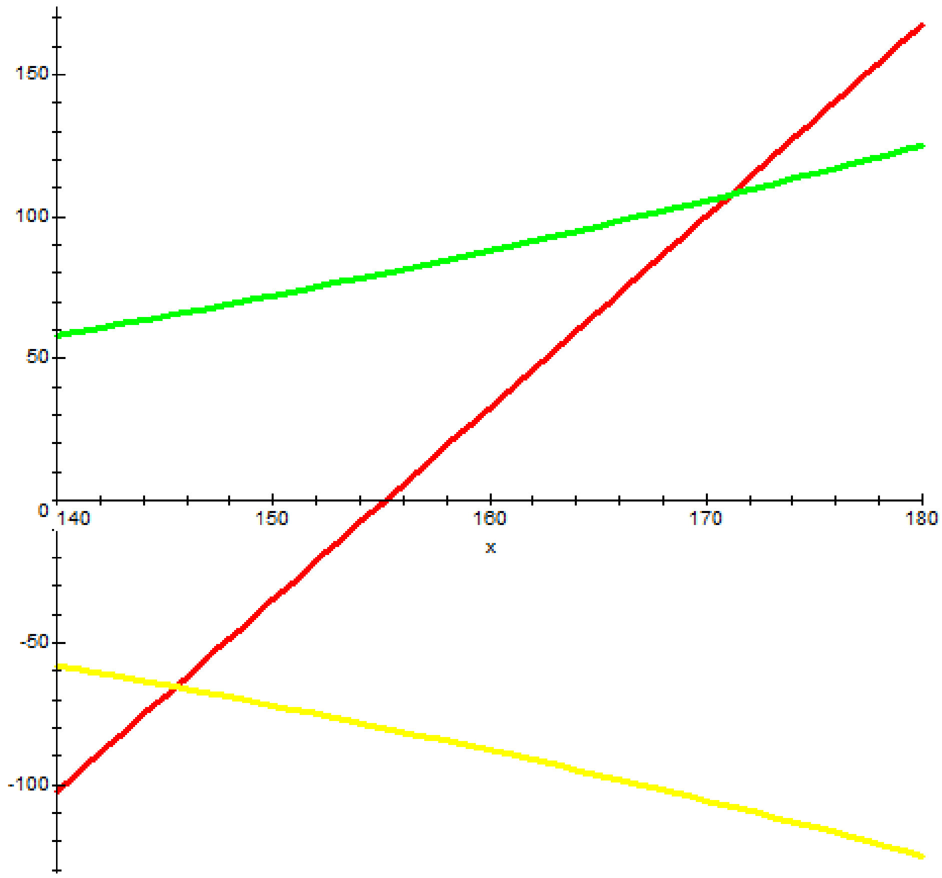
Figure 7 - Front view .

Then solving the problem on Maple, we obtain:

```
>
  f2:=0.009*x*x-1.21*x+51.139;
  f1:=-73.58+((130.45+73.58)*(x-144.27)/(174.5-144.27));
  f3:=-(0.009*x*x-1.21*x+51.139);

  S1:=abs(evalf(int(f2-f1,x=144.27..174.5)));
  S2:=abs(evalf(int(f1-f3,x=144.27..174.5)));
  k:=S1/(S1+S2);
  plot([f1,f2,f3],x=140..180,thickness=3);
```

$$f2 := .009\,x^2 - 1.21\,x + 51.139$$
$$f1 := -1047.295121 + 6.749255706\,x$$
$$f3 := -.009\,x^2 + 1.21\,x - 51.139$$
$$S1 := 1788.58052$$
$$S2 := 3507.76056$$
$$k := .3377011588$$




```
>
> f:=0.009*x*x-1.21*x+51.139;
  ff:=diff(f,x$1);
  SS2:=abs(evalf(2*Pi*int(f*sqrt(1+ff*ff),x=144.27..174.5)));

  S:=SS1+k*SS2;
>
```

$$f := .009\,x^2 - 1.21\,x + 51.139$$

$$ff := .018\,x - 1.21$$

$$SS2 := 32627.95977$$

$$S := 29991.17005$$

```
>
```

So we get a coefficient of proportionality

$$k \approx 0.3377$$

The area of the surface shape of a truncated:

$$S_2 = k * 32627.95977 = 11018.462\ м^2$$

And the final value of the surface area of the Shang Shatyr:

$$S = S_1 + S_2 = 18972.67 + 11018.462 = 29991.17\ м^2$$

Bibliography

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Addition 1

```
unit Unit1;
interface
uses
  Windows, Messages, SysUtils, Variants, Classes, Graphics, Controls, Forms,
math,  Dialogs, ExtCtrls, StdCtrls, jpeg, Grids;

type
  TForm1 = class(TForm)
    Image1: TImage;
    Memo1: TMemo;
    Edit1: TEdit;
    RadioGroup1: TRadioGroup;
    Button1: TButton;
    Memo2: TMemo;
    Button2: TButton;
    Memo3: TMemo;
    Memo4: TMemo;
    procedure Image1MouseMove(Sender: TObject; Shift: TShiftState; X,
      Y: Integer);
    procedure FormCreate(Sender: TObject);
    procedure Button1Click(Sender: TObject);
    procedure Image1Click(Sender: TObject);
    procedure Button2Click(Sender: TObject);
  private
    { Private declarations }
  public
    { Public declarations }
  end;

var
  Form1: TForm1;
 b0:boolean;
 t:array[0..8] of tpoint;
i,xx,yy:integer;
d,k:real;
sx,sy,sxn,syn:real;
l1,l2,r1,r2,cosfi,sinfi,ax,ay,bx,by:real;
a00,a1,a2,aa,bb,cc:real;
txn,tyn, tx,ty:array[0..8] of real;

implementation
```

```
{$R *.dfm}

procedure TForm1.Button1Click(Sender: TObject);
begin
b0:=true;
radiogroup1.ItemIndex:=0;
memo2.Clear;
end;

procedure TForm1.Button2Click(Sender: TObject);
var b0,a0,ff:real;
begin
d:=150;
k:=d/sqrt(sqr(t[0].X-t[1].X)+sqr(t[0].y-t[1].y));
//переводим все в метры

for I := 0 to 8 do
begin
tx[i]:=t[i].x*k;
ty[i]:=t[i].y*k;
end;

 // находим точку S

a0:=tx[3]-tx[0];
b0:=ty[3]-ty[0];
sx:=(ty[2]-ty[0]+(b0/a0)*tx[0]+(a0/b0)*tx[2])/(b0/a0+a0/b0) ;
sy:=b0/a0*(sx-tx[0])+ty[0];

// сдвиг начала координат в точку 0

for I := 1 to 8 do
begin
tx[i]:=tx[i]-tx[0];
ty[i]:=ty[i]-ty[0];
end;
sx:=sx-tx[0];
sy:=sy-ty[0];
tx[0]:=0;
ty[0]:=0;

//повернем систему координат
```

```
ax:=1;
ay:=0;
bx:=tx[3]-tx[0];
by:=ty[3]-ty[0];
cosfi:=abs(ax*bx+ay*by)/( sqrt(sqr(ax)+sqr(ay))*sqrt(sqr(bx)+sqr(by))  );
ff:=-(arccos(cosfi));
cosfi:=cos(ff);
sinfi:= sin(ff);

// поворот
for I := 0 to 8 do
begin
txn[i]:=tx[i]*cosfi+ty[i]*sinfi;
tyn[i]:=-tx[i]*sinfi+ty[i]*cosfi;
end;
sxn:=sx*cosfi+sy*sinfi;
syn:=-sx*sinfi+sy*cosfi;

for I := 0 to 8 do
memo3.Lines.Add(inttostr(i)+' /       x = '+floattostr(int(txn[i]*100)/100)+'   y =
'+floattostr(int(tyn[i]*100)/100));

a00:=tyn[6]/((txn[6]-txn[8])*(txn[6]-txn[4]));
a1:=tyn[8]/((txn[8]-txn[6])*(txn[8]-txn[4]));
a2:=tyn[4]/((txn[4]-txn[6])*(txn[4]-txn[8]));

aa:= a00+a1+a2;
bb:=-(a00*(txn[8]+txn[4])+a1*(txn[6]+txn[4])+a2*(txn[6]+txn[8]));
cc:=a00*(txn[8]*txn[4])+a1*(txn[6]*txn[4])+a2*(txn[6]*txn[8]);

memo4.Clear;
memo4.Lines.Add('y          =      '+floattostr(int(aa*1000)/1000)+'      x^2      +(
'+floattostr(int(bb*1000)/1000)+' ) x +( '+floattostr(int(cc*1000)/1000)+' )');
end;

procedure TForm1.Image1Click(Sender: TObject);
var i:integer;
begin
if b0 then
begin
i:= RadioGroup1.ItemIndex;
t[i].X:=xx;
t[i].y:=yy;
```

```
memo2.Lines.Add(inttostr(i)+' /     x = '+inttostr(xx)+'  y = '+inttostr(yY))
;  inc(i);
radiogroup1.ItemIndex:=i;
if i=9 then  b0:=false;

end;
end;

procedure TForm1.Image1MouseMove(Sender: TObject; Shift: TShiftState; X,
  Y: Integer);
begin

xx:=x;
yy:=500-y;    memo1.Text:='x = '+inttostr(xx)+'  y = '+inttostr(yy);
end;

end.
```