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Optimal Placement of Capacitor in Radial Distribution System Using Real Coded Genetic Algorithm

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ABSTRACT: Capacitors in power systems are generally used to supply reactive power to reduces losses and improvement of the voltage profile.The appropriate placement of capacitors is also important to ensure that system power losses and total capacitor costs can be reduced. This paper presents a new constraint objective cost function to maximize the net annual savings by minimizing real power loss and optimizing the purchase cost of shunt capacitors while maintaining a better node voltage profile of the system.Here two steps solution methodology i.e. loss sensitivity factors for identification of candidate buses to place the capacitors and real coded genetic algorithm to determine the value of optimal capacitor have been adopted.The effectiveness of the proposed method has been tested on 33-bus IEEE standard radial distribution system.Result shows that after placement of capacitor in candidate buses the energy loss hasbeen reduced.

Keywords: Radial Distribution System, Capacitor Placement, Loss Sensitivity Factors and Real Coded Genetic Algorithm (RGA).

Nomenclature

S_i: Complex power at the i^{th} bus
P_i: Real power at the i^{th} bus
Q_i: Reactive power at the i^{th} bus
V_i: Bus voltage at the i^{th}bus
B: Branch current
I: Equivalent current injection
BIBC: Bus injection to bus current
BCBV: Branch current to bus voltage
B_k: Line section located between bus i and bus j
m: number of branches
n: number of buses
ΔV: Voltage variation for each regulation
Ii: The current of the section-i
Ri: The resistance of section-i
T: The time duration
Ce: The energy rate.
ELC: The energy loss cost
Cci: The constant installation cost of capacitor
Ccv: The rate of capacitor per kVAr
Qck: The rating of capacitor on bus-k in kVAr
S: Cost function
X_N: An N -dimensional vector of optimization parameters
g_{ij}: A gene
L_i: The length of the code string of the ith optimization parameter
N_{pop}: Population size
P_{cross}: Crossover probability
P_{mut}: Mutation probability

I. INTRODUCTION

The distribution system provides the link between bulk power system and consumer. A distribution circuit normally uses primary or main feeders and lateral distributors. A main feeder originates from the substation and passes through the major load centres. Lateral distributors connect the individual load points to the main feeder and are defined as radial distribution systems. Radial systems are popular because of their simple design and low cost.

A distribution system connects consumers to the high-voltage transmission system. Because of lower voltage, and hence higher current, the I^2R loss in a distribution system is significantly high compared to that in a high-voltage transmission system. Studies have indicated that as much as 13% of total power generated is wasted in the form of losses at distribution level. The pressure of improving the overall efficiency of power delivery has forced the power utilities to reduce the loss, especially at the distribution level.

The I^2R loss in a distribution system can be reduced by reconfiguring the network. The reconfiguration process changes the path of power flow from the source to the loads. The loss can also be reduced by adding shunt capacitors to supply part of the reactive power demands. Shunt capacitors not only reduce the loss but also improve the voltage profile, power factor and stability of the system.Economic benefits of the capacitor depends mainly on where and how many capacities of the capacitor are installed and proper control schemes of the capacitors at different load levels in the distribution system.

In general, a distribution system is fed at only one point and the structure of the network is mainly radial. For such a system all active power demands and losses must be supplied by the source at the root bus. However, addition of shunt capacitors can generate the reactive power and therefore it is not necessary to supply all reactive power demands and losses by the source. Thus, there is a provision to minimise the loss associated with the reactive power flow through the branches.

The capacitor placement problem is a well-researched topic and has been addressed by many authors in the past. Bhutad and kulkarni [1] reviewed various methods for the load flow of distribution systems are reviewed and the performance of these methods is compared for various parameters. Shirmohammadi *et al.* [2] has proposed a load flow method for distribution networks using a multi-port compensation technique and basic for mulations of Kirchhoff's Laws.

Rajicic and Tamura [3] have modified the fast decoupled load flow method to suit high R/X ratio nature of distribution system. Ghosh and Das [4] have proposed a method for the load flow of radial distribution network using the evaluation based on algebraic expression of receiving end voltage. Teng [5] has proposed the load flow of radial distribution system employing bus-injection to branch-current (BIBC) and branch-current to bus-voltage (BCBV) matrices. Capacitor placement problem is well researched topic. Neagle and Samson [6] consider ed loss reduction by one capacitor bank placed along the feeder by considering uniformly distributed loads, uniformly decreasing loads and equally distributed loads along the feeder. A general application curves for selecting location and size of single capacitors to minimize loss has been presented. Cook [7] considered the effects of fixed capacitors on radial distribution network with distributed loadsand considered the reduction in energy loss. A methodology has been used to determine the ratings and location of fixed capacitors on the radial feeder for periodic load cycle. Maxwell [8] suggested there are several benefits of capacitor placement which include: (i) reduced kVA input to feeder (ii) reduced I2 R loss and energy losses (iii) reduced I 2X losses (iv) reduced regulation cost (v) increased revenue as a result of increased voltage levels. Major benefits are due to the reduction in kVA input, kW demand and energy loss. Schmill [9] considered feeders with uniformly distributed and randomly distributed loads. A simplified method for capacitor application has been developed. Bae [10] presented an analytical method for capacitor allocation, under the assumptions (i) capacitor banks optimally located for specific load levels (ii) voltage regulation is not considered (iii) loads are assumed to be uniformly distributed along the feeder with the size of capacitor banks assumed equivalent (iv) only losses due to reactive current component are considered. The equations to determine the best capacitor locations and the loss reduction under varying load conditions has also presented. Grainger and Lee [11] considered the problem as non-linear programming problem, the capacitor sizes have been considered as continuous variables and iterative solution scheme has been proposed. K. Prakash and M. Sydulu [12] an effective topological and primitive impedance based distribution power flow algorithm is developed for both balanced and unbalanced distribution systems.

Using this concept and primitive impedances of the lines, only diagonal elements of the Distribution Load Flow (DLF) matrix are computed and stored in single dimension vectors to obtain the distribution load flow solution. Sattianadan *et al.* [13] load flow technique for a radial distribution using BIBC and BCBV has been used optimizing the overall economy calculated considering the energy loss cost and capacitor cost. Y. Mohamed Shuaib and C.Christober Asir Rajan [14] have solved the general capacitor placement problem in a distribution system using a genetic algorithm. Sensitivity analysis has been used to select the candidate locations of capacitors. Sundhararajan and Pahwa [15] also proposed shunt capacitor problem using Genetic Algorithm. In this paper a recent load flow technique for a radial distribution using BIBC and BCBV has been used. The candidate buses for shunt capacitor placement have been identified using Loss Sensitivity Factors. The sizes of the capacitors have been found using RGA, while optimizing the overall economy calculated considering the energy loss cost and capacitor cost. The developed algorithm is tested for 33-bus radial distribution systems while taking the different step sizes for capacitors.

II. LOAD FLOW OF RADIAL DISTRIBUTION SYSTEM

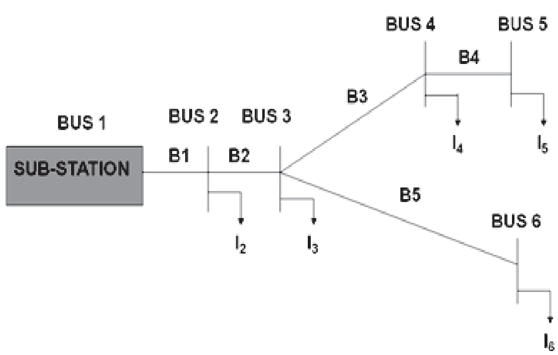

Fig. 1. A sample Distribution System.

The power flow is an important tool for power system analysis. The efficiency of the optimization problem of distribution system depends on the load flow algorithm because load flow solution has to run for many times. Therefore, the load flow solution of distribution system should have robust and time efficient characteristics. A method which can find the load flow solution of radial distribution system directly by using topological characteristic of distribution network is used [4].

The load flow of radial distribution system is carried out using the equivalent current injections,bus-injection to branch current matrix (BIBC) and branch current to bus voltage (BCBV). For distribution systems, at a bus i the complex load is expressed by (1) And the corresponding equivalent current injection is given By (2)

$S_i = (P_i + jQ_i)$ i = 1,2,...,n ...(1)

$I_i = (S_i / V_i)^*$ ...(2)

For the system shown in Fig. 1, apply Kirchhoff's current law (KCL), the branch currents can be expressed in terms of equivalent current injections as

$B_1 = I_2 + I_3 + I_4 + I_5 + I_6$ ...(3)

$B_2 = I_3 + I_4 + I_5 + I_6$ ...(4)

$B_3 = I_4 + I_5$ ...(5)

$B_4 = I_5$ ...(6)

$B_5 = I_6$ ...(7)

$$\begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix}$$

The above branch current equations can be rearranged in the generalized form as below

[B] = [BIBC][I] ...(8)

A. Algorithm for formation of BIBC matrix

The power injections can be converted to the equivalent current injection and the relationship between the bus-current injection and branch-current injections are obtained by Kirchhoff's current law (KCL) to the distribution network. The branch currents are formulated as equivalent of current injection.

Step (1): Create a null matrix of dimension m*n–1

Where, m = number of branches

n = number of buses

Step(2): If a line section (B_k) is located between Bus i and Bus j, copy the column of the i-th bus of the BIBC matrix to the column of the j-th bus and fi ll a + 1 in the position of the k-th row and the j-th bus column.

Step (3): Repeat Procedure (2) until all the line sections are included in the BIBC matrix.

The building Procedure for BIBC matrix shown in Fig.2. The algorithm can be easily expanded to a multi-phase line section or bus.

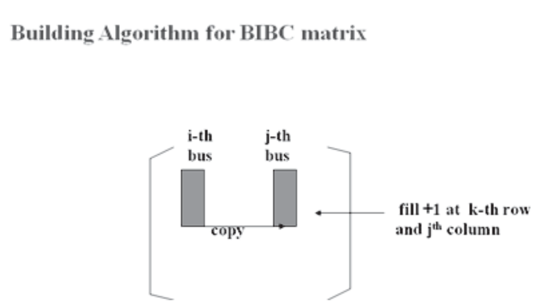

Fig. 2. BIBC matrix.

B. Algorithm for formation of BCBV matrix.

The BCBV matrix is responsible for the relations between the branch currents and bus voltages The corresponding variation of the bus voltages, which is generated by the variation of the branch currents, can be found directly by using the BCBV

$V_2 = V_1 - B_1 Z_{12}$ ...(9)
$V_3 = V_2 - B_2 Z_{23}$ ...(10)
$V_4 = V_3 - B_3 Z_{34}$ ...(11)

Substituting eqs. (9), (10) into eq. (11), the voltage of bus 4 can be written as
$V_4 = V_1 - B_1 Z_{12} - B_2 Z_{23} - B_3 Z_{34W}$

$$\begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} Z_{12} & 0 & 0 & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & Z_{34} & 0 & 0 \\ Z_{12} & Z_{23} & 0 & 0 & Z_{36} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix}$$

Step (1): Create a null matrix of dimension n-1*m
m = number of branches
n = number of buses

Step (2): If a line section (B_k) is located between Bus i and Bus j, copy the row of the i-th bus of the BCBV matrix to the row of the j-th bus and fi ll the line impedance (Z_{ij}) in the position of the j-thbus row and the k-th column.

Step (3): Repeat Procedure (2) until all the line sections are included in the BCBV matrix shown in Fig.3.

In the general form, we have
$[\Delta V] = [BCBV]\,[B]$ ...(12)

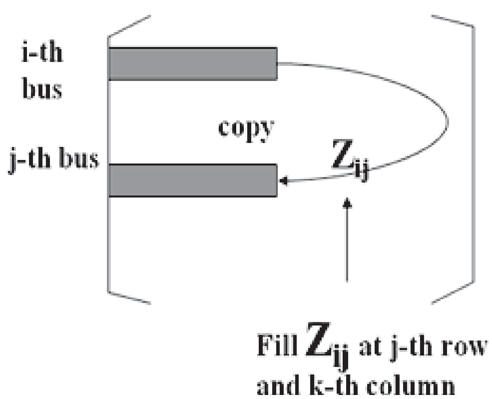

Fig. 3. BCBV matrix.

C. Algorithm for Distribution System Load Flow

A brief idea of how bus voltages can be obtained for a radial system is given below,
1. Input data.
2. Form the BIBC matrix.
3. Form the BCBV matrix.
4. Form the DLF matrix.
5. Iteration k = 0.
6. Iteration k = k + 1.
7. Solve the equations iteratively and update voltages

$I_i^k = (P_i + Q_i / V_i)$
$[\Delta V^{k+1}] = [DLF]\,[I^k]$
If $I_i^{k+1} - I_i^k >$ tolerance, go to step (6) else print result.

III. LOSS SENSITIVITY FACTOR AND CANDIDATE BUS SELECTION

The loss sensitivity factor is able to predict which bus will have the biggest loss reduction when a capacitor is placed [14]. Therefore, these sensitive buses can serve as candidate buses for the capacitor placement. The estimation of these candidate buses basically helps in reduction of the search space for the optimization problem. As only few buses can be candidate buses for compensation, the installation cost on capacitors can also be reduced. Real power loss in the line given by $[I^2_k]*[R_k]$, which can also be expressed as,

Pline loss[j] = $(P^2[j] + Q^2[j]) * [R_k] / (V[j])^2$
Similarly the reactive power loss in the k^{th} line is given by,
Qline loss[j] = $(P^2[j] + Q^2[j]) * [X_k] / (V[j])^2$

The Loss Sensitivity Factors can be calculated as:
∂Pline loss[j]/∂Q[j] = $(2 * Q[j] * [R_k] / (V[j])^2$ …(13)

A. Candidate Bus Selection Using Loss Sensitivity Factor

The Loss Sensitivity Factor (∂Pline loss [j]/∂Q[j]) as given in eq. (13) has been calculated from the base case load flows. The values of loss sensitivity factors have been arranged in descending order and correspondingly the bus numbers are stored in bus position'bpos [i]' vector. The descending order of (∂Pline loss [j]/∂ Q[j]) elements of 'bpos [i]' vector will decide the sequence in which the buses are to considered for compensation. At these buses of 'bpos[i]' vector, normalized voltage magnitudes are calculated by considering the base case voltage magnitudes given as below:
norm [i] = |V[i]| / 0.95 …(14)
The 'norm [i]' decides whether the buses need reactive compensation or not. The buses whose norm [i] value is less than 1.01 can be selected as the candidate buses for capacitor placement. The following are the steps to be performed to find out the potential buses for capacitor placement:
Step 1: Calculate the Loss Sensitivity Factor at the buses of distribution system using Eq. (13).
Step 2: Arrange the value of Loss Sensitivity Factor in descending order. Also store the respective buses into bus position vector
bpos [i].

Step 3: Calculate the normalized voltage magnitude norm [i] of the buses of bpos [i] using Eq. (8).
Step 4: The buses whose norm[i] is less than 1.01 are selected as candidate buses for capacitor placement.

B. Energy cost calculation

If Ii is the current of section-i in time duration T, then energy loss in section-i is given by:
$EL_i = I_i^2 * R_i * T$
The Energy loss (EL) in time T of a feeder with n sections can be calculated as:
$EL = \sum EL_i$ ...(15)
The Energy loss cost (ELC) can be calculated by multiplying eq.(15) with the energy rate(C_e)
$ELC = C_e \times EL$ ...(16)
Where,
EL_i, is energy loss (kW) in section–i in time duration T.
I_i is the current of the section-i
R_i is the resistance of section-i.
T is the time duration.
C_e is the energy rate.
ELC is the energy loss cost.

Capacitor Cost (CC)

Capacitor cost is divided into two terms constant installation cost and variable cost which is proportional to the rating of capacitors. Therefore capacitor cost is expressed as:
$CC = C_{ci} + (C_{cv} \times Q_{ck})$ …(17)
where,
C_{ci} is the constant installation cost of capacitor.
C_{cv} is the rate of capacitor per kVAr.
Q_{ck} is the rating of capacitor on bus-k in kVAr.
"The cost function is obtained by combining eqs. (16) and (17)........" This cost function is considered as the objective function to beminimized in the present work. The cost function 'S' is therefore expressed as:
Minimize
$S = C_e * \sum EL_i + \sum C_{ci} + (C_{cv} * Q_{ck})$

IV. REAL-CODED GENETIC ALGORITHM

The genetic algorithm (GA) initiates the mechanism of the natural selection and evolution and aims to solve an optimization problem [17] with object function f(x) where
$x = [x_1, x_2, . . . ,x_N]$ is an N -dimensional vector of optimization parameters. It has proved to be an effective and powerful global optimization algorithm for many combi- natorial optimization problems, especially for problems with discrete optimization parameters, non-differentiable and/or a discontinuous objective function.
Genes and chromosomes are the basic building blocks of the GA. The conventional standard GA (SGA) encodes the optimization parameter into a binary code string. A gene in SGA is a binary code. A chromosome is a concatenation of genes that takes the form chromosome = $[g^1_1 g^1_2 . . . g^1_{L1}\ g^2_1 g^2_2 . . . g^2_{L2} ... g^N_1\ g^N_2 . . . g^N_{LN}] = [x_1\ x_2 . . . x_N]$ (1)
where$g^i_j$is a gene, and L_i is the length of the code string of the ith optimization parameter and $x_k = [g^k_1\ g^k_2 . . . g^k_{Lk}]$. (2)
The genetic algorithm used in this paper is RCGA. Real number encoding has been confirmed to have better performance than either binary or gray encoding for con- strained optimization problems.Then, in RCGA, a gene is the optimization parameter itself selected from the alphabet set. The chromosome takes the form:
chromosome = $[x_1 x_2 . . . x_N]$. (3)
The RCGA structure is summarized as follows

1) Initial population: The RCGA operates on a population of N_{pop} chromosomes simultaneously. The initial population of real numbered vectors is created randomly. Each of these vectors represents one possible solution to the search problem. The population size (N_{pop}) generally varies from 2 to 2.5 times the number of genes. Once the initialization is completed, the population enters the main GA loop and performs a global optimization for searching the optimal solution of the problem. In a GA loop, the stages 2 to 7 are carried out in turn. The GA loop continues until the termination conditions in stage 3 are fulfilled.

2) Scaling: The scaling operator, a preprocessor, is usually used to scale the object function into an appropriate fitness function. It aims to prevent premature convergence in the early stages of the evolution process and to speed up the convergence in the more advanced stages of the process.

3) **Termination criterion:** After the fitness has been calculated, it has to be determined if the termination criterion has been met. This can be done in several ways. The algorithm used here stops when a finite generation number has been reached and the best fit among the population is declared the winner and solution to the problem.

4) Selection: The selection (or reproduction) operator selects good chromosomes on the basis of their fitness values and produces a temporary population, namely, the mating pool. This can be achieved by many different schemes, but the most common method is the roulette wheel selection. The roulette wheel is biased with the fitness of each of solution candidates. The wheel is spun M-times where M is the number of strings in the population. This operation generates a measure that reflectsthe fitness of the previous generation's candidates.

5) Crossover: The crossover operator is the main search tool. It mates chromosomes in the mating pool by pairs and generates candidate offspring by crossing over the mated pairs with probability P_{cross} .

The probability of parent-chromosome crossover is assumed to be between 0.6 and 1.0. Many variations of crossover have been developed, *e.g*, one-point, two-point and N -point, and random multipoint crossover. Here, the arithmetical one-point crossover is used and introduced.

6) Mutation: After crossover, some of the genes in the candidate offspring are inverted with probability P_{mut}. This is the mutation operation for the GA.

The mutation operator is included to prevent premature convergence by ensuring the population diversity. A new populationis therefore generated. In this paper, the probability of mutation (P_{mut}) is assumed to be between 0.01 and 0.1.

7) Elitism: The postprocessor is the elisit model. The worst chromosome in the newly generated population is replaced by the best chromosome in the old population if the best number in the newly generated population is worse than that in the old population. It is adopted to ensure the algorithm convergence. This method of preserving the elite parent is called elitism.

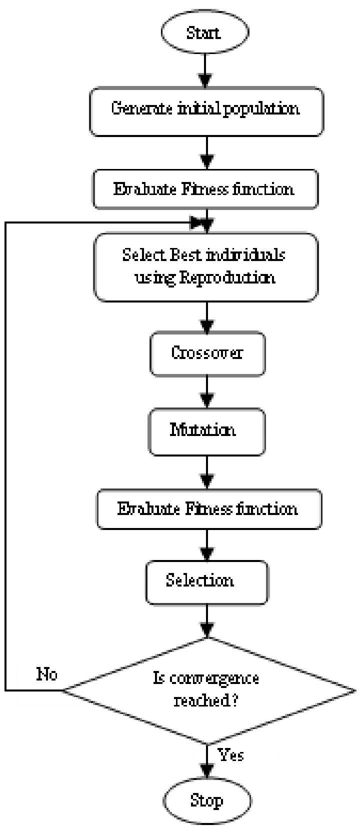

Fig. 4. Flow chart for Genetic Algorithm.

Steps for Genetic Algorithm:
Step 1: Randomly generate initial population strings.
Step 2: Calculate the fitness value for each string in the population.
Step 3: Create the pool after selection.
Step 4: Create offspring's through crossover andmutation operation.
Step 5: Evaluate the offspring's and calculate the fitness value for each solution.
Step 6: If the search goal is achieved, or an allowable generation is attained, return the best chromosome as the solution; otherwise go to step3.

V. RESULT AND DISCUSSION

In present work Real Coded Genetic Algorithm has been successfully applied to determine the optimal value of capacitors on the potential buses for IEEE 33 bus radial distribution system [13]. Further program has been coded in matlab7.10(R2010a) and results are simulated on 4GB (RAM).Result shows that for the adopted test system the potential buses identified for the placement of capacitor are [29, 30, 7, 12, 28] and their respective capacitor sizes are [100,100,100,100,100] KVAr. The fig.5 shows the convergence characteristics for iteration v/s power loss. The total power loss, Annual Energy cost and capacitor cost with capacitor are 172.56 KW $90,700 and $2800respectively, which are superior from the results as obtained without capacitor i.e. power loss 203.4609(KW) and annual energy cost$1,06,940.

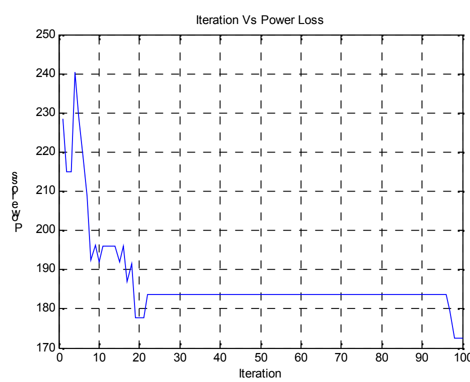

Fig.5. Iteration v/s power loss.

VI. CONCLUSION

In present work the optimal value of capacitor that are need to be placed on potential buses of IEEE 33 Bus Radial distribution system has been successfully determined with the help of real coded genetic algorithm and results obtained in terms of power loss is better in comparison to the results without capacitor. Here voltage profile has also been improved.

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