THE BASIC PROPERTY OF ITERIATIVE PROCESS OF BALANCING THE UNSUSTAINABLE PRODUCTION AND CONSUMPTION MODEL

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Abstract: In this paper, a special case of unbalanced model of production and consumption has been explained. The proposed method consists of introducing balance certain ordering relationship in the set of consumers. This method may help decision-makers in the management of essential logistics processes.

Key words: iteriative process, production and consumption model

Introduction

In [1], [2] an iterative method for balancing the production and consumption model has been described. In this model we have the following:

a) supply-demand vector $\mathbf{p} = (p_1, p_2, \ldots, p_n)$, where $p_i > 0$ for $i = 1, 2, \ldots, m < n$ (p_i - demand of i th contractor), $p_j < 0$ for $j = m+1, m+2, \ldots, n$ (supply of j th contractor) such that $p_1 + p_2 + \ldots + p_n < 0$,

b) maximum concession vector $\mathbf{u} = (u_1, \ldots, u_n)$ ($u_i \geq 0$ - the size of the maximum possible concession of i th contractor), such that $p_1 + u_1 + p_2 + u_2 + \ldots p_n + u_n \geq 0$

c) vector $\mathbf{a} = (a_1, a_2, \ldots, a_n)$, a_i - concession weight of i th contractor.

Taking $\mathbf{p}^o = \mathbf{p}$, $\mathbf{a}_o = \mathbf{a}$ dla $s \geq 1$ we defining the size of

$$r^s = -\left(p_1^{s-1} + \ldots + p_n^{s-1}\right) \tag{1}$$

$$N^s = \left\{ i \leq n : \sum_{j \leq s} a_i^{j-1} r^j \leq u_i \right\} \tag{2}$$

$$\mathbf{a}^s = \left(a_1^s, \ldots, a_n^s\right) \tag{3}$$

where

$$a_i^s = \begin{cases} 0 & dla \;\; i \notin N^s \\ a_i^{s-1}\left(B^{s-1}\right)^{-1} & dla \;\; i \in N^s \end{cases} \tag{4}$$

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$$B^{s-1} = \sum_{j \in N^{s-1}} a_j^{s-1} \tag{5}$$

$$\overline{N}^{s} = N^{s-1} \setminus N^{s} \tag{6}$$

Where $N^o = \{1, \ \ldots, n\}$ (7)

$$\mathbf{p}^s = \left(p_1^s, \ldots, p_n^s\right) \tag{8}$$

where

$$p_i^s = \begin{cases} p_i^{s-1} + r^s \cdot a_i^{s-1} & dla \quad i \in N^s \\ p_i + u_i & dla \quad i \in \overline{N}^s \\ p_i^{s-1} & dla \quad i \in \bigcup_{j \langle s} \overline{N}^j \end{cases} \tag{9}$$

Iteration process, described above, ends for *s*-*th* iteration, such that $r^s = 0$.

Iteration steps and number of contractors

In this article, we prove that the number of iteration steps does not exceed the number of contractors.

Statement 1.

If $\overline{\overline{N}}^o = n \Rightarrow \exists \ s \ \leq n \ i \ \overline{N}^s \neq \emptyset$ the N^o set can be presented in the form of disjointed subsets.

$$N^o = \overline{N}^1 \cup \overline{N}^2 \cup \ldots \cup \overline{N}^{n-1} \cup N^{n-1} \tag{10}$$

(see [2]).

If for every $i \leq n-1 \ \overline{N}^i \neq \emptyset$, set N^{n-1} can include at least one element. If $N^{n-1} = \emptyset$, then $\overline{N}^n$ i san empty set $\left(\overline{N}^n \subset N^{n-1}\right)$. If $N^{n-1} \neq \emptyset$, then $N^{n-1} = N^n \cup \overline{N}^n = \{i_o\}$

Because for every $j \leq n-1 \ \overline{N}^j \neq \emptyset$, then:

$$\mathrm{p}_i^{n-1} = \mathrm{p}_i^o + \mathrm{u}_i \ \text{for} \ i \neq i_o \tag{11}$$

and

$$\mathrm{p}_{io}^{n-1} = \mathrm{p}_i^o + \sum_{j \leq n-1} \mathrm{r}_j \ \mathrm{a}_{j_o}^{j-1} \tag{12}$$

In [3] demonstrated that

$$\overline{S}^{n-1} + r^n \ a_{i_o}^{n-1} = r^1 \tag{13}$$

$$\sum_{j=1}^{n} u_j - u_{i_o} + \sum_{j=1}^{n-1} r^j a_i^{j-1} + r^n a_{i_o}^{n-1} = -\sum_{j=1}^{n} p_j \tag{14}$$

$$\text{or, } \sum_{j=1}^{n}\left(u_j + p_j\right) = u_{i_o} - \sum_{j=1}^{n} r^j a_{i_o}^{j-1} \tag{15}$$

Thus, from the set properties $N^s \sum_{j=1}^{n} r^j a_{i_o}^{j-1} \le u_{i_o}$ it follows that $i_o \in N^n$ thus the set $\overline{N}^n = \emptyset$. What ends the proof of the statement 1.

Statement 2.

If $\overline{\overline{N}}^o = n \Rightarrow \exists\ s \le n: p_1^s + p_2^s + \ldots + p_n^s = 0$.

The proof

To determine r^s, $\mathbf{a}^s$, $\mathbf{p}^s$ and adding and subtracting

$$0 = \sum_{i \in \overline{N}^s}\left(p_i^{s-1} - p_i^{s-1} + r^s\ a_i^s - r^s a_i^s\right) \tag{16}$$

We have:

$$\sum_{i=1}^{n} p_i^s = \sum_{i \in N^o} p_i^{s-1} + \sum_{i \in N^{s-1}} r^s a_i^{s-1} + \sum_{i \in \overline{N}^s}\left(u_i + p_i^o - p_i^{s-1} - r^s a_i^{s-1}\right) =$$

$$= \sum_{i \in \overline{N}^s}\left(u_i - r^s\ a_i^{s-1} + \left(p_i^o - p_i^1\right) + \ldots + \left(p_i^{s-2} - p_i^{s-1}\right)\right) =$$

$$= \sum_{i \in \overline{N}^s}\left(u_i - r^s a_i^{s-1} - r^1 a_{i_o}^o - \ldots - r^{s-1} a_i^{s-2}\right) = \sum_{i \in \overline{N}^s}\left(u_i - \sum_{j \le s} r^j a_i^{j-1}\right) \tag{17}$$

To determine the set N^s and $\overline{N}^s$ shows, that for every $i \in \overline{N}^s$ there is inequality $u_i \rangle \sum_{j \le s} r^j a_i^{s-1}$. Therefore $\sum_{i=1}^{n} p_i^s = 0 \Leftrightarrow \overline{N}^s = \emptyset$.

From the statement 1 shows that there is $s \le n$, such as $\overline{N}^s = \emptyset$ What ends the proof of the statement 2.

Statement 3.

If $\sum_{i=1}^{n} p_i^s = 0$, to $\mathbf{p}^s = \mathbf{p}^{s+1}$

Proof:

If $\sum_{i=1}^{n} p_i^s = 0$, to $r^{s+1} = 0$,

Therefore for every $i \in N^o, r^{s+1} \cdot a_i^s = 0$ and $\sum_{j \le s+1} r^j a_i^{j-1} \le u_i \Leftrightarrow \sum_{j \le s} r^j a_i^j \le u_i$

Therefore, N^s and N^{s+1} sets are identical and $\overline{N}^s$ and $\overline{N}^{s+1}$ sets are identical as well.

Therefore

$$p_i^{s+1} = p_i^s + r^{s+1} a_i^s = p_i^s \text{ for } i \in N^s \quad (18)$$

$$p_i^{s+1} = p_i^0 + u_i = p_i^s \text{ for } i \in \overline{N}^s \quad (19)$$

$$p_i^{s+1} = p_i^s \quad \text{for } i \in \bigcup_{j \langle s} \overline{N}^j \quad (20)$$

Therefore, for $i \in N^o$ $p_i^s = p_i^{s+1}$ what ends the proof of the statement 3.

Summary

Practically, this means that of the vector $\mathbf{p}$ is fully corrected, every next iteration does not bring any changes in the demand and supply of contractors, and, moreover, the algorithm should be included in the moment of overall vector correction $\mathbf{p}$. The steps of iteration does not exceed the number of contractors.

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PODSTAWOWA WŁASNOŚĆ PROCESU ITERACYJNEGO RÓWNOWAŻĄCEGO NIEZBILANSOWANY MODEL PRODUKCYJNO-KONSUMPCYJNY

Streszczenie: W artykule tym zostały udowodnione podstawy teoretyczne dotyczące liczby algorytmów iteracyjnych, które prowadzą do zrównoważenia niezbilansowanego modelu produkcyjno-konsumpcyjnego. Zaproponowana metoda dotyczy równoważenia pewnych związków tworzenia zamówień w grupie konsumentów. Ta metoda może pomóc decydentom w zarządzaniu podstawowymi procesami logistycznymi.
Słowa kluczowe: proces iteracyjny, model produkcyjno-konsumpcyjny

平衡不可持續的生產和消費模式的 ITERIATIVE 過程的基本特性

摘要：摘要：在本文中，生產和消費的不平衡模型的特例已經解釋。該方法包括引入的一組消費者的餘額一定順序的關係。這種方法可以幫助決策者在必要的物流過程的管理。
關鍵詞：iteriative 過程中，生產和消費模式