THE UNSUSTAINABLE PRODUCTION AND CONSUMPTION MODEL

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Abstract: The article presents the relationships on the line producer-consumer-contractor represented by the unsustainable production and consumption model. It also presents a range of solutions to the problem of determining the scope of concessions contractors.

In every society there are producers and consumers who need to satisfy their needs. The natural balance is when all production is absorbed by the market and contractors are not interested in changes in supply and demand. Because of these differences there is often the conflict of interests on the producer (increasing production) - consumer line (demand reduction). This problem can be presented in the form of unsustainable production and consumption model.

Suppose that the model is $n \geq 2$ of contractors, including $m \geq 1$ producers and $n - m \geq 1$ consumers of the goods. We assume that total supply of the goods is less than the total amount of demand over a specified period of time i.e. one year. Presenting the supply and demand in a n-dimensional vector, this problem can be written using uncorrected vector.

Definition 1. Vector $p = (p_1, \ldots, p_n)$ called uncorrected vector $\iff$

1) $\bigwedge_{i \in N} i \leq m \langle n \Rightarrow p_i \rangle 0$

2) $\bigwedge_{i \in N} m+ \leq i \leq n \Rightarrow p_i \langle 0$

3) $(p, e_n) \langle 0$

where $e_n = \underbrace{(1, \ldots, 1)}_{n}$, $(p, e_n) = \sum_{i=1}^{n} p_i e_i = \sum_{i=1}^{n} p_i$

The first m coordinates of a p vector expressing supply of a m manufacturers, the end $n - m$ mean coordinates of consumer demand. From the conditions (1) and (2) of the above definition shows that the case $p_i = 0$, is not considered, because this condition means there is a lack of demand for the goods by the i-receiver $(i \rangle m)$ or the inability to produce the goods by the i-producer $(i \leq n)$. Condition (3) of the definition indicates that the total production in a given period of time does not cover the demand for the commodity. At this

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2011

vol. 4

POLISH JOURNAL OF MANAGEMENT STUDIES
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interpretation of the uncorrected vector, definition 1 presents the unsustainable production and consumption model.

In order to achieve sustainability of the model (total supply equals total demand) have to be concessions on the part of contractors. The maximum possible concession contractors can be expressed by the vector $u = (u_1, \ldots, u_n)$, which we will call the uncorrected vector amendment $p = (p_1, \ldots, p_n)$.

Definition 2. Vector $u = (u_1, \ldots, u_n)$ called uncorrected vector amendment $p = (p_1, \ldots, p_n) \Leftrightarrow$

1) $\bigwedge\limits_{i \in n} u_i \geq 0$

2) $(p + u, e_n) \geq 0$

For $i \leq m$, u_i means the maximum possible increase in production of the i-producer. For $i \rangle m$, u_i means the maximum possible reduction in demand by the i-customer. Condition (1) of the above definition allows for a case $u_{i1} = u_{i2} = \ldots = u_{ik} = 0$. This means that some contractors will not increase their production or reduce the demand for the goods. The contractors will not take an active role in balancing the model. However the condition (2) gives the guarantees of possibility of sustainable model

The problem arises to determine which contractor should be reviewed and if its demand or its supply. Let the vector $x = (x_1, \ldots, x_n)$ means the actual contractor's concessions leading to sustainable the model. Then the problem of the unsustainable production and consumption model can be written as conditions

1) $(p, e_n) \langle 0$

2) $(p + u, e_n) \geq 0, u_1 \geq 0$

3) $(p + x, e_n) = 0$

4) $0 \leq x_i \leq u_i$

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vol. 4

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This problem in the rest of the work will be marked (A).

Where is $\langle + p, e_n \rangle = 0$ then the problem (A) has exactly one solution $x = u$. If, however, $\langle + p, e_n \rangle > 0$, then the problem (A) has no unique solution. One possible solution is to impose the full weight of imbalance with contractors having "the earlier number". The contractor gives a sufficiently to balance the model (if he can do so), or gives to its limits. In this case, the problem (A) has the following solution:

$$x_1 = \begin{cases} -\langle p, e_n \rangle & \text{when} \quad -\langle p, e_n \rangle \leq u_1 \\ u_1 & \text{when} \quad -\langle p, e_n \rangle > u_1 \end{cases}$$

$$x_2 = \begin{cases} 0, & \text{when} \quad x_1 = -\langle p, e_n \rangle \\ -\langle p, e_n \rangle - u_1 & \text{when} \quad -\langle p, e_n \rangle - u_1 \leq u_2 \\ u_2 & \text{when} \quad -\langle p, e_n \rangle - u_1 > u_2 \end{cases}$$

for $3 \leq k \leq n$:

$$x_k = \begin{cases} 0, & \text{when } x_{k-1} = 0 \quad \text{or} \quad x_{k-1} = -\langle p, e_n \rangle - \langle, e_{k-2} \rangle \\ -\langle p, e_n \rangle - \langle, e_{k-1} \rangle & \text{when} \quad -\langle p, e_n \rangle - \langle, e_{k-1} \rangle \leq u_k \\ u_k & \text{when} \quad -\langle p, e_n \rangle - \langle, e_{k-1} \rangle > u_k \end{cases}$$

It is obvious that $0 \leq x_i \leq u_i$. It's needed to show that $\langle + p, e_n \rangle = 0$.

a) Suppose $k = \min_i \{ x_i = 0 \text{ and } u_i > 0 \}$. If $k \geq n$, then $x_i = 0$ for $i \geq k$.

$$\langle + p, e_n \rangle = \langle, e_n \rangle + \langle p, e_n \rangle = \langle, e_{k-1} \rangle + \langle p, e_n \rangle =$$
$$= \langle p, e_n \rangle + \langle, e_{k-2} \rangle + x_{k-1} = \langle p, e_n \rangle + \langle, e_{k-2} \rangle -$$
$$- \langle p, e_n \rangle - \langle, e_{k-} \rangle = \langle - u, e_{k-} \rangle = \langle, e_{k-} \rangle = 0$$

b) Suppose for every $i \leq n$ the following implication is true

$$u_i \rangle 0 \implies x_i \rangle 0, \text{ to } x_n = - \langle p, e_n \rangle - \langle, e_{n-} \rangle,$$

$$\langle + p, e_n \rangle = \langle p, e_n \rangle + \langle, e_{n-1} \rangle + x_n = \langle p, e_n \rangle + \langle, e_{n-1} \rangle -$$

$$- \langle p, e_n \rangle - \langle, e_{n-} \rangle = \langle - u, e_{n-} \rangle = \langle, e_{n-} \rangle = 0$$

Which completes the proof of equality $\langle p + x, e_n \rangle = 0$.

Due to the randomly numbered of these contractors the allocation method may be considered as discriminatory by some of them.

Let the vector $z = u - x$ determines of stocks concession contractors, if a problem's (A) solution is the vector x. It can be, therefore, possible to looking for a solution $x = \langle_1, \ldots, x_n \rangle$, including the weight of individual contractors, to maximize the stock vector z, means, attach a condition to the problem (A).

$$\sum_{i=1}^{n} w_i \langle_i - x_i \rangle = \max \tag{5}$$

when w_i is a weight of i-contractors, $w_i \geq 0$ and $\langle w, e_n \rangle = 1$.

Solving the problem (A) with an attached condition (5) can be found using linear programming methods. From a practical point of view, a major difficulty, however, is balancing weights contractors. If we assume that all contractors are equal to each other (i.e. $w_i = n^-$), the condition (5) does not bring anything new to the problem (A), since for any solution $x = \langle_1, \ldots, x_n \rangle$ there is an equality:

$$\sum_{i=}^{n} n^- \langle_i - x_i \rangle = \text{const}.$$

Due to possible contractors' discrimination, it is appropriate to attach the condition (6) to the problem (A) instead of condition (5)

$$\sum_{i=}^{n} \langle_i - x_i \rangle u_i^- = \max \tag{6}$$

assuming $\langle_i - x_i \rangle u_i^- = \rangle$, if $u_i = \rangle$.

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POLISH JOURNAL OF MANAGEMENT STUDIES
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Quantity $100 \, (c_i - x_i) u_i^{-}$ determines the percentage of the i-contractor's stock, if the solution of the problem (A) is presented by the $x = (x_1, \ldots, x_n)$ vector.

If, however, in the contractors' collection will be large disparities between the maximum potential concessions, the condition (6) will make that the contractors who have a high potential concessions will give way to in the first place. However, if their concessions already sufficient to balance the model, then there will be a sub collection of the contractors who could step down, although did not give up at all. This type of situation can sometimes be inappropriate, because in cases of conflict, each contractor should step down (if he can do so) and have been involved (even symbolically) to balance the model.

As mentioned before, number $z_i = 100 \, (c_i - x_i) u_i^{-}$ determines the percentage of the number of remaining stocks concession to the i-contractor, only if the vector $x = (x_1, \ldots, x_n)$ is a solution to the problem (A). Let $z = (z_1, \ldots, z_n)$. If its requested that each contractor gives "evenly", the good measure of the solution to the problem (A) is a measure of

$$m(x) = \left| n^{-1} \sum_{i=1}^{n} z_i - n^{-1} \sum_{i=1}^{n} z_i \right|.$$

This measure determines the percentage deviation from the average stock per one contractor only if the vector $x = (x_1, \ldots, x_n)$ is a solution to the problem (A). It is, therefore, necessary to find such a vector x to achieve $\min_{x} m(x)$.

The solution (A) may be the solution, in which each contractor gives the same amount (if he can do so). Not always, however, it is appropriate to pass on an imbalance of all contractors equally.

The above examples show that quite difficult (if at all possible?) is to introduce a measure of evaluating the solution of the problem (A). Clearly, the contractors agreeing to increase supply or reduce demand to bet certain conditions, which should also influence the choice of solution (A).

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MODEL NIEZRÓWNOWAŻONEJ PRODUKCJI I KONSUMPCJI

Streszczenie: Artykuł przezentuje relację na linii producent-konsument-kontrahent reprezentowaną przez model niezrównoważonej produkcji i konsumpcji. Przedstawia także zakrez rozwiązań problemu określania zakresu ustępstw kontrahentów.

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