International Journal of Engineering Research and General Science Volume 8, Issue 3, May-June, 2020
ISSN 2091-2730

Performance Analysis of Topological Creative Spaces for Distributional Generalized Transform

Padmaja G, Gulhane A

Assistant Professor, Mathematics, drpadmajaarungulhane@gmail.com

Abstract— In this paper a novel scheme and study for spaces that makes the Gelfand Shilov technique to generalize the Laplace Stieltjes transform a simple objective function a combination of two different transforms in the Distributional Generalized sense appropriate domains for harmonic analysis needs to be taken into consideration during the planning modeling operating Cauchy problems and performing various operations due to wide spread applicability to solve the PDE involving distributional condition.

Keywords— Laplace Stieltjes Transform, Continuous Linear Functional, Transmission Image, Multinormed Space, Distributional Generalized Sense, The Cauchy Problems, Gelfand Shilov Spaces.

INTRODUCTION

The theory of distributional integral transform that objects which generalize functions have its origin in the work of Schwartz [1]. The roots of the method is stressed back to the work of Oliver Heaviside [1890]; due to the concept of derivative to all the integrable functions used to solve formulate generalized solutions of partial differential equations involving distributional conditions like the equation of propagation of heat in cylindrical coordinates imposing generalized boundary conditions, especially in the quantum field theory.

They play a crucial role in mathematical analysis, mathematical physics, and engineering where many continuous, non-continuous problems naturally lead to differential equations whose solutions are distributions, such as the Dirac-delta which is used to model the density of an idealized point mass or point charge as a function to zero everywhere except for zero and it's integral over the entire real line is one. Recently, the theory of distributions devised by L.Schwartz is used in the microlocal analysis, signal processing, image processing, wavelets, the Schwartz spaces, as we need by Laurent Schwartz. The linear parts of such equations have motivational distributional generalized Laplace Stieltjes transform defined in [2, 5].

We studied a class of Gelfand Shilov spaces [7, 8] and their closed subspaces of analytic signals which are almost exponentially localized in time and frequency variables. The description of progressive Gelfand-Shilov type spaces [18] was given by almost exponential decay in time and frequency variables, which do not contain explicit regularity conditions. The spaces have gained more attention in connection with the modulation spaces [14] localization operators [9, 15], the corresponding pseudo-differential calculus in [6, 16, 17, 20] the projective descriptions of a general class of Gelfand Shilov spaces of Roumieu type are indispensable for achieving completed tensor product representations of different important classes of vector valued ultra-differentiable functions [4, 10, 13] of Roumieu. The main interest comes historically from Quantum Mechanics, where the exponential decay of eigen functions have intensively studied. Gelfand Shilov type spaces [11,12, 19] in which the topology of bounded convergence is assigned to the dual function study with the Symbol-Global operator's type in the context of time-frequency analysis.

We start by recalling some facts about LS type spaces Gelfand Shilov having the approach to solve differential equations mainly Cauchy's obtained in the modeling of various technical fields transform the defined LS domain next obtain solution by applying to related distributional generalized Laplace Stieltjes transform defined in [3] which involves both integral and differentiation concept under one umbrella.

A function $\varphi(t, x)$ defined for all positive values of $t,\ x$ having continuous derivatives upto some desired order over some domain C^{∞} is a member of $LS_{a,\ \alpha}$ $(\alpha \geq 0)$ if for each nonnegative integer $l,\ q$

$$\gamma_{a,\,k,\,l,\,q}\varphi = \sup_{\substack{0<t<\infty \\ 0<x<\infty}} \left| e^{at}(1+x)^{k} D_t^{l} (xD_x)^{q} \varphi(t,\ x) \right|$$

$$\leq C_q A^k k^{k\alpha}$$

as the constants A and C_q depend on the everywhere differentiable testing function φ and $a \in R$. We get $k^{k\alpha} = 1$ for $k = 0$
With this topology $LS_{a,\alpha}$ of the multinormed space generated by the countable multiform $\{\gamma_{a,k,l,q}\}_{l,q=0}^{\infty}$
is a countably multiform space complete normed real (or complex). Hence a sequence $\{\varphi_\nu\}_{\nu=1}^{\infty}$ should converge to a single point φ in the space $LS_{a,\alpha}$ for each mentioned earlier nonnegative integers the ideal property helps to establish application aspect in the transmission of image over wireless network based on the embedded system for $\gamma_{a,k,l,q}(\varphi_\nu - \varphi) \to 0$ as $\nu \to \infty$. Multiplication by factor $(1+x)^k$ is a continuous operator due to which suggest translation by a positive factor. Also additional benefit give rise due to multiplication by exponential function e^{at} which is again a series of combinations of infinitely many $t's$. So, translation effect goes on increasing in the defined space. This property allows to rotate move skew elements if 2D or 3D are applied to transformation.
Let us denote $e^{at}(1+x)^k D_t^l (xD_x)^q \varphi_\nu(t, x)$ by $\psi_\nu(t, x)$.

If a sequence $\{\varphi_\nu\}_{\nu=1}^{\infty}$ is a Cauchy sequence in $LS_{a,\alpha}$, then $\{\psi_\nu\}_{\nu=1}^{\infty}$ is a uniform Cauchy sequence on $0 < t < \infty,\ 0 < x < \infty$ for each l, q it can be easily seen that the sequence $\{\psi_\nu(t, x)\}_{\nu=1}^{\infty}$ converges uniformly on defined interval which explain the existence of smooth function $\varphi(t, x)$ defined on $0 < t < \infty,\ 0 < x < \infty$ such that $\psi_\nu(t, x) \to \psi(t, x)$ and $D\psi_\nu \to D\psi$. Moreover there exists a real number $N_{l,q}$ such that for every $\nu, \eta \geq N_{l,q}$ $|\psi_\nu(t, x) \to \psi_\eta(t, x)| \leq \varepsilon$ for a given $\varepsilon > 0$ as ν and η tends to infinity independently.We obtain $|\psi_\nu(t, x) \to \psi(t, x)| \leq \varepsilon$ if $\eta \to \infty$ and $\gamma_{a,k,l,q}(\varphi_\nu - \varphi) \to 0$ as $\nu \to \infty$ for all l, q which results $\varphi_\nu \to \varphi$ in $LS_{a,\alpha}$ as it can be easily proved that $\varphi(t, x)$ is in $LS_{a,\alpha}$.

For $a_1 \leq a_2 \leq a_3 \leq ------$

$$\gamma_{a,k,l,q}\varphi = \sup_{\substack{0<t<\infty \\ 0<x<\infty}} \left| e^{a_1 t}(1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right|$$

$$\leq \sup_{\substack{0<t<\infty \\ 0<x<\infty}} \left| e^{a_2 t}(1+x)^k D_t^l (xD_x)^q \varphi(t, x) \right|$$

and soon for $m = 1, 2, 3, ------$, $\gamma_{a,k,l,q}\varphi \leq \infty$. Hence $LS_{a_{m+1},\alpha}$ is a subspaceof $LS_{a_m,\alpha}$. The topology of each $LS_{a_{m+1},\alpha}$ is stronger than the topology induced on it by $LS_{a_m,\alpha}$.

Let ω denote either a real number or $-\infty$

Let $\{a_\nu\}_{\nu=1}^{\infty}$ be a sequence of real numbers with $a_\nu > \omega$ and such that $a_\nu \to \omega+$ as $\nu \to \infty$

We define the countable union space

$$LS(\omega, \alpha) = \bigcup_{\nu=1}^{\infty} LS_{a_\nu,\alpha}$$

The topology for $LS_{a,\alpha}$ is the strongest possible one.Such the induction map from $LS_{a_\nu,\alpha}$ to $LS_{a,\alpha}$ is continuous for every choice of $\nu > 0$. Since Cauchy sequence for each ν in a countably multi-normed space is convergent countable union space $LS(\omega, \alpha)$ is complete space. A complete countably multi-norm space is a Fréchet space.

$LS_{a,\alpha,A}$ be the space of testing function $\varphi(t, x)$ in $LS_{a,\alpha}$ get by changing A to $(A+\delta)$ due to conversion of polynomial for any $\delta > 0$.

We have $LS_{a,\alpha,A_1} \subset LS_{a,\alpha,A_2}$ if $A_1 < A_2$, topology of LS_{a,α,A_2} is stronger than that induced by LS_{a,α,A_1}

$LS_{a,\alpha} = \bigcup_{A=1}^{\infty} LS_{a,\alpha,A}$ which is a countable union space and therefore the projective limit of Gelfand-Shilov type spaces.

The space $LS^{*}_{a,\alpha}$ which contains set of all continuous linear functional f defined on $LS_{a,\alpha}$ is the dual space of the space $LS_{a,\alpha}$.

If $\gamma_{a,k,l,q}\varphi \le C_q B^{q'} q^{q'\beta}$ where q' is a function depend on l, q which arise as an application of differentiable functions whose derivatives do or donot exist in the classical sense the space LS_a^{β} for $\beta \ge 0$ have the origin. For the change of from too we have a countable union space $LS_a^{\beta} = \bigcup_{B=1}^{\infty} LS_a^{\beta,B}$

Let there be given $\alpha, \beta \ge 0$. Gelfand Shilov type space $LS_{a,A}^{\beta,B} = LS_{a,A}^{\beta,B}\left(R^d\right)$ is defined by set of all functions $\varphi(t,x)$ defined for all positive values of t, x belongs to $C^{\infty}\left(R^d\right)$ such that $\exists C_q > 0$ satisfying

$$\sup_{\substack{0<t<\infty \\ 0<x<\infty}} \left| e^{at}(1+x)^k D_t^l (xD_x)^q \varphi(t,x) \right| \le C_q A^k B^{q'} k^{k\alpha} q^{q'\beta} \quad \text{for } A, B \ge 0$$

The Gelfand Shilov type spaces LS_a^{β} and $\sum_a^{\beta}$ are given by the union and intersection for $A, B \ge 0$ of $LS_{a,A}^{\beta,B}$ and its topology is given by the inductive and projective limits:

$LS_a^{\beta} = ind \lim_{A,B>0} LS_{a,A}^{\beta,B}$ and $\sum_a^{\beta} = proj \lim_{A,B>0} LS_{a,A}^{\beta,B}$

LS_a^{β} and $\sum_a^{\beta}$ are nontrivial if $\alpha + \beta \ge 1$ and $\alpha\ \beta > 0$ the union and intersection for $A, B \ge 0$ of $LS_{a,A}^{\beta,B}$.

and the topology for LS_a^{β} is the strongest possible one such that the inclusion map from $LS_{a,A}^{\beta,B}$ to $LS_{a,A}^{\beta,B}$ is contuous for every choice of $A, B \ge 0$.

The Gelfand Shilov type distributional spaces $\left(LS_a^{\beta}\right)'$ and $\left(\sum_a^{\beta}\right)'$ are given by the intersection and union for $A, B \ge 0$ of $\left(LS_{a,A}^{\beta,B}\right)'$ and its topological sense is given by the projective and inductive limits:

$\left(LS_a^{\beta}\right)' = \bigcap_{A,B>0} \left(LS_{a,A}^{\beta,B}\right)'$ and $\left(\sum_a^{\beta}\right)' = \bigcup_{A,B>0} \left(LS_{a,A}^{\beta,B}\right)'$

Here, $\left(LS_a^{\beta}\right)'$ is the dual of LS_a^{β} and $\left(\sum_a^{\beta}\right)'$ is the dual of $\sum_a^{\beta}$.

The Gelfand Shilov spaces and its distributional spaces possess several properties. For $\alpha, \beta \ge 0$ the Laplace Stieltjes transform is a topological isomorphism between LS_a^{β} and LS_{β}^{a} which can be extended to a continuous linear transform from LS_a^{β} onto LS_{β}^{a}. For a given sequence of positive numbers $\left(\alpha_p\right)_{p\in N_0}$ and $\left(\beta_q\right)_{q\in N_0}$ a broader family of Gelfand Shilov type spaces $LS_{a_p}^{\beta_q}$ and $\sum_{a_p}^{\beta_q}$ are given by the union and intersection for $A, B \ge 0$ of $LS_{a_p,A}^{\beta_q,B}$ and its topology is given by the inductive and projective limits:

$LS_{a_p}^{\beta_q} = ind \lim_{A,B>0} LS_{a_p,A}^{\beta_q,B}$ and $\sum_{a_p}^{\beta_q} = proj \lim_{A,B>0} LS_{a_p,A}^{\beta_q,B}$

and their topologies are defined in analogous ways as for the spaces. Here $\left(LS_{a_p}^{\beta_q}\right)'$ is the dual of $LS_{a_p}^{\beta_q}$ and $\left(\sum_{a_p}^{\beta_q}\right)'$ is the dual of $\sum_{a_p}^{\beta_q}$ which are introduced to study the Cauchy problems in PDEs respectively, the spaces of ultradistributions of Beurling and Roumieu type.

CONCLUSION

The proposed approach and technique with topological properties of involved transforms using Gelfand Shilov spaces is a powerful mathematical tools and classification used to improve employ very effective fast solution more effectively.

REFERENCES:

[1] L Schwartz “Theorie des Distributions”, Hermann, Paris, 1950.

[2] I M Gelfand, G E Shilov, “generalized Laplace functions”, Vol 2, Academic Press, New York, 1968.

[3] P A Gulhane, A S Gudadhe, “On L.S. Spaces of Gelfand Shilov Technique”, Bulletin of Pure & Applied Sciences, Vol 25E, No. 1, pp 351- 354, 2006.

[4] M C Gómez Collado, “Almost Periodic Ultradistributions of Beurling and Roumieu Type”, Proceedings of the American Mathematical Society, Vol. 129, No. 8, pp. 2319-2329, August 2001.

[5] V Geetha, N R Mangalambal, “The Laplace-Stieltjes Transformation on Ordered topological Vector Space of Generalized functions” Acta Math. Univ. Comenianae Vol. LXXX, 2, pp. 243–249, 2011.

[6] Ahmed Abdeljawad, Marco Cappiello and Joachim Toft: Pseudo-Differential Calculus in Anisotropic Gelfand-Shilov setting, Integral Equations, and Operator Theory (2019) 91:26 (pages from 1-33), and localization operators, Functional Analysis, Approximation and Computation 7 (2), pp 135–158, 2015.

[7] Spaces Todor Gramchev, Stevan Pilipovi´c, Luigi Rodino, “Classes of Degenerate Elliptic Operators in Gelfand–Shilov”, Operator Theory: Advances and Applications, Vol. 189, pp 15–31 c, Birkhauser Verlag Basel/Switzerland, 2008.

[8] S.J.L. van Eijndhoven , “Functional analytic characterizations of the Gelfand-Shilov spaces”, Proceedings A 90 (2), June 22, 1987, Communicated by Prof. N.G. de Bruijn at the meeting of January 26, pp 133-144, 1987.

[9] Nenad Teofanova, “Gelfand-Shilov spaces, and localization operators”, Functional Analysis, Approximation and Computation 7 (2), pp 135–158, 2015.

[10] Hamed M. Obiedat Ibraheem Abu Falahah, “Structure Of (W1, W2)-Tempered Ultradistribution Using Short-Time Fourier Transform”, Italian Journal of Pure and Applied Mathematics – N. 39 pp 154–164, 2018.

[11] Z Lozanov Crvenkovic, D.Peri Si C, M.Taskovi C, “Gelfand-Shilov Spaces Structural and Kernel Theorems”, arXiv:0706.2268v2, pp 1-11, 16 Jan 2007.

[12] N Teofanov, “Gelfand-Shilov spaces, Gevrey classes, and related topics”, The Prague seminar on function spaces pp 1-57, 22/11/17.

[13] M Cappiello, R Schulz,, “Microlocal analysis of quasianalytic Gelfand-Shilov type ultra-distributions”, Complex Var. Elliptic Equ. 61 (4), pp 538-561, 2016.

[14] H G Feichtinger, K Grochenig, and D Walnut, “Wilson bases and modulation spaces,” Math. Nachr. 155, pp 7–17, 1992.

[15] E Cordero, S Pilipovic, L Rodino, N Teofanov, “Quasianalytic Gelfand-Shilov spaces with application to localization operators”, Rocky Mountain Journal of Mathematics, 40 (4), pp 1123-1147 2010.

[16] J Toft, “Multiplication properties in Gelfand-Shilov pseudo-differential calculus, Pseudo differential operators”, Generalized Functions and Asymptotics, 117172, Oper. Theory Adv. Appl., 231.

[17] N. Teofanov, “Modulation spaces, Gelfand-Shilov spaces and Pseudo-differential operators”, Sampling Theory in Signal and Image Processing 5(2), pp 225–242, 2006.

[18] Dusan Raki, Nenad Teofanov, “Progressive Gelfand-Shilov Spaces and Wavelet Transforms”, Hindawi Publishing Corporation Journal of Function Spaces and Applications Volume 2012, Article ID 951819, 19 pages DOI:10.1155/2012/951819.

[19] Jaeyoung Chung, Soon Yeong Chung, Dohan Kim, “Characterizations of The Gelfand-Shilov Spaces Via Fourier Transforms”, Proceedings of The American Mathematical Society Volume 124, Number 7, pp 2101-2108, July 1996.

[20] J. Toft, “The Bargmann transform on modulation and Gelfand-Shilov spaces, with applications to Toeplitz and Pseudo-differential operators”, J. Pseudo-Differ. Oper. Appl. 3, pp 145–227, 2012.