International Journal of Engineering Research and General Science Volume 7, Issue 3, May-June, 2019
ISSN 2091-2730

Expansion of $(a+b+c+d+e+\ldots\ldots)^n$ using Symmetry and principles of homogeneity of the sum of "m" elements raised to the power of a +ve integer "n"

Prof. C. Santhamma, B.Sc. (Hons), M.Sc., D.Sc., FAPASc.

Prof. C. Santhamma, Emeritus Professor, Centurion University of Technology and Management AP, Visakhapatnam.
Mob : +91 7799384448, email_id : santhammachilukuri2018@gmail.com

Abstract- A method for the expansion of $(a+b+c+d+e+\ldots\ldots \text{ m terms})^{n\ (+ve\ integer)}$ raised to the power of any +ve integer is carried out using the basic principles laid down by Jagadguru Sankaracharya of Puri Mutt (Sri Bharati Krishna Tirtha Maharaja). The procedure adopted by considering the sum of seven elements being raised to the positive integer four. This finds application in determining roots of numbers including decimals and also of the imperfect roots. The method is applicable in the determination of roots of polynomials and is most general for any number of elements the sum of which being raised to any +ve integer.

Introduction

Jagadguru Shankaracharya Sri Bharati Krishna Tirtha Swamiji of Puri Mutt, in his series of works on Vedic Mathematics consisting of application of 29 Sutras solved many problems in Algebra, Geometry, Conics, determination of higher order roots for numbers, extending them to polynomials, recurring decimals, Squarings, Cubing, new theories such as Duplex concept, Straight Division method, theory of Osculators, Auxiliary fractions, concept of Vinculum methods, Solving higher order equations, factorization by differentiation, different types of equations, integration, differentiation, partial fractions, partial derivatives, etc. The author has extensively worked on these methods and published the results in detail into 5 Volumes as the Lecture Notes and compared them with the method in vogue. This paper deals with the general expansion of the addition of seven elements raised to the positive integer four. Such expansions have application in the problems related to the root determination of number (perfect, imperfect and decimals), polynomials, and also in determining contribution of terms of expansion pertaining to a particular decimal position.

The method developed to arrive at the expansion of
$(a+b+c+d+e+\ldots\ldots \text{ m terms})^{n\ (+ve\ integer)}$ inclusive of the coefficients of each term is worked out for m=7 and n=4 leading to the most general expansion of m terms and n being any +ve integer, together with the corresponding coefficient of each term in the final expansion.

Expansion of any number of terms raised to any power of positive integer is shown to be carried out using the basic principles laid down by Shri Bharathi Krishna Tirtha Swamiji.

Considering the Expression as $(a+b+c+d+e+f+g)^{4 = \text{Power}}$, The terms are classified into various sets following symmetry and homogeneity and the terms of each set will have the same coefficient and is different from set to set which can be determined using combination principle. The sets are given in the Table1

$(a+b+c+d+\ldots\ldots\ldots)^n$ ---- General, when m is the no of elements and n is the power.

Evaluation of the total number of terms in the expansion is $\frac{m(m+1)(m+2)(m+3)\ldots}{n!}$

Hence, the no of terms in the expansion = $\frac{7X8X9X10}{4X3X2X1} = 210$

Table 1

$(a+b+c+d+e+f+g)^4 =$

I Set	1 (Q_1)	$a^4, b^4, c^4, d^4, e^4, f^4, g^4$							7 terms
II Set	4(Q_2)	$a^3b, ab^3, a^3c, ac^3, a^3d, ad^3, a^3e, ae^3, a^3f, af^3, a^3g, ag^3, b^3c, bc^3, b^3d, bd^3, b^3e, be^3, b^3f, bf^3, b^3g, bg^3, c^3d, cd^3, c^3e, ce^3, c^3f, cf^3, c^3g, cg^3, d^3e, de^3, d^3f, df^3, d^3g, dg^3, e^3f, ef^3, e^3g, eg^3, f^3g, fg^3$							42 terms
III Set	6(Q_3)	$a^2b^2, a^2c^2, a^2d^2, a^2e^2, a^2f^2, a^2g^2, b^2c^2, b^2d^2, b^2e^2, b^2f^2, b^2g^2, c^2d^2, c^2e^2, c^2f^2, c^2g^2, d^2e^2, d^2f^2, d^2g^2, e^2f^2, e^2g^2, f^2g^2$							21 terms
IV Set	12(Q_4)	a^2bc	b^2cd	c^2de	d^2ef	e^2fg	f^2ag	g^2af	105 terms
		a^2bd	b^2ce	c^2df	d^2eg	e^2af	f^2bg	g^2ab	
		a^2be	b^2cf	c^2dg	d^2fg	e^2ag	f^2cg	g^2ac	
		a^2bf	b^2eg	c^2ef	d^2ab	e^2ab	f^2dg	g^2ad	
		a^2bg	b^2de	c^2eg	d^2ac	e^2ac	f^2eg	g^2ae	
		a^2cd	b^2df	c^2fg	d^2ae	e^2ad	f^2ab	g^2bc	
		a^2ce	b^2dg	c^2ad	d^2af	e^2bc	f^2ac	g^2bd	
		a^2cf	b^2ef	c^2ae	d^2ag	e^2bd	f^2ad	g^2be	
		a^2cg	b^2eg	c^2af	d^2be	e^2bf	f^2ae	g^2bf	
		a^2de	b^2fg	c^2ag	d^2bf	e^2bg	f^2bc	g^2cd	
		a^2df	b^2ac	c^2bd	d^2bg	e^2cd	f^2bd	g^2ce	
		a^2dg	b^2ad	c^2be	d^2ce	e^2cf	f^2be	g^2cf	
		a^2ef	b^2ae	c^2bf	d^2cf	e^2cg	f^2cd	g^2de	
		a^2eg	b^2af	c^2bg	d^2cg	e^2df	f^2ce	g^2df	
		a^2fg	b^2ag	c^2ab	d^2bc	e^2dg	f^2de	g^2ef	
		15 terms	15 terms	15 terms	15 terms	15 terms	15 terms	15 terms	
V Set	24 (Q_5)	abcd, abce, abcf, abcg, abde, abdf, abdg, abef, abeg, abfg, acde, acdf, acdg, acef, acfg, aceg, adef, adeg, adfg, aefg, bcde, bcdf, bcdg, bcfg, bcef, bceg, bdef, bdeg, bdfg, befg, cdef, cdeg, cdfg, cefg, defg							35 terms
		TOTAL							210 terms

Coefficients by Combination Method

Table 2

a^4	nc_n	1	Q_1
a^3b	nc_3 x $(n-3)c_1$	4	Q_2
a^2b^2	nc_2 x $(n-2)c_2$	6	Q_3
a^2bc	nc_2 x $(n-2)c_1$ x $(n—3)c_1$	12	Q_4
abcd	nc_1 x$(n-1)c_1$ x $(n-2)c_1$ x $(n-3)c_1$	24	Q_5

Coefficients of each set are Q_1, Q_2, Q_3, Q_4, Q_5

The expansion is
I Set Sum + 4 (II Set Sum) + 6 (III Set Sum) + 12 (IV Set Sum) + 24 (V Set Sum)

A method is derived for the first time to give out the expansion terms as explained by Shri Bharati Krishna Tirtha Swamiji. This can be extendable for any number of terms to any positive power.

Consider the number as abcdefg and its expansion to the power 4 is

$$(a+b+c+d+e+f+g)^4$$

The expansion consists of different sets of terms following the principle of Symmetry, and homogeneity. The different sets consist of single element, combination of two elements, combination of three elements, combination of four

International Journal of Engineering Research and General Science Volume 7, Issue 3, May-June, 2019
ISSN 2091-2730

elements. These are shown in Table 1 under 5sets. The coefficients of the terms in each set could be evaluated using combination theory. The results of such are given in the Tabel 2. The final expansion is
Q_1Set1(sum of terms)+Q_2Set2(sum of terms)+Q_3Set3(sum of terms)+Q_4Set4(sum of terms)+Q_5Set5(sum of terms)

Application in identifying the contribution of terms to a particular decimal accuracy.

The positions of a b c d e f g in the given number are $10^0,10^1,10^2,10^3,10^4,10^5,10^6$ respectively. Keeping this in view, the decimal contribution of terms in the expansion upto 12th decimal is as follows (Table 3).

Table 3

1st Decimal	a^3b. b is calculated from the Common Divisor using Swamiji's Straight Division method.
2nd decimal	a^3c, a^2b^2
3rd decimal	a^3d, a^2bc, ab^3
4th decimal	a^3e, a^2c^2, a^2bd, b^2ac, b^4, b^3c
5th decimal	a^3f, a^2cd, b^2ad, a^2be, c^2ab, b^3c, a^2be
6th decimal	a^3g, a^2d^2, a^2bf, a^2ce, b^2ae, abcd, c^3, b^2c^2, b^3d
12th decimal	d^4, ae^3, c^3g, $a^2g^2, b^2f^2, c^2e^2, b^2eg, c^2ef, d^2ag, d^2bf, d^2ce$, fab

Similary one can read from the terms, the contributions to various other decimals as well

Verification $(c+b+a)^3$

Consider the number (3 digits)

c	b	a
3	2	1

$(c+b+a)^3$ a – 10^0, b – 10^1, c – 10^2

On the basis of the present procedure, the terms are

a^3	$3a^2b$	$3ab^2+3a^2c$	b^3+6abc	$3ac^2+3b^2c$	$3bc^2$	c^3
10^0	10^1	10^2	10^3	10^4	10^5	10^6

By Vedic Method :

a^3	=	1
$3a^2b$	=	60
$3ab^2$	=	1200
$3a^2c$	=	900
b^3	=	8000
6abc	=	36000
$3ac^2$	=	270000
$3b^2c$	=	360000
$3bc^2$	=	5400000
c^3	=	27000000
$(321)^3$	=	33076161

By General Method (existing:

$(321)^2$	=	10341
321	X	321
$(321)^3$	=	33076161

Acknowledgements

The author is deeply indebted to Prof. GSN Raju, the Vice Chancellor of Centurion University of Technology and Management AP, Visakhapatnam and also equally acknowledges Prof. D.N.Rao, the Vice President of the Institution for their interest in this line of Investigation. The author extends her special thanks to Prof. Mukti Kanta Mishra, the President of the Institution for giving the position of Emeritus Professor in the Institution.

REFERENCES

1. Vedic Mathematics by Jagadguru Swamiji, Sri Bharati Krishna Tirthaji Maharaja, Publishers : Motilal Banarsidas Pvt. Ltd, New Delhi (Latest Edition – 2013).
2. Lecture Notes (in 5 Volumes) by Prof. C. Santhamma et.al published by Bharateeya Vidya Kendram, Visakhapatnam (2002-2007)