Modelling and Simulation of a Rectangular Microstrip in MATLAB Working Environment

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Abstract Modelling and simulation of a rectangular microstrip patch antenna on MATLAB platform is presented in form of three algorithms. The design algorithm which computes the dimensions of the rectangular microstrip patch antenna is based on the transmission line model. The cavity model algorithm analysis the rectangular microstrip patch antenna. The radiation pattern algorithm plots the radiation patterns of the antenna. The algorithms developed were used to model and simulate three different specifications of rectangular patch antenna. The results obtained are found to compare favourably with existing records.

Keywords Antenna, Microstrip patch, Transmission line model, Cavity model, Radiation pattern

Introduction Microstrip antenna technology began its rapid development in the late 1970s. By the early 1980s, primary microstrip antenna elements and arrays were relatively well established in terms of design and modeling [1]. Microstrip antennas have received a lot of attention because of their many advantages. These advantages include low profile, low cost, conformability to planar and non-planar surfaces and ease of integration with active devices [2].

The microstrip patch antenna consists of three basic parts; the radiating patch, the dielectric substrate and the ground plane as illustrated in Fig. 1. The dielectric substrate is located between the radiating patch and the ground plane. The radiating patch is made of any conducting material and it can take up any shape depending on the design requirements. Some of the most common shapes of radiating patch are shown in Fig 2. The ground plane is a metal plate bonded to one side of the dielectric substrate.

Figure 1: Microstrip patch antenna

Figure 2: Common shapes for radiating patch.
It is necessary to predict exactly how an antenna will behave prior to actual fabrication. Modelling and simulation predict the radiation characteristics of an antenna and suggest necessary modifications before fabrication. There are currently several full electromagnetic modelling tools and products such as Ansoft Ensemble, High Frequency Structure Simulation (HFSS) and Computer Simulation Technology and Microwave Studio (CST MS) that are used in the modelling and simulation of complex circuits and antennas [3]. However, a lot of these softwares come at a very steep price and are sometimes proprietary [4]. It is possible to implement the modelling and simulation entirely on MATLAB platform and this makes it much more accessible and affordable to students and designers. MATLAB is a ubiquitous math, data manipulation, signal processing, and graphics software package [5]. It can be used effectively in the design and analysis of antenna. In this work, modelling and simulation of a rectangular microstrip antenna are implemented in MATLAB working environment.

**Figure 3: Sequence of steps involved in the modelling and simulation of the microstrip patch antenna**

**Microstrip Patch Antennas**

Microstrip patch antennas are analysed using any of the three following models; the transmission line model, the cavity model and the full wave model [6]. The transmission line model is the easiest to implement and it gives adequate insight into the physical attributes of the antenna [7]. The cavity model is much more accurate and much more complex than the transmission line model. The full wave model is the most accurate and most complex of the three models as it can be used to analyse all shapes and all arrangements of patch element [8]. The transmission line model is selected in this work to calculate the physical dimensions of the antenna. The cavity model is used to analyse and plot the radiation patterns of the antenna. Fig. 3 illustrates the sequence of steps involved in the modelling and simulation of microstrip patch antenna.

**Transmission Line Model.**

The top view of a typical rectangular microstrip antenna patch is shown in Fig 4. W is the width of the patch, L is the actual length of the patch, \( w_f \) is the width of the feed and \( \varepsilon_r \) is the dielectric constant of the substrate. \( h \) is the distance between the patch and the ground plane which may represent the thickness of the substrate. The fringing effect comes into play as the dimensions of the radiating patch are finite and the fields at the edge undergo fringing [9]. Fringing makes the microstrip look wider electrically compared to its physical dimensions and since some of the waves travel in the substrate and some in air, an effective dielectric constant \( \varepsilon_{reff} \) is introduced to account for this fringing effect and the wave propagation in the line. This would make the
microstrip patch appear as shown in Fig. 5. The transmission line equations that govern the transmission line model are given by Eqns. (1), (2), (3), (4) and (5) [6].

$$W = \frac{v_0}{2f_r} \sqrt{\frac{\epsilon_r+1}{\epsilon_r+1}}$$  \hspace{1cm} (1)

where $v_0$ is the speed of light in free space and $f_r$ is the resonant frequency.

$$\epsilon_{\text{eff}} = \frac{\epsilon_r+1}{2} + \frac{\epsilon_r+1}{2} \left( 1 + 12 \frac{\hbar}{W} \right)^{-1/2}$$  \hspace{1cm} (2)

$$\Delta L = 0.412h \left( \frac{\epsilon_{\text{eff}} + 0.3}{\epsilon_{\text{eff}} - 0.258} \right) \left( \frac{W+0.264}{W-0.8} \right)$$  \hspace{1cm} (3)

$$L = \frac{v_0}{2f_r \sqrt{\epsilon_{\text{eff}}}} - 2\Delta L$$  \hspace{1cm} (4)

where $L$ is the actual length of the microstrip.

$$L_{\text{eff}} = L + 2\Delta L$$  \hspace{1cm} (5)

Furthermore, the transmission line model estimates the impedance of the microstrip at the feed point. The two widths of the patch are treated as slots and each radiating slot is represented by a parallel equivalent admittance $Y$ (with conductance $G$ and susceptance $B$) as illustrated in Fig. 6 [10]. The two radiating slots are similar and have equal conductance ($G$) and equal susceptance ($B$) as described in Eqn. (6). $G$ and $B$ are as given by Eqns. (7) and (8) respectively [6]. The mutual conductance $G_{12}$ and input resistance $R_{in}$ are given by Eqns. (9) and (10) [6].

$$G_2 = G_1 = G; \ B_2 = B_1 = B$$  \hspace{1cm} (6)

$$G = \frac{W}{120\lambda_0} \left( 1 - \frac{1}{24} (k_0h)^2 \right) \hspace{1cm} f o r \hspace{0.5cm} \frac{h}{\lambda_0} < \frac{1}{10}$$  \hspace{1cm} (7)

where $\lambda_0$ is free space wavelength.

$$B = \frac{W}{120\lambda_0} \left( 1 - 0.636\ln(k_0h) \right) \hspace{1cm} f o r \hspace{0.5cm} \frac{h}{\lambda_0} < \frac{1}{10}$$  \hspace{1cm} (8)
\[G_{12} = \frac{1}{240\pi^2} \int_0^\pi \left( \frac{\sin \left( \frac{k_0 W \cos(\theta)}{\cos(\theta)} \right)}{\cos(\theta)} \right)^2 J_0(k_0 L \sin(\theta)) \sin^3(\theta) d\theta \]  

(9)

where \(J_0\) is a Bessel function of the first kind of order zero. \(\theta\) is one of the angles that describes the H plane and E plane.

\[R_{in} = \frac{1}{2(G_{11}+G_{12})} \]  

(10)

Another way to change the resonant input resistance is by an inset feed recessed a distance \(y_0\) and is shown in Fig 7. Varying input resistance is just to achieve matching.

Finding the inset feed distance \(y_0\) entails knowing the characteristic impedance of the microstrip line feed. Its impedance \(Z_c\) is a function of \(w_0\), \(h\) and \(\varepsilon_{reff}\) [6]. \(w_0\) is subject to the condition in Eqn. (11). Equations (12) and (13) give \(Z_c\) [6]. Eqn. (14) gives the admittance while Eqn. (15) calculates the input resistance. Finding the inset distance involves solving an optimization problem that minimizes the objective function of Eqn. (16).

Based on these mathematical equations, a computer program was developed to be used in the first phase of the microstrip antenna modelling and the flow chart is shown in Fig 8.

\[w_0 \ll W \]  

(11)

\[Z_c = \frac{60}{\sqrt{\varepsilon_{reff}}} \ln \left[ \frac{8h}{w_0} + \frac{w_0}{4h} \right] \quad \text{If } \frac{w_0}{h} \leq 1 \]  

(12)

\[Z_c = \sqrt{\varepsilon_{reff}} \left[ \frac{W_0}{h} + 1.393 + 0.667 \ln \left( \frac{W_0}{h} + 1.444 \right) \right] \quad \text{If } w_0 > 1 \]  

(13)

\[Y_c = \frac{1}{Z_c} \]  

(14)
\[ R_{in}(y = y_0) = \frac{1}{2(G_1 \pm 0.12 \, \text{o})} \left( \cos \left( \frac{\pi}{L} y_0 \right) + \frac{G_1 + \beta}{L} \sin \left( \frac{\pi}{L} y_0 \right) - \frac{G_1}{L} \sin \left( \frac{2\pi}{L} y_0 \right) \right) \]  

(15)

\[ \min Z_0 - R_{in} \text{ such that } 0 \leq y_0 \leq L/2 \]  

(16)

Cavity Model

Microstrip antennas resemble dielectric-loaded cavities, and they exhibit higher order resonances [11]. The normalized fields within the dielectric substrate (between the patch and the ground plane) can be found more accurately by treating that region as a cavity bounded by electric conductors (above and below it) and by magnetic walls (to simulate an open circuit) along the perimeter of the patch [12]. This is an approximate model which in principle leads to a reactive input impedance (of zero or infinite value of resonance) and it does not radiate any power. Although, the actual fields are approximate to those generated by such a model, the computed pattern, input admittance, and resonant frequencies compare well with measurements. This model is suitable for finding the radiation pattern of the microstrip antenna.

In the TM_{010} mode, the total field is the sum of the fields due to two elements array (each representing a slot) [13]. Since the slots are identical, this can be accomplished using an array factor for the two slots. In spherical coordinates, the far-zone electric field is given by Eqn. (17). Using the array factor (AF) of Eqn. (18) gives the total radiated field of Eqn. (19).

\[ E_r = 0; \ E_\theta = 0; \ E_\phi = J \frac{k_0 h W e^{-j k_0 r}}{2 \pi r} \left\{ \sin \theta \sin(X) \sin(Z) \right\} \]  

(17)

where \( X = \frac{k_0 h}{2} \sin \theta \cos \phi \) and \( Z = \frac{k_0 W}{2} \cos \theta \)

\[ (AF)_y = 2 \cos \left( \frac{k_0 L_y}{2} \right) \sin \theta \sin \phi \]  

(18)

where \( L_y \) is the separation distance in the y direction and the effective length.

\[ E'_\phi = J \frac{k_0 h W e^{-j k_0 r}}{2 \pi r} \left\{ \sin \theta \sin(X) \sin(Z) \right\} 2 \cos \left( \frac{k_0 L_y}{2} \right) \sin \theta \sin \phi \]  

(19)

Therefore the E-plane field is obtainable by substituting \( \theta = 90^\circ, 0^\circ \leq \phi \leq 90^\circ \), \( 270^\circ \leq \phi \leq 360^\circ \) in Eqn. (19). The H-plane field is obtainable by substituting \( \phi = 0^\circ, 0^\circ \leq \theta \leq 180^\circ \) in Eqn. (19). Eqns (20), (21) and (22) define important metrics of microstrip antenna. The flow chart detailing the cavity model analysis is shown in Fig 9. Fig. 10 shows the flow chart of a separate algorithm which is developed to plot of the radiation properties such as the electric field plot and magnetic field plot.

**Return loss, RL (dB) = 10 \log_{10} \left( \frac{P_i}{P_r} \right) **

where \( P_i \) = incident power and \( P_r \) = reflected power.

**Reflection coefficient, RC = \frac{Z_l - Z_s}{Z_l + Z_s} **

where \( Z_l \) = load impedance and \( Z_s \) = source impedance.

\[ Voltage \ Standing \ Wave \ Ratio, VSWR = \frac{1 + RC}{1 - RC} \]  

(22)

Results

Transmission Line Model Results

Three separate microstrip antennas, Antenna A, B and C were modelled and simulated with the developed algorithms. Antenna A was designed to operate at a frequency of 10 GHz and it has a substrate dielectric constant of 2.2 with a chosen height of 0.1588 cm. Antenna B was designed to operate at a frequency of 2.4 GHz with dielectric constant of 3.4 and a height of 0.1500 cm. Antenna C was designed to operate at a frequency of 2.25 GHz with a dielectric constant of 4.4 and a height of 0.1600 cm. The dimensions and parameters of the three antennas are obtained with the aid transmission line models as presented in Table 1.

**Variation of Input impedance with Inset distance**

The input impedance for varying inset distances \( 0 \leq y_0 \leq L/2 \) for the inset feed for the three antennas A, B and C are shown in the figure 11 (a), (b) and (c) respectively. It can be seen that the input impedance continues
to decrease until it gets to a desired minimum for all the three antennas. This minimum inset distance is selected as the insertion point, $y_o$ for each Antenna as recorded in Table 1.

**Radiation Pattern**

The electric field plot and the magnetic field plot are both derived for antennas A, B and C. These are presented in Figures 12 and 13. Table 2 shows the values of reflection coefficient (RC), return loss (RL) and VSWR at different insertion points ($y_o$). The directivity of Antennas A, B and C are obtained as 5.4376 dB, 4.3944 dB and 4.0551 dB respectively.

**Figure 9: Flow chart showing the cavity model**

```
START

\[ \theta = 30 \text{deg}, \phi = 30 \text{deg} \]
\[ X = (\cos(\theta)) \sin(\text{theta}) \cos(\phi) \]
\[ Z = (\cos(\theta)) \cos(\text{theta}) \cos(\phi) \]
\[ \rho = \beta^{i + j} \hat{a} + \beta^{j + i} \hat{b} \]

\[ \rho = \pi^{i + j} \hat{a} + \pi^{j + i} \hat{b} \]
\[ \rho = (\pi^{i + j} \hat{a} + \pi^{j + i} \hat{b}) \]
\[ \rho = (\pi^{i + j} \hat{a} + \pi^{j + i} \hat{b}) \]

STOP
```

where thetha = $\theta$, pi = $\pi$

**Figure 10: Flow chart showing the code for radiation pattern plotting**

```
START

Enter phi, gain, range1, increment, rays NO

G = 30*(\pi/9) (gain)

G = range2

Yes

G = range1 + G = G

No

PHI0 = 9*(cos(\theta)), PHI1

\[ x0 = \sin(\theta) \]
\[ x0 = 0.35 \times x0 \]
\[ x0 = 0.35 \times x0 \]

\[ END \]
```

where thetha = $\theta$, pi = $\pi$
Table 1: Microstrip Design Parameters for Antenna A, B and C

<table>
<thead>
<tr>
<th>Description</th>
<th>Antenna A</th>
<th>Antenna B</th>
<th>Antenna C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design Specifications</td>
<td>Frequency</td>
<td>10.00 GHz</td>
<td>2.40 GHz</td>
</tr>
<tr>
<td></td>
<td>Substrate Permittivity, $\varepsilon$</td>
<td>2.2</td>
<td>3.4</td>
</tr>
<tr>
<td></td>
<td>Height, h</td>
<td>0.1588 cm</td>
<td>0.1500 cm</td>
</tr>
<tr>
<td>Patch's Specifications</td>
<td>Length of Patch, L</td>
<td>0.9061 cm</td>
<td>3.3471 cm</td>
</tr>
<tr>
<td>Dimensions and Parameters</td>
<td>Effective Length of Patch, $L_e$</td>
<td>1.0683 cm</td>
<td>3.4914 cm</td>
</tr>
<tr>
<td></td>
<td>Width of Patch, W</td>
<td>1.1859 cm</td>
<td>4.2137 cm</td>
</tr>
<tr>
<td></td>
<td>Substrate Effective Permittivity, $\varepsilon_{reff}$</td>
<td>1.9716</td>
<td>3.2045</td>
</tr>
<tr>
<td></td>
<td>Transmission Line Width, $w_o$</td>
<td>0.0237 cm</td>
<td>0.0843 cm</td>
</tr>
<tr>
<td></td>
<td>Insertion Point at $Z_c = 50$ $\Omega$, $y_o$</td>
<td>0.2962 cm</td>
<td>1.1807 cm</td>
</tr>
</tbody>
</table>

Figure 11: Variation of input impedance with inset distance

Figure 12: E-Field plot of Antenna A, B and C modelled

Figure 13: H-Field plot of Antenna A, B and C modelled

Conclusion
The modelling and simulation of a rectangular microstrip antenna in MATLAB has been presented. Antennas are successfully modelled and simulated to operate at specified frequencies with specified dielectrics were modelled and simulated. For specified antenna parameters, appropriate dimensions and parameters of the patch antenna are obtained with the aid of transmission line model. Cavity model gives the radiation pattern analysis in form of the electric and magnetic field plots.
Table 2: Variations of reflection coefficient, return loss and VSWR with inset distance for Antennas A, B and C

<table>
<thead>
<tr>
<th>Antenna A</th>
<th>Antenna B</th>
<th>Antenna C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_0$ (cm)</td>
<td>RC (dB)</td>
<td>VSWR</td>
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<tr>
<td>0</td>
<td>0.7890</td>
<td>8.4800</td>
</tr>
<tr>
<td>0.0453</td>
<td>0.7538</td>
<td>7.1245</td>
</tr>
<tr>
<td>0.0906</td>
<td>0.7012</td>
<td>5.6937</td>
</tr>
<tr>
<td>0.1359</td>
<td>0.6246</td>
<td>4.3276</td>
</tr>
<tr>
<td>0.1812</td>
<td>0.5192</td>
<td>3.1599</td>
</tr>
<tr>
<td>0.2265</td>
<td>0.3948</td>
<td>2.3050</td>
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<td>0.2718</td>
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<td>1.8464</td>
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<td>0.3624</td>
<td>0.3856</td>
<td>2.2550</td>
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<tr>
<td>0.4077</td>
<td>0.5101</td>
<td>3.0822</td>
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<tr>
<td>0.4530</td>
<td>0.6176</td>
<td>4.2296</td>
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References