STATISTICAL MODELS FOR THE WORKING PROCESS CARRIED OUT
BY THE ORGANIC FERTILIZER SPREADING MACHINE

MODELE STATISTICE PENTRU PROCESUL DE LUCRU, REALIZAT DE CÂTRE
MAŞINA PENTRU ADMINISTRAT ÎNGRĂŞĂMINTE ORGANICE

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ABSTRACT
This article presents the results of statistical modeling using experimental results obtained with an organic fertilizer spreading machine having a spreading device that consists of vertical helical rotors. Exposed statistical models are embodied in functions that connect input parameters, command parameters and output parameters of the process in order to estimate machine qualities and optimize the working process. It also shows global qualitative results, along with the profiling of some research methods on experiment groups characterized by different sets of values of the considered working regime parameters.

REZUMAT
Acest articol prezinta rezultatele modelarii statistice folosind rezultatele experimentale obtinute cu o maşină de administrat îngrăşăminte organice al cărui dispozitiv de împrăştiere este format din rotoare elicoidal se dispune în plan vertical. Modelele statistice expuse se concretizează în funcții care legă parametrii de intrare, parametrii de comandă și parametrii de ieșire ai procesului, în scopul estimării calităților mașinii și a optimizării procesului de lucru realizat. De asemenea, se expun și rezultate calitative globale, alături de profilarea unor metode de studiu pe grupuri de experimente, caracterizate de seturi diferite de valori ale parametrilor regimului de lucru considerați.

INTRODUCTION
The working processes of the solid organic fertilizer spreading machines are characterized by the random variation of some of the process parameters as recently been shown (Popa et al., 2008; Pylypaka et al., 2017; Stefan et al., 2015). Among these parameters are those that characterize the material and its interaction with the mechanical distribution device. The distributor mode of action on the fertilizer block and on its structure produces fragments of particles with very different shapes, sizes and implicitly different masses.

The process of throwing the material into space leads to collisions accompanied by deformations and disintegrations of the spread material fragments. Moreover, the friction between the spread material and the spreading device has the same random character, taking into account the non-homogeneous composition of the material and the humidity that is not uniform in the block spread (Stefan et al., 2015; Stefan et al., 2018). As a result, a deterministic description of the particle trajectory in space is accompanied by a high degree of uncertainty.

According to Cox (2006), a statistical model is a mathematical model that includes a set of statistical assumptions on data generation, samples (and similar data from a larger population). A statistical model is often, in a considerably idealized form, the process of generating data. A statistical model is usually specified as a mathematical relation between one or more random variables and other non-random variables. Therefore, a statistical model is “a formal representation of a theory” (Ader et al., 2008).

In this research the results of which are presented in this article, the main purpose was to develop useful regressions (Freedman, 2009), for predictions and estimation of the influence of some parameters in the process.

Keywords: fertilizer, organic, statistical models, working process, agriculture, spreader

MATERIALS AND METHODS

The basic material used in this work is the experimental data presented in (Stefan et al., 2019). In the same article were made the first researches useful for the statistical modeling, respectively the calculation of the correlation between the experimental data series corresponding to the seven process parameters monitored. The organic fertilizer machine is presented in figure 1.

The method used is the classical method of statistical modeling, based on the study of the correlation between experimental data, on linear and nonlinear regressions.

The process parameters considered in this study are given in Table 1, taken from Stefan et al. (2019). According to the same work, we reproduce Table 2, which gives the statistical characteristics that rank the connections between the quality parameter, $\sigma$ and the other parameters considered.

![Fig. 1 - Organic fertilizer spreader MG](image)

**Table 1**

<table>
<thead>
<tr>
<th>Parameter name</th>
<th>Notation</th>
<th>M</th>
<th>L</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>The abscissa of the measuring point on the axis perpendicular to the working direction, originating in the projection on the ground of the intersection between the axis of the working direction and the axis perpendicular to it.</td>
<td>$x$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Rotors speed</td>
<td>$\omega$</td>
<td>0</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Machine mass flow rate</td>
<td>$q$</td>
<td>1</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>Rotors inclination angle</td>
<td>$\beta$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Spread material density</td>
<td>$\rho$</td>
<td>1</td>
<td>-3</td>
<td>0</td>
</tr>
<tr>
<td>Spread material linear density</td>
<td>$\sigma$</td>
<td>1</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>Machine drive speed</td>
<td>$v$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Effective width of material distribution (machine working width)</td>
<td>$B$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Conveyor constructive width</td>
<td>$B_r$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Total width of distribution</td>
<td>$b$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Fertiliser theoretical rate</td>
<td>$N$</td>
<td>1</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>Real rate administered</td>
<td>$N_r$</td>
<td>1</td>
<td>-2</td>
<td>0</td>
</tr>
<tr>
<td>Conveyor linear speed</td>
<td>$v_t$</td>
<td>0</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>Height of the material in the bucket</td>
<td>$h$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Thickness of material layer entrained by a row of rotor teeth</td>
<td>$s$</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Number of rotor teeth rows</td>
<td>$z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 2**

<table>
<thead>
<tr>
<th>Variation $\sigma$, with:</th>
<th>Covariance</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>-0.297</td>
<td>-0.055</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.256</td>
<td>-0.023</td>
</tr>
<tr>
<td>$q$</td>
<td>9.894</td>
<td>0.565</td>
</tr>
<tr>
<td>$v$</td>
<td>-0.024</td>
<td>-0.053</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-0.006</td>
<td>-0.057</td>
</tr>
<tr>
<td>$\rho$</td>
<td>61.230</td>
<td>0.270</td>
</tr>
</tbody>
</table>
By the variant and the correlation coefficients indicate a tight connection between the spread material linear density and the machine mass flow rate, \( q \), respectively, the spread material density, \( \rho \). The other dimensions taken into consideration in the experimental working process are insignificantly related to the main quality parameter of the machine, the spread material linear density, \( \sigma \).

This result shows that any attempt to mathematically model the connection between the main parameter of interest, \( \sigma \), as the main quality parameter of the natural fertilizer spreading machine, and the process input and control parameters, must take into account first of all the two parameters emphasized by covariance and correlation: \( q, \rho \).

Elementary reasons require some calculation relations that help making some useful estimates for designing the working capacity and working regimes of organic fertilizer spreading machines. Such global calculation relations are given in the literature, for example in Şandru et al., (1983) and Scripnic and Babiciu, (1979). According to the quoted sources, and working in the International System of Units - SI, one can write the relation:

\[
q = B v N
\]  

(1)

According to Sandru et al. (1983) the achievement of the flow (1) is ensured by adjusting the control parameters: the speed of the conveyor and the height of the material, with the relation:

\[
q = B_t h v_t \rho
\]  

(2)

By equalizing the two expressions of the flow (1) and (2), one can obtain, for example, the working speed of the aggregate:

\[
v = v_t \frac{B_t h \rho}{N}
\]  

(3)

According to Scripnic and Babiciu (1979), the feed (the thickness of the material layer entrained by a rotor teeth row) is given by the relation:

\[
s = v_t \frac{2\pi}{2\omega}
\]  

(4)

In fact, the distribution of the spread material is not uniform on the working width of the machine (see for example Figure 2 and Figure 3). For this reason, relations (1) - (3) change. Instead of theoretical and constant rate \( N_t \) is considered an average norm calculated using the spread material linear density, \( \sigma \):

\[
N_r = \frac{1}{b} \int_{-b/2}^{b/2} \sigma(\xi) d\xi
\]  

(5)

The total distribution width, \( b \), is determined experimentally and measured between the extreme throwing points on a perpendicular to the aggregate travel direction, mediating on a path of the order of at least 10 bucket widths, for example, every one meter, or chosen on a given travel length, randomly.

RESULTS

The research method on experimental data through linear and nonlinear regressions leads to results that allow estimating the quality of material distribution on the soil, but also the influence of adjustment and control parameters on the quality function \( \sigma \).

Linear and square regression of global data

The simplest mathematical models of the distribution function of the material spread by the machine equipped with the device described are obtained by direct interpolation (using the method of least squares). The method is simple, but writing all the equations is a complicated, routine operation that is done automatically in the commercial processing programs (for polynomials).

This first attempt to obtain a mathematical model is done using the polynomial interpolation program provided by Mathcad software, 2001.

From a physical point of view, there are some objections to this approach. The first is that angular variables usually appear as arguments of trigonometric functions, while in the following regressions they appear directly as polynomial arguments. This observation refers to the variable \( \beta \) and, partially to the angular velocity \( \omega \). A second observation refers to the fact that the spatial distribution of the material administered – essentially the dependence of function \( \sigma \) on variable \( x \) – is, at least partially, of a different nature than the dependence of the same function on the qualities of the administered material or on the geometry and kinematics of the distribution mechanism.

The expression of the linear regression of the distributed material mass linear density is given by the formula:

\[
\sigma = 0.438 - 0.026x - 0.001838\omega + 0.077q - 0.361v - 1.363\beta + 0.0007509\rho
\]  

(6)
The full results provided by the regression coefficients computation program are given in Table 4.

### Table 4

<table>
<thead>
<tr>
<th>Terms</th>
<th>Coefficient value</th>
<th>Standard deviation of regression coefficient distribution</th>
<th>Minimum limit of confidence interval 95% CI</th>
<th>Maximum limit of confidence interval 95% CI</th>
<th>Variance inflation factor - The measure of regression coefficient inflation due to multicollinearity</th>
<th>Significance test with Student’s t-distribution</th>
<th>Probability of rejecting the term based on t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant term</td>
<td>0.438</td>
<td>0.464</td>
<td>-0.474</td>
<td>1.35</td>
<td>NaN</td>
<td>0.944</td>
<td>0.255</td>
</tr>
<tr>
<td>$x$</td>
<td>-0.026</td>
<td>0.014</td>
<td>-0.054</td>
<td>2.219·10$^{-5}$</td>
<td>1</td>
<td>-1.808</td>
<td>0.078</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-1.838·10$^{-7}$</td>
<td>7.034·10$^{-8}$</td>
<td>-0.016</td>
<td>0.012</td>
<td>1.001</td>
<td>-0.261</td>
<td>0.385</td>
</tr>
<tr>
<td>$q$</td>
<td>0.077</td>
<td>4.709·10$^{-6}$</td>
<td>0.067</td>
<td>0.086</td>
<td>1.149</td>
<td>16.274</td>
<td>0</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-0.361</td>
<td>0.174</td>
<td>-0.703</td>
<td>-0.019</td>
<td>1.05</td>
<td>-2.074</td>
<td>0.047</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.363</td>
<td>0.698</td>
<td>-2.733</td>
<td>7.519·10$^{-7}$</td>
<td>1.031</td>
<td>-1.953</td>
<td>0.059</td>
</tr>
<tr>
<td>$\rho$</td>
<td>7.509·10$^{-4}$</td>
<td>3.667·10$^{-4}$</td>
<td>3.094·10$^{-4}$</td>
<td>1.471·10$^{-4}$</td>
<td>1.169</td>
<td>2.048</td>
<td>0.049</td>
</tr>
</tbody>
</table>

It is noted that the values of the coefficients of the parameters considered in the process (Table 4 column 2) are included in the confidence intervals with the threshold of 0.05, 95% probability (Table 4, columns 4 and 5). It can be seen that the two variables the closest enclosed are, the flow, $q$ and the density of the administered material. The significance test relative to Student’s t-distribution shows that the same two variables have the greatest significance. Also, regression terms containing the two variables have the lowest probability of being rejected (the last column of Table 4).

For the estimation of a measure of the function error (6) reported to the experimental values the quantity is calculated:

$$
\varepsilon_1 = \sqrt{\frac{\sum_{k=1}^{n}(\sigma_k - \sigma(x,\omega,q,v,\beta,\rho))^2}{n}}
$$

where $\varepsilon_1$ is the measure of the approximation global error (6) of the administered material mass linear density, $\sigma(x,\omega,q,v,\beta,\rho)$ is function (6), $n$ is the number of experimental data, while the string $\{\sigma_k\}_{k=1,...,n}$ is the string of the mass linear densities determined experimentally. For linear regression (6), the value of global error measure (7) is 0.049 kg/m$^2$, namely 2.465% of the average value of the experimental data string, $\{\sigma_k\}_{k=1,...,n}$.

### Table 5

<table>
<thead>
<tr>
<th>Terms</th>
<th>Coefficient value</th>
<th>Standard deviation of regression coefficient distribution</th>
<th>Minimum limit of confidence interval 95% CI</th>
<th>Maximum limit of confidence interval 95% CI</th>
<th>Variance inflation factor - The measure of regression coefficient inflation due to multicollinearity</th>
<th>Significance test with Student’s t-distribution</th>
<th>Probability of rejecting the term based on t statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant term</td>
<td>27.531</td>
<td>1.711e6</td>
<td>-3.36e6</td>
<td>3.36e6</td>
<td>NaN</td>
<td>1.609e-5</td>
<td>0.399</td>
</tr>
<tr>
<td>$x$</td>
<td>0.073</td>
<td>0.077</td>
<td>-0.077</td>
<td>0.224</td>
<td>91.708</td>
<td>NaN</td>
<td>0.958</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.061</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>7.092e13</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>$q$</td>
<td>0.091</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>$\nu$</td>
<td>-0.463</td>
<td>2.491e6</td>
<td>-4.892e6</td>
<td>4.892e6</td>
<td>5.004e14</td>
<td>-1.856e-7</td>
<td>0.399</td>
</tr>
<tr>
<td>$\beta$</td>
<td>-1.517</td>
<td>9.409e6</td>
<td>-1.848e7</td>
<td>1.848e7</td>
<td>1.217e14</td>
<td>-1.613e-7</td>
<td>0.399</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-0.089</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
</tr>
<tr>
<td>$x\omega$</td>
<td>-7.866e-4</td>
<td>1.166e-3</td>
<td>-3.077e-3</td>
<td>1.503e-3</td>
<td>47.037</td>
<td>-0.675</td>
<td>0.318</td>
</tr>
<tr>
<td>$xq$</td>
<td>-2.547e-3</td>
<td>7.589e-4</td>
<td>-4.037e-3</td>
<td>-1.056e-3</td>
<td>5.905</td>
<td>-3.356</td>
<td>1.489e-3</td>
</tr>
<tr>
<td>$x\nu$</td>
<td>0.011</td>
<td>0.025</td>
<td>-0.038</td>
<td>0.061</td>
<td>12.096</td>
<td>0.452</td>
<td>0.36</td>
</tr>
<tr>
<td>$x\beta$</td>
<td>0.032</td>
<td>0.117</td>
<td>-0.197</td>
<td>0.262</td>
<td>8.764</td>
<td>0.275</td>
<td>0.384</td>
</tr>
<tr>
<td>$x\rho$</td>
<td>-3.413e-5</td>
<td>6.041e-5</td>
<td>-1.528e-4</td>
<td>8.449e-5</td>
<td>25.975</td>
<td>-0.565</td>
<td>0.34</td>
</tr>
<tr>
<td>$\omega q$</td>
<td>5.26e-4</td>
<td>3.728e-4</td>
<td>-2.059e-4</td>
<td>1.258e-3</td>
<td>53.507</td>
<td>1.411</td>
<td>0.147</td>
</tr>
<tr>
<td>$\omega \nu$</td>
<td>-9.12e-4</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
<td>NaN</td>
</tr>
</tbody>
</table>

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The second order regression analysis shows that the second order term in $x$ cannot be rejected. It must also be taken into account the produced term $xq$. The importance of considering the dependence of function $\sigma$ of coordinate on the axis perpendicular to the tangent of the aggregate trajectory is clearly evidenced and must be nonlinear as the experimental data show.

The second order regression can be written in the form (8),

$$
\sigma = 27.531 + 0.073x + 0.061\omega + 0.091q - 0.463v - 1.517\beta - 0.089\rho - 0.0007866x\omega \\
- 0.002574xq + 0.011xv + 0.032x\beta - 0.00003413x\rho + 0.000526a_qq \\
- 0.000912aw + 0.027aw - 0.00007387aw - 0.042qv + 0.002324q\beta \\
+ 0.00005188q\rho - 0.304v\beta - 0.0001475v\rho - 0.002782q\rho - 0.096x^2 \\
- 0.0008211\omega^2 - 0.0004262q^2 + 0.399v^2 + 6.476\beta^2 + 0.00007082\rho^2
$$

(8)

but the table (5) is sufficient to draw the necessary conclusions. For the second order linear regression with the coefficients given in Table 5, the value of the global error measure (7) is 0.027 kg/m², namely 1.469% of the average value of the experimental data string, $\{x_i, y_i, \ldots, x_n\}$.

Figure 2 shows the graphical representation of linear and second order regressions compared to the distribution of experimental data along the perpendicular to the aggregate trajectory.

![Graphical representation of linear and square regressions compared to experimental data](image_url)
It is noted that linear regression is not suitable for estimating the form of administered material distribution along the axis perpendicular to the aggregate trajectory. The second order regression reflects to some extent the form of the actual distribution, the approximate symmetry with respect to the longitudinal axis of the machine, but falls below zero beyond the 4 m limit reported to the middle of the distance between the central rotors. This aspect is not in line with reality. However, the 7 - 8 meters for which the square regression is positive show that the working width is somewhere in this area. Taking into account that, according to (Stefan et al, 2019), the standards for the administration of the materials with which the experiments were performed fall between 10 and 80 t/ha (namely between 1 and 8 kg per m²); it can be noticed that, depending on the required standard, working widths greater than 4 m cannot be achieved according to the second order regression. At the same time, we can observe the relatively large non-uniformity of the distribution experimental, practical data.

A measure of the uniformity of experimental data very commonly used in agricultural science statistics is given, for example, in (Gheres, 2007) by the formula:

\[
U = \left[ 1 - \frac{\sum_{j=1}^{n} (\hat{\sigma}_j - \bar{\sigma})^2}{n-1} \cdot \frac{1}{\bar{\sigma}} \right] \cdot 100
\]

where:
- \( U \) is the \textit{degree of distribution uniformity}, in %,
- \( n \) - number of experimental data;
- \( \bar{\sigma} \) - average value of distributed material linear density, \( \sigma \).

For the experimental data in Figure 2, the value \( U = 15.019 \% \) is obtained. The value obtained for the degree of distribution uniformity, gives the work an unsatisfactory qualitative character (it is higher than 75%), according to Sandru et al. (1983). Also according to Sandru et al. (1983), the working width of the machine can be 6 - 7 m. This uniformity is a uniformity in relation to the average value of the experimental data string, in this case having a value of 1.843 Kg/m² (18.43 t/ha). Another uniformity can be calculated in relation to a standard specified by the beneficiary. In this case, however, an appropriate setting of the machine’s working parameters must be sought in order to achieve the required norm.

Another way of illustrating the administered material distribution uniformity can be given by considering the distribution of the material on the axis perpendicular to the travel direction, having the origin in the middle of the machine (the point on the posterior side of the bucket, in the middle of it).

![Fig. 2 - The variation of the “norm achieved” along the axis perpendicular to the trajectory of the bucket middle point (its projection on the soil)](image)

The approximate calculation of a “norm achieved” is sought, so called to distinguish it from the norm required by agronomic needs. The variation of the “norm achieved” along the axis perpendicular to the trajectory of the bucket middle point (its projection on the soil) is given in Figure 3 compared to the minimum and maximum limits of the norm required for agronomic considerations.
A more extensive study on the distribution of material spread by the MG 5 fertilizer spreading machine is required because the administrable manure norm can differ for the same working regimes depending on the material and its effects on agricultural crops. Thus, a degree of uniformity as high as possible is desirable, but it is good if the maximum amounts administered in addition are estimated in order to avoid values that may adversely affect the development of crops. For example, if it is desired to apply the norm specified in Figure 3, in the working regime chosen, on the centre of the segment representing the working width, we obtain values 3-4 times higher, which can endanger the health of the culture.

CONCLUSIONS

Statistical modelling of experimental data and regression analysis are mandatory stages in the experimental study of complex working processes characterized by intense random components. The results presented in this article show that, through statistical modelling and regression analysis, one can observe important aspects of the phenomenon studied:

- independent parameters ranking according to their influence on dependent parameters;
- removing insignificant parameters from the study;
- retaining or elaborating qualitative or quantitative parameters of the studied process;
- aspects related to the need for detailed studies (on subset of parameters that characterize well-defined relative working regimes).

Within the same statistical modelling and regression analysis activities, parameters that characterize the studied process in terms of classical literature were introduced. Taking into account the distribution non-uniformity that characterizes these processes, a new parameter, called the actual norm administered, was introduced, a parameter that will be used in the subsequent studies for the purpose of estimating as accurately as possible the values of the administrable norms according to the values of the adjustable regime parameters.

These parameters open the way to other process quality parameters: uniformity of material distribution, working capacity, energy consumption, shredding degree of the spread material.

It is noted that two of the independent parameters have the greatest influence on the qualitative parameter of the direct process experimentally determined, the linear density of the administered material, $\sigma$. These parameters are the material flow provided by the machine or the flow provided by the conveyor in the bucket, $q$, respectively the mass density of the material administered, $\rho$.

It was found that linear models (linear regression) only have the role of characterizing the intensity of the dependence between the dependent parameter and the independent parameters; it cannot characterize the nonlinear form of the material distribution along the normal line direction to the forward direction of the aggregate.

The results of this study highlight the ways to be addressed further:

- estimation of the work qualitative parameters on working regimes (the statistical model of the linear density of the spread material distribution, determination of the working width, the working uniformity and the production capacity, eventually of the energy consumption);
- variation of distribution linear density at spreading level according to the geometric and kinematic characteristics of the spreading device (rotors);
- formulation, in a more distant term, of similar results (regression analysis), based on non-polynomial formulas, on functions with substantiation by dimensional analysis and physical analysis of the process, in which, for example, angular or kinematic angular parameters appear, preferably as arguments of trigonometric functions. It is also desirable to avoid results that require additional explanation of the physical dimensions of some coefficients appearing in the regression formulas.

REFERENCES


