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RESEARCH ARTICLE

Optimistic Results of HCS Transform

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Abstract

In this paper, have study useful results of half canonical sine transform of generalized function.Also given that definition of testing function,and use ful results for that transform like Time Reverse Property, Parity Linearity Property Addition property Separability.

Keyword: Canonical transform, half canonical sine transform, Integral transform, generalized function testing function space.

1. Introduction

Now a days, fractional integral transforms play an key role in signal processing, image reconstruction, pattern recognition, accostic signal processing [1],[2].A new generalized integral transform was obtained by Zayed[10]. Bhosale and Chaudhary [3], Chavhan S B [6],[7][8],[9]. Had extended fractional Fourier transform to the distribution of compact support. Chavhan S B [4],[5]. had define the half canonical sine transform as

$$\{HCSTf(t)\}(s) = -i\sqrt{\frac{2}{\pi ib}}\, e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \int_0^\infty \sin\left(\frac{s}{b}t\right) e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \qquad \text{for } b \neq 0$$

Notation and terminology as per Zemanian [11] .This paper is organized as section 2 definition of testing function space .Section 3 definition of half canonical sine transform. Section 4 useful results of half canonical sine transform and lastly conclusion is stated.

2. Definition of testing function space *E*:

An infinitely differentiable complex valued function ϕ on R^n belongs to $E(R^n)$, if for each compact set. $I \subset s_a$ where $s_a = \{t :\in R^n,\ \ |t| \le a, a > 0\}$ and for $k \in R^n$,

$$\gamma_{E,k}\phi(t) = \sup_{t \in I} \left|D^k\, \phi(t)\right| < \infty \qquad \text{k=0,1,2,3.....}$$

Note that space E is complete and a Frechet space,let E' denotes the dual space of E

3. Definition of half canonical sine transform:

Half canonical sine transform of $f(t)$ is given by

$$\{HCSTf(t)\}(s) = -i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2} \qquad \text{for } b \neq 0$$

$$= \sqrt{d}\,.e^{\frac{i}{2}cds^2} f(d.s) \qquad \text{for b = 0}$$

Where, $$K_{HS}(t,s) = -i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2} \sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{a}{b}\right)t^2}$$

Hence half canonical sine transform of $f(t)$ is defined as

$$\{HCST\,f(t)\}(s) = \langle f(t), K_{HS}f(t,s)\rangle$$

Since the range of integration for the half canonical sine transform is just $[0,\infty]$ and not $(-\infty,\infty)$ using half canonical sine transform is more convenient than using the canonical transform to deal with the odd function.

4. Valuable properties of half canonical sine transform:

4.1 Time Reverse Property:

If $\{HCST\,f(t)\}$ is half canonical sine transforms of $f(t)$, $f(t) \in E^1(R^1)$ then

$$\{HCST\,f(-t)\}(s) = \{HCST\,f(t)\}(s)$$

Proof : Using definition of half canonical sine transforms of $f(t)$

$$\{HCST\,f(t)\}(s) = -i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2} f(t)\,dt$$

$$\{HCST\,f(-t)\}(s) = -i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \sin\left(\frac{-s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)t^2} f(-t)\,dt$$

Put $-t = x \qquad \therefore t = -x$

Hence $dt = -dx,$ also $t \to 0\ to\infty,\ x \to \infty, 0$

$$\{HCST\,f(-t)\}(s) = i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_\infty^0 \sin\left(\frac{s}{b}x\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)(-x)^2} f(x)\,dx$$

$$\{HCST\,f(-t)\}(s) = -i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \sin\left(\frac{s}{b}x\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)(x)^2} f(x)\,dx$$

Replacing x by t again

$$\{HCST\,f(-t)\}(s) = -i\sqrt{\frac{2}{\pi ib}}e^{\frac{i}{2}\left(\frac{d}{b}\right)s^2}\int_0^\infty \sin\left(\frac{s}{b}t\right)e^{\frac{i}{2}\left(\frac{d}{b}\right)(t)^2} f(t)\,dt$$

$$\therefore \{HCST\,f(-t)\}(s) = \{HCST\,f(t)\}(s)$$

4.2 Parity :

If $\{HCST\,f(t)\}(s)$ is half canonical cosine transform of $f(t) \in E^1(R^1)$ then

$$\{HCST\,f(-t)\}(s) = -\{HCST\,f(t)\}(-s)$$

4.3 Linearity Property :

If C_1, C_2 are constant and f_1, f_2 are functions of t then

$$\{HCST[C_1 f_1(t) + C_2 f_2(t)]\}(s) = C_1\{HCST\, f_1(t)\}(s) + C_2\{HCST f_2(t)\}(s)$$

4.4 Addition property: If $\{HCST\, f(t)\}(s)$ and $\{HCST\, g(t)\}(s)$ are half canonical cosine transform of $f(t)$ and $g(t)$ then

$$\{HCST[f(t) + g(t)]\}(s) = \{HCST\, f(t)\}(s) + \{HCST\, g(t)\}(s)$$

4.5 Separability : If $f(t,x) = f_1(t).f_2(x)$ then $\{HCST\ f(t,x)\}(s,w) = \{CST\ f_1(t)\}(s).\{CST\ f_2(x)\}(w)$

5. Conclusion

In this paper half canonical sine transforms is generalized in the form the distributional sense, we have obtained optimistic results for this transform are proved.

6. References

1. Alieva Tatiana and Bastianas Martin J . On Fractional Fourier transform moments," IEEE signal processing letters, 2000, Vol. 7, No.11, Nov. page no.?
2. Alieva Tatiana and Bastiaans Martian J. Wigner Distribution and Fractional Fourier Transform for 2-Dimentional Symmetric Beams", JOSA A, 2000, Vol.17, No.12, P.2319 – 2323.
3. Bhosale BN and Choudhary MS.Fractional Fourier transform of distributions of, compact support, *Bull.Cal.Math.Soc.*, 2002, Vol. 94, No.5,: 349-358.
4. Chavhan SB. Modulation of Generalized Canonical CS-transform, IJARCET, 2012, volume 12, issue 9, November, page no.???
5. Chavhan SB and Borkar VC. Two- dimensional generalized Canonical cosine-cosine transforms, IJMES, 2012, volume 1 issue 4, pp 48-52.
6. Chavhan SB. Generalized two dimensional canonical transform IOSR Journal of Engineering, 2012, Vol.2,issue 6 pp:1487—1491.
7. Chavhan SB and Borkar VC. Operational calculus of Canonical cosine transform research inventy: IJES, 2012, volume 1 issue 7, pp1-5.
8. Chavhan SB. Canonical Sine Transform and their Unitary Representation, Int. J. Contemp. Math. Sciences, 2012, Vol. 7, 2012, no. 15, 717 - 725
9. Chavhan SB. Generalization of two dimensional canonical SS-transform. Archives of Applied Science Research, 2013, 5 (2):203-207
10. Zayed A. A convolution and product theorem for the fractional Fourier transform", IEEE sign. proc. Letters, 1998, Vol.5, No.4: 101-103.
11. Zemanian AH. Generalized Integral Transform", Inter Science Publisher's New York, (1968).