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Travelling wave solutions of potential KdV equation through tanh-coth method

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ABSTRACT

In this work, establish travelling wave solutions of the potential kdv equation using travelling wave transform and tanh-coth method. Through tanh-coth method, we obtained travelling wave and kink solutions of the potential kdv equation. Applied method proved to be reliable, effective and easy in handling a large number of nonlinear partial differential equations (PDE's).

Keywords : Tanh-coth method, potential kdv equation, travelling wave solution, kink solution.

INTRODUCTION

Travelling waves propagating in homogeneous media have attracted considerable attention of researcher in recent years. Finding exact solutions of nonlinear dispersive and dissipative equations is an important subject. Many researchers developed and used several new methods for finding solutions of such nonlinear PDE's. which based on some previous methods. The tanh-coth method is improved tanh method and extended tanh method. the inverse scattering method, the backland transformation method, sin-cosine method, tanh-sech method, Hirota's bilinear technique, Adomian decompositiopn method are some another methods that are used to handled non linear PDE's.

In solid physics, plasma physics, fluid dynamics etc., the potential kdv equation is written as

$$v_t + \alpha v_x^2 + v_{xxx} = 0 \quad (1)$$

The potential kdv equation is obtained from the pioneer model kdv equation by using transformation $v = \vartheta_x$ and integration once.

The tanh-coth method:

The wave variable

$z = x - ct$

convert the PDE

$$Q(v, v_x, v_t, v_{xx}, v_{tt}, v_{xxx}, \dots\dots\dots) = 0 \qquad (2)$$

to ordinary differential equation(ODE)

$$R(v, v', v'', v''', \dots\dots\dots\dots) = 0 \qquad (3)$$

Integrating equation (3) as long as terms contain derivatives and the constant of integration consider as zero.
Introducing a new independent variable

$Y = \tanh(\mu z)$, $\quad$ $where z = x - ct$

and μ is wave number.

In tanh-coth method used the finite expansion

$$v(x,t) = S(Y) = \sum_{i=0}^{N} a_i Y^i + \sum_{i=1}^{N} b_i Y^i \qquad (4)$$

where N is positive integer and its value to be determined in most of cases. By substituting (4) in Ode (3), obtained value of N by balance method. Usually balance the highest order nonlinear terms with the linear terms of highest order using the following scheme

$v \to N$

$v^2 \to 2N$

$v^r \to rN$

$v' \to N+1$

$v'' \to N+2$

$v^{(r)} \to N+r$

$(v')^2 \to (N+1)^2$

$(v')^r \to (N+1)^r$

$(v'')^2 \to (N+2)^2$

$(v'')^r \to (N+2)^r$

Applications:

The potential kdv equation is

$$v_t + \alpha v_x^2 + v_{xxx} = 0 \qquad (5)$$

Substitute the wave variable

$z = x - ct$ in (5).

We look for a travelling wave solution in the form

$$v(x,t) = v(z) = v(x - ct) \qquad (6)$$

where c is the speed of soliton.

Substitute (6) in (5) ,we have

$$-cv' + \alpha(v')^2 + v''' = 0 \qquad (7)$$

Balancing the nonlinear term $(v')^2$, that has exponent $(N+1)^2$, with the highest order derivative term v''' , that has the exponent $N+3$, this gives

$$(N+1)^2 = N+3$$

$$N^2 + 2N + 1 = N + 3$$

$N = 1, -2$

Case (i) Let $N = 1$

Then by tanh-coth method formula (4), the method admits the substitution

$$v(x,t) = S(Y) = a_0 + a_1 + b_1 Y^{-1} \qquad (8)$$

As $Y = \tan(\mu z)$

Therefore,

$$\frac{d}{dz}(.) = \mu(1-Y^2)\frac{d}{dY}(.)$$

$$\frac{d^2}{dz^2}(.) = -2\mu^2 Y(1-Y^2)\frac{d}{dY}(.) + \mu^2(1-Y)^2\frac{d^2}{dY^2}(.)$$

$$\frac{d^3}{dz^3}(.) = 2\mu^3(1-Y^2)(3Y^2-1)\frac{d}{dY}(.) - 6\mu^3 Y(1-Y^2)\frac{d^2}{dY^2}(.) + \mu^3(1-Y^2)^3\frac{d^3}{dY^3}(.)$$

Hence,

$$v' = \frac{d}{dz}(v)$$
$$= \mu(1-Y^2)\frac{d}{dY}(v) \qquad (9)$$
$$= \mu(1-Y^2)(a_1 - b_1 Y^{-2})$$

$$v''' = \frac{d^3}{dz^3}(v)$$
$$= 2\mu^3(1-Y^2)(3Y^2-1)\frac{d}{dY}(v) - 6\mu^3 Y(1-Y^2)\frac{d^2}{dY^2}(v) + \mu^3(1-Y^2)^3\frac{d^3}{dY^3}(v)$$
$$= 2\mu^3(1-Y^2)(3Y^2-1)(a_1 - b_1Y^{-2}) - 6\mu^3 Y(1-Y^2)(2b_1Y^{-3}) + \mu^3(1-Y^2)^3(-6b_1Y^{-4}) \qquad (10)$$

Putting (9)and (10) in equation (7),we have

$$-c\,\mu(1-Y^2)(a_1-b_1Y^{-2}) + \alpha\mu^2(1-Y^2)^2(a_1-b_1Y^{-2})^2 + 2\mu^3(1-Y^2)(3Y^2-1)(a_1-b_1Y^{-2}) - 6\mu^3Y(1-Y^2)(2b_1Y^{-3}) + \mu^3(1-Y^2)^3(-6b_1Y^{-4}) = 0$$

$$-c\,\mu(a_1-b_1Y^{-2}) + \alpha\mu^2(1-Y^2)(a_1-b_1Y^{-2})^2 + 2\mu^3(3Y^2-1)(a_1-b_1Y^{-2}) - 6\mu^3Y(2b_1Y^{-3}) + \mu^3(1-Y^2)^2(-6b_1Y^{-4}) = 0$$

$$-c\,\mu a_1 + cb_1\mu Y^{-2} + \alpha\mu^2(a_1-b_1Y^{-2})^2 - Y^2(a_1-b_1Y^{-2})^2 + 2\mu^3(3Y^2-1)(a_1-b_1Y^{-2}) - 6\mu^3Y(2b_1Y^{-3}) + \mu^3(1-Y^2)^2(-6b_1Y^{-4}) = 0$$

$$(-c\,\mu a_1 + 2a_1b_1 - 6b_1\mu^3 - 2a_1\mu^3 + \alpha\mu^2a_1^2) + (c\mu b_1 - 2\alpha\mu^2a_1b_1 - b_1^2 + 2b_1\mu^3)Y^{-2} + (-a_1 + 6a_1\mu^3)Y^2 + (\alpha\mu^2b_1^2 - 6b_1\mu^3)Y^{-4} = 0 \qquad (11)$$

This gives

$$-c\,\mu a_1 + 2a_1b_1 - 6b_1\mu^3 - 2a_1\mu^3 + \alpha\mu^2a_1^2 = 0$$
$$c\mu b_1 - 2\alpha\mu^2a_1b_1 - b_1^2 + 2b_1\mu^3 = 0 \qquad (12)$$
$$-a_1 + 6a_1\mu^3 = 0$$

$$\alpha\,\mu^2 b_1^2 \;\; - 6b_1\mu^3 = 0$$

From this, we obtained the following sets of solutions.

(i) $a_0 = R$ *, where R is any constant*

$$\mu = \frac{\sqrt{c}}{2}, c > 0\,, a_1 = \frac{3\sqrt{c}}{a}\,, b_1 = 0$$

(ii) $a_0 = R$ *, where R is any constant*

$$\mu = \frac{\sqrt{c}}{2}, c > 0\,, a_1 = 0\,, b_1 = \frac{3\sqrt{c}}{a}$$

This gives two solutions from (8) ,

(i) $v_1(x,t) = R + \frac{3\sqrt{c}}{a}\,Y$

$$= R + \frac{3\sqrt{c}}{a}\,\tanh(\mu z)$$

$$= R + \frac{3\sqrt{c}}{a}\,\tanh\left(\frac{\sqrt{c}}{2}(x-ct)\right), \text{ where } c > 0$$

This is kink solution.

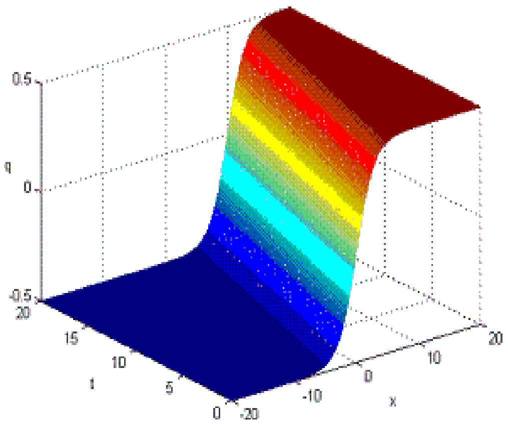

Fig.-1: Kink solution $v_1(x,t)$ for a= 0.5, c= 1.5, R=0

(ii) $v_2(x,t) = R + \frac{3\sqrt{c}}{a}\,Y^{-1}$

$$= R + \frac{3\sqrt{c}}{a}\,[\tanh(\mu z)]^{-1}$$

$$= R + \frac{3\sqrt{c}}{a}\,\coth(\mu z)$$

$$= R + \frac{3\sqrt{c}}{a}\,\coth\left(\frac{\sqrt{c}}{2}(x-ct)\right), \text{ where } c > 0$$

This is travelling wave solution.

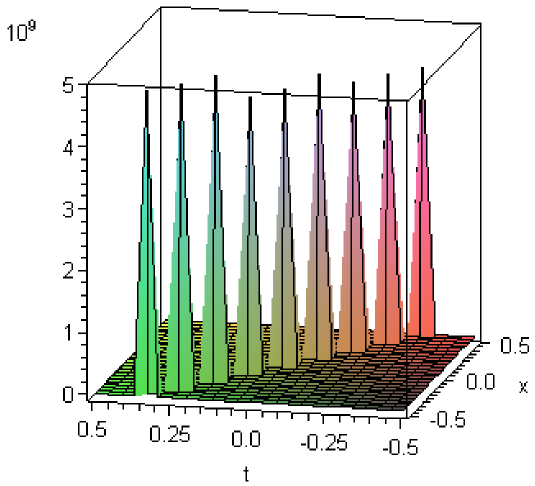

Fig. -2: Travelling wave solution $v_2(x,t)$ for a= 0.5, c= 1.5, R=0

If $N = -2$, then one can obtained another some special type sets of solutions.

CONCLUSION

In this paper, we obtain the kink solution and travelling wave solution of the potential kdv equation by using tanh-coth method. Using travelling wave transformation the nonlinear partial differential equation is transformed into ordinary differential equation. The used tanh-coth method is effective, reliable and gives multiple solutions. This work emphasized that tanh-coyh method is powerful technique to solve nonlinear partial differential equation that admits travelling wave and solitary wave solutions..

Conflicts of interest: The authors stated that no conflicts of interest.

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