A Study the Axial and Radial Rotor Stability of the Turbo Machinery with Allowance the Geometry of the Surface and Properties of the Lubricating Fluid

E.A. Zadorozhnaya, S.V. Cherneyko, M.I. Kurochkin, N.A. Lukovich

Department of Motor Transport, South Ural State University, Chelyabinsk, Russia.

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ABSTRACT

An ensuring of the axial and radial rotor stability is one of the main objectives in the design and operation of the small-sized turbo-machinery. Hydrodynamic thrust bearings must securely restrain unbalanced axial forces, arising during any possible operating modes. Using of the different types of intermediate elements provides a stable position of the rotor in the radial bearings. Recently flexible rotors have been widely used in small-sized turbo-machines. The complexity of the implementation of methods for calculating the dynamics of these rotors is to solve a system of motion equations. The system of motion equations, which is presented in this work, contains the motion equation for the each bearing element, including the floating rings and elements of the rotor (wheels of the turbine and compressor, the central mass). The load acting on the bearings includes the masses of elements, the reaction of lubricating layers and the rotor imbalance, potential impact loads and forces torques, acting on the rotor from the other elements of the rotor. To calculate the reactions of lubricating layers for two-layer bearings the hydrodynamic pressure field and the friction losses were considered for a real bearing design, on the surface of which the sources of the lubrication is always located. The thrust bearings with segments of various designs have been considered, including laser texturing surface. The trajectories of the rotor elements of the turbo-machinery for various rotation velocities, as well as the elastic line of the rotor at different time points are represented as the results of the calculation.

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1. INTRODUCTION

The multilayer bearings are widely used to enhance the stability of the rotor movement of the turbo-machines. The rotating and non-rotating rings are used as intermediate elements of bearings. The number of intermediate elements, their design and the way of their installation in to the hull of the turbo-machine depends on operating conditions and design parameters. The bearings with three layers are the most promising. It provides the greatest stability of the rotor motion.

At the same time, the using of multigrade oils that are prevalent in recent years, leads to a change of the hydro-mechanical characteristics of the bearings. This is due to the rheological
properties of lubricants. Consideration of these properties helps us in the mathematical models to approach to real operating conditions of units of friction.

This study is part of the work to create the methodology for calculating hydro-mechanical characteristics of multilayer radial and thrust bearings, operating on non-Newtonian lubricants.

To reduce rotor vibrations of a turbocharger the most common way is to use intermediate floating elements in the form of rotating floating (RF) (Fig. 1) or a non-rotating floating (NF) (Fig. 2) rings. Working surfaces of each ring with surfaces of the hull and the rotor are formed the several lubricating layers.

![Fig. 1. Bearing with rotating rings: 1 - rotor, 2 - hull, 3 - rotating rings, 4 - stopper ring.](image1)

![Fig. 2. Bearing with non-rotating ring: 1 - rotor, 2 - hull, 3 - non-rotating ring.](image2)

Edgar J. Gunter presented some results of the linear and nonlinear dynamics of the rotor, which rotates in the RF ring bearings for the typical turbocharger [6]. He computed the linearized stability of the system for the various ratios of inner and outer clearances. He also represented the analysis of the critical speed and showed several fluctuation forms of the rotor.

![Fig. 3. The package of the floating ring: 1 - rotor, 2 - hull, 3, 4 - non-rotating rings, 5 - journal.](image3)

A. Tartara [7] had conducted experiments and concluded, that remarkable effects of stabilizing system can be expected, when the bearings with floating rings are used. Namely, the ring starts rotating as soon as the whirl occurs, and the further stable operation is realized as the journal speed increases. The stabilizing effect of the floating-ring bearing is conspicuous when the ratio between inner and outer clearances and the absolute clearances are large.

The bearing design with three lubrication layers provides the more stability of the rotor. It was shown in [8]. An example of the calculation of the dynamics of the flexible asymmetric rotor is presented in [9]. Newtonian and non-Newtonian fluids have been considered as a lubricant the bearings of the rotor.

However, theoretical researches of stability of a rotor taking into account properties of lubricant are developed insufficiently.

2. THEORETICAL ANALYSIS

The problem of the hydrodynamic theory of friction units is characterized by a set of methods for solving several interrelated tasks: the determination the conditions of stability and parameters of the journal nonlinear oscillations
on the lubricating film; the calculation of its trajectory; the calculation of the field of hydrodynamic pressure in the lubricating layer, which separates the friction surfaces of the journal and bearing, taking into account an arbitrary law of their relative motion; the calculation of the temperature of the lubricating film. The design of the turbocharger rotor with three layers bearings is shown at Fig. 4.

**Fig. 4.** The design of the rotor on multilayer bearings: 1 – compressor’s wheel, 2 – floating rotating ring, 3 – floating non-rotating ring, 4 - stopper ring, 5 – rotor, 6 – hull, 7 – turbine’s wheel.

### 3. THE DYNAMICS OF THE SYSTEM

The dynamics of the rotor is considered as the movement of the journal at the multilayer bearings, depending on the operating forces and initial conditions. The stability of the motion of each movable element is a prerequisite for the effectiveness of the system as a whole. As a rule, the system is stable, if it deviates from its equilibrium position on an arbitrarily small value for any sufficiently small change of the load. The effectiveness of different designs of bearings with intermediate elements is estimated by the calculating trajectories of the geometric center of the journal and rings, which are moving under the action of loads. Besides the characteristics of stability and a set of hydro-mechanical characteristics are calculated.

A dynamic model of the rotor of the turbocharger with asymmetrically arranged wheels is presented in the form of five masses, connected by weightless flexible rods. The rotor leans on two three-layered bearings (Fig. 5). The rotor, rings and the hull are separated by thin layers of lubricants. Misalignment axis of the journal and rings are excluded. The axis OZ of the inertial coordinate system OXYZ is drawn through the geometric center of the bearing hull. The motion of the flexible rotor represents a superposition of the motion of the rotor axis as a rigid body within clearances of bearings and the elastic displacements of all elements of the rotor relative to the rigid axis.

**Fig. 5.** The dynamic model of the turbocharger rotor.

The system of equations of the motion for a flexible asymmetric rotor, which leans on multilayer bearings, is presented by V. Prokopiev and others [10], P. Taranenko and others [11].

An analysis of the trajectories of each element of the turbocharger, which are obtained by direct numerical integration of the motion equations, is the most accurate method for evaluating of the performance of bearings with intermediates elements. For arbitrary initial values of the coordinates and velocities of the centres of mobile elements we can not anticipate the kind of trajectories at which they move, and therefore we can not specify the sufficient of time steps for the analysis of these movements. We have developed an algorithm for integrating the motion equations, which contains a specific procedure for the automatic determination of the stop of calculations. According this procedure, the individual turns of the trajectory must be placed inside a closed rectangular region. The position of this rectangular region doesn’t change in time.

### 4. PROPERTIES OF THE LUBRICANT

The lubricant is considered as a design element of any friction units. At present, multigrade engine oils are widely used in the operation of the internal combustion engine and turbochargers of system of the pressurization of
It is known that such oils are named as non-Newtonian fluids. The main feature of the rheological behaviour of these oils is the non-linearity of the viscosity as a function of a shear rate. In other words, the viscosity is a function not only of the temperature and pressure, but and shear rates, the values of which reach 3·10^8 s^{-1} for the rotor bearing of the turbochargers. For these values of shear rates the viscosity of the lubricant may be significantly reduced. This fact must be taken into account when the hydro-mechanical characteristics of the bearings of the rotor are calculated.

Thus, the need for a comprehensive rheological model of the lubricant emerged. This model must take into account the viscosity as function of the lubrication temperature, the hydrodynamic pressure in a thin lubricating layer, the shear rates, the relaxation (delay of viscosity change with rapid growth of hydrodynamic pressures) and the highly viscous boundary layer. The following viscosity model was proposed:

\[
\mu^*(T, p, \dot{\gamma}) = \begin{cases} 
\mu_1 \cdot C_1 e^{(C_2/(T + C_3))} - \beta(T_p)\mu, \\
1 \leq \dot{\gamma} \leq 10^2; \\
(1) \\
10^2 \leq \dot{\gamma} \leq 10^6; \\
\mu_2 \cdot C_1 e^{(C_2/(T + C_3))} - \beta(T_p)\mu, \\
\dot{\gamma} > 10^6,
\end{cases}
\]

where \( \dot{\gamma} = \sqrt{I_1 + I_2} = (\partial V_x/\partial y)^2 + (\partial V_z/\partial y)^2 \).

At section 1 in the range of shear rates from 1 to 10^2 s^{-1} the oil behaves as a Newtonian fluid with a viscosity \( \mu_1 \). At section 2 in the range of shear rates from 10^2 to 10^6 s^{-1} it is characteristic the decrease in viscosity as a power law. In section 3, where the shear rate is greater than 10^6 s^{-1}, the oil behaves as a Newtonian fluid with the viscosity \( \mu_2 \).

Accounting for the highly viscous boundary layer is calculated as follows [12,13]:

\[
\mu_l = \mu^*(T, p, \dot{\gamma}) + \mu_5 \exp\left(-\frac{h_l}{l_h}\right),
\]

where \( l_h \) is a characteristic parameter with dimension of a length, the value of which is specific to each combination of the lubricating oil and the solid surface; \( \mu_5 \) is a parameter representing the equivalent viscosity at infinitely small distance from the boundary surface; \( \mu_0 \) is the oil viscosity in the volume.

### 5. The Calculation of the Temperature of the Lubricating Film

Thermal processes in the oil film of the bearing are based on the solutions of the generalized energy (heat) equation for the thin layer of the viscous incompressible fluid, which is between two moving surfaces. The generalized energy equation takes into account both the convective heat transfer, which are implemented in the lubricant, and the heat transfer by conduction. The temperature distribution in the lubricating film is described as [10]:

\[
\rho C_0 \frac{\partial T}{\partial t} + \rho C_0 \left(V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \frac{\partial T}{\partial z}\right) - \lambda_0 \frac{\partial^2 T}{\partial y^2} = D.
\]

(3)

Where \( \rho \) is the density of lubricant; \( c_0, \lambda_0 \) is a specific heat capacity and a thermal conductivity of oil (usually assumed as constant); \( T(x, y, z, t) \) is the temperature at coordinates \( x, y, z \); \( t \) is time; \( V_x, V_y, V_z \) are velocity components of the unit volume of lubrication, located between two moving surfaces of the tribo-unit; \( D \) is a dissipation function.

Depending on the assumptions used for the temperature distribution in the thin lubricating layer, three approaches can be applied to integrate the energy equation: thermo-hydrodynamic (non-isothermal), adiabatic, isothermal.

For the isothermal approach the three-constant Fogel’s formula works very well:

\[
\mu(T) = C_1 \cdot \exp\left(C_2/(T + C_3)\right),
\]

(4)

where \( C_1, C_2, C_3 \) are constants, which reflect empirical features of lubricant.

When thermo-hydrodynamic approach is used the temperature change is expected in all directions [14], including across the lubricating layer. In this case, the boundary conditions are the most adequate to the actual thermal processes. This approach provides the information on the local properties of the
temperature field of the lubricant layer: the maximum of instantaneous average temperatures, areas of increased thermal stress.

To determine the temperature variation in the thickness of the lubricating layer we performed a discretisation across the layer. This allows us to take into account the temperature field, the dependence of the lubricant viscosity on the second invariant of the shear rates, and the effect highly-viscous boundary layer, which adsorbed on the friction surfaces. So we can consider the non-Newtonian properties of the lubricant.

6. DETERMINATION OF THE HYDRODYNAMIC PRESSURE IN THE LUBRICATING LAYER

Among the techniques which take into account properties of the lubricants, is best known an approach by modifying the equations for the field of hydrodynamic pressure through a variety of rheological laws of the lubricant behaviour. For example, the equation of Reynolds considering the power law of the viscosity takes following form:

\[
\begin{align*}
1 \frac{\partial}{r^2 \partial \phi} \left[ \left( \phi_2 - \phi_0 \phi_1 \right) \rho \frac{\partial p}{\partial \phi} \right] + \\
+ \frac{\partial}{\partial \alpha} \left[ \left( \phi_2 - \phi_0 \phi_1 \right) \rho \frac{\partial p}{\partial \alpha} \right] = \\
= \frac{\partial}{\partial t} \left( \bar{h} \rho \right) + \frac{\partial}{\partial \phi} \left[ \alpha_1 + \left( \frac{1 - \phi_1}{h \phi_0} \right) \omega_2 \right] \rho \bar{h},
\end{align*}
\]

(5)

Where \( \bar{h}_k = \int_0^{1} \! \frac{\partial \bar{h}}{\partial \phi} \, d\gamma \), \( \bar{h}(\phi, \bar{e}) = 1 - \chi \cos(\phi - \bar{\phi}) \) is the dimensionless film thickness; \( \chi \) is the relative eccentricity.

Equation (5) is integrated with the boundary conditions of Swift-Shtiber:

\[
\begin{align*}
\bar{p}(\phi, \bar{z} = \pm a) = 0; \quad \bar{p}(\phi, \bar{z}) = \bar{p}(\phi + 2\pi, \bar{z}); \\
\bar{p}(\phi, \bar{z}) \geq 0; \quad \bar{p}(\phi, \bar{z}) = \bar{p}_s.
\end{align*}
\]

(6)

Where \( (\phi, \bar{z}) \in \Omega_s \), \( S = 1, 2 \ldots S^* \) is the area of the lubrication source in which the pressure is constant and equal to the supply pressure \( \bar{p}_s \); \( S^* \) is the number of sources.

We defined the trajectory of moving parts, the position of the rotor axis and the hydro-mechanical characteristics of friction units when we solve together the equations of motion, the equations for determining the pressure field (5) and the generalized energy equation (3) for finding the temperature in each film of lubricant.

7. MATHEMATICAL MODEL FOR CALCULATING THRUST BEARINGS

Usually the thrust bearing consists of the shoe rotating together with the rotor and a motionless part. Hydrodynamic thrust bearings with macro-profiling of the friction surface were widely adopted. The thrust bearing is carried out in the form of following segments: inclined surfaces, "Rayleigh's steps" or other profile. At the same time micro-profiling of surfaces of friction receives bigger distribution for increase of the bearing capacity [15,16]. Advantages and disadvantages of traditional and modern approaches of surface analysis based on concepts of roughness and texture were discussed in [17].

Laser texturing (creation of certain properties of a micro-profile of the friction surface by means laser impulses) is the most perspective for increase of the load-capacity of various tribo-units. Therefore we used this type of profiling for calculation the thrust bearings of the turbocharger. Calculations of the load capacity and load characteristics for the four types of surfaces of the hydrodynamic thrust bearing segment were performed.

A scheme of the hydrodynamic thrust bearing is shown at Fig. 6. The rotor rotates at a constant speed relative to the turbocharger hull. The axial load on the rotor is constant and counterbalances the reactions of the lubricating layer. The axis of the thrust bearing coincides with the rotational axis of the rotor.

The surface of the thrust bearing consists of eight blocks (segments). The lubricant under pressure moves through an internal ring groove of the thrust bearing. The pressure on the external radius of the thrust bearing is accepted equal to the atmospheric. Blocks are divided by radial grooves. The pressure in grooves is distributed according to the linear law. The load capacity of
the bearing is formed due to formation of system of hydrodynamic wedges, when lubricant is involved in the narrowed axial gap.

The planes of blocks can be processed by means of various technologies. We considered 4 types of bearings (Fig. 7). A design 1 is Rayleigh’s step \( \left( \frac{h}{h_y} = 1.87, \quad l_2 = l/3.59 \right) \). A design 2 is a block with two inclined planes in the circumferential direction \( \left( \gamma = 0.3°, \quad \frac{h}{h_y} = 1.87, \quad l_2 = l/3.59 \right) \). A design 3 is the inclined plane in the circumferential direction \( \left( \gamma = 0.3° \right) \). A design 4 is the block surface with the laser texturing. Extensive experimental and theoretical studies of thrust bearings have been performed by I. Etsion and his colleagues [15]. The textured surface represents the plane with micro-deepenings, which have the diameter, depths and density of their distribution. The laser texturing can be used for part of the bearing or for all plane of the bearing.

The following parameters of laser texturing were accepted in calculations: socket depth \( h_p = 50 \times 10^{-6} \text{m} \), socket radius \( r_p = 50 \times 10^{-6} \text{m} \), distribution density of sockets \( S_p = 0.6 \), \( l_p = 1.06 \).

To determine the pressure, we also used the Reynolds equation, which has been successfully applied to a thrust bearing by other authors [18].

8. RESULTS

The equation (5) was written in term of finite differences and integrated by a Gauss-Seidel method using of three levels of grids. The finest grid contains 96 intervals on an axis \( \varphi \) and 25 ones on an axis \( z \). Each time, when condition \( p > 0 \) during calculation is not satisfied in any point of a difference grid, the pressure in this point was considered as zero. All integrals were evaluated by Simpson method.

The distribution of the hydrodynamic pressure in the lubricant layer with sources of lubricants on the bearing surface is shown at Fig. 8.
Amplitudes of the rotor fluctuation. The speed of the rotor rotation also influences to amplitudes of rotor fluctuations. Additionally, with increase in frequency of rotation the rotor fluctuations form changes too. Changing the position of the rotor elements of the turbocharger with time is shown at Fig. 10. At increase the speed of the rotor rotation from 3000 to 9000 1/s amplitudes of fluctuations of its elements decreased by 2.5 times. The form of fluctuations changes from conic to the cylindrical. The further increase in the rotor speed leads to sharp increase in amplitudes.

During numerical researches of the thrust bearing the new technical solution was proposed. It combines advantages of "Rayleigh's step" and the friction surface with laser texturing. The schematic diagram of new segment is submitted at Fig. 12.

Advantage of the developed design was shown at the example of comparison of calculation results of the load-capacity and the maximum pressure for four forms of segments: I – laser texturing of the surface, II – the inclined surface, III – "Rayleigh's step", IV – "Rayleigh's step" with laser texturing of the surface (Table 1).

### Table 1. Results of calculation of the thrust bearing.

<table>
<thead>
<tr>
<th>characteristics</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{max}/P_{maxIV}$</td>
<td>0.62</td>
<td>0.66</td>
<td>0.99</td>
<td>1</td>
</tr>
<tr>
<td>$W/W_{IV}$</td>
<td>0.47</td>
<td>0.59</td>
<td>0.60</td>
<td>1</td>
</tr>
</tbody>
</table>

9. CONCLUSION

Non-Newtonian properties of the lubricant have a significant impact on the stability of flexible rotors of turbo-machines. The calculation results showed that the properties of the lubricant affect on the trajectory of moving parts of the bearing and on its hydro-mechanical
Characteristics. The threshold speed, which corresponds to the transition from the conical shape to a cylindrical shape of the oscillation, is reduced by using non-Newtonian fluids.

Accounting for one of the properties of the lubricant does not reflect the processes taking place in the thin lubricating film. Each of these properties of lubrication and the dependence of the viscosity on one (or more) of the parameters improves or worsens the hydro-mechanical characteristics of friction units. Therefore, the choice of the rheological models used in the calculation of friction units of machines, depending on the type, operating conditions and lubricant tribo-unit.

Results of the authors and other researchers show that the properties of lubricants affect the characteristics of friction units as much as a geometric factor. However, the parameters of rheological models should be determined on the basis of experimental data for each grade of lubricants.

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