Revista de Sistemas de Informação da FSMA
n. 19 (2017) pp. 53-61

SISTEMAS DE INFORMAÇÃO
http://www.fsma.edu.br/si/sistemas.html

TRILHA PRINCIPAL

Solution Methods for the Integrated Production Routing Problem

Henrique Hiroshi Motoyama Watanabe, *Instituto de Ciências Matemáticas e de Computação - USP*,
Rafael Bernardo Zanetti Cirino, *Instituto de Ciências Matemáticas e de Computação - USP*,
Willy Alves de Oliveira Soler, *Instituto de Matemática - UFMS*,
Maristela Oliveira Santos, *Instituto de Ciências Matemáticas e de Computação - USP*

Solution Methods for the Integrated
Production Routing Problem

***Abstract*—This paper addresses the Production Routing Problem (PRP). In this problem we make integrated decisions about the number of products to be manufactured, the number of products to be delivered to each customer and the routes of the vehicles used to deliver the manufactured products. The integrated problem attempts to minimize both production and distribution costs, over all periods of a finite planning horizon. An Integer Programming Model is presented and model-based heuristics are proposed to tackle the PRP. Computational results are shown for both benchmark and proposed instances, in order to compare the proposed solution method with a high-performance commercial solver. The results show that our approach outperforms the commercial solver in getting high-quality solutions in instances with more than 10 customers and 5 products.**

***Index Terms*—Production Planning, Distribution Scheduling, Vehicle Routing, Production Routing Problem, Integer Programming.**

I. Introduction

THE lot sizing and distribution problems, found in various industries, are classical integer programming problems that were introduced over 50 years ago by [28] and [14], respectively. The increased competition between companies lead to a search for more efficient solutions, culminating in an integration of both problems - the Production Routing Problem (PRP). The PRP consists of minimizing both production and distribution costs, over all periods of a finite planning horizon ensuring that the customer demands are met while respecting the limits of production capacity and vehicles capacity. This problem has been addressed many times in literature, for example [1], [5], [7], [8], [9], [10], [12].

The lot sizing problem defines the size of each production lot, in order to meet customers demand at minimal cost, which usually comprises production, inventory hold and setup costs. An extensive review of lot sizing problems can be found in [20]. The lot sizing part of the problem dealt with in this paper considers a finite horizon, with multiple products and a single capacitated production plant. The demand is dynamic (varies over the planning horizon) and previously known (deterministic) and backlogging is not allowed.

On the other hand, the distribution part of the problem consists of determining the quantity of each product to be delivered at each customer for each period of the planning horizon as well as defining the routes to be adopted for each vehicle of a known fleet aiming to minimize the costs.

When the lot sizing and distribution problems are solved hierarchically, i.e., a model is used to determine the size of the production lots and the solution found solving this lot sizing model is used as input data for another model with the aim of determining the distribution scheduling, we usually arrive at a suboptimal solution when looking at a complete integrated approach, which can provide a more efficient production plan and distribution schedule. Thus, the PRP has been addressed more often in the literature, since its solution can lead to higher profits in the supply chain.

The PRP can be classified by some characteristics, as the number of products (single or multiple), the number of production plants (single or multiple), the production capacity (capacitated or uncapacitated) and the type of distribution fleet (homogeneous, heterogeneous or outsourced). In particular, this paper addresses the problem with a single production plant with a limited capacity to production which has to be used to manufacture multiple products which are delivered to customers using homogeneous vehicle fleet without backlogging or partial delivery. These features also were considered in [6].

For each period in the planning horizon the decisions to be made are the following: (i) how much of each product to be manufactured; (ii) how much of each product should be delivered at each customer; and (iii) the route of each vehicle in the fleet, i.e., which customer will be served by each vehicle. The problem aims to meet the customer demands with no delay, taking into consideration production and vehicle capacity aiming to minimize costs (inventory, setup, and distribution). Vehicles have a route

Correspondece author: Henrique Hiroshi Motoyama Watanabe, *hhwatanabe@usp.br*

maximum length and cannot be refilled in the production plant during a period and each customer can be visited, at maximum, one time per period.

One of the contributions by this paper is the proposed model-based heuristics for the PRP. Initially, a solution is given by a relax-and-fix procedure, followed by a fix-and-optimize improvement procedure applied in the initial solution enhancing it iteratively by solving small subproblems. We could not find a benchmark of instances available in the literature with more than one product. So in order to test our solution method, a set of instances with multiple products was generated (with up to 15 customers, 14 periods and 5 products). Computational tests show that, given a time limit, the proposed solution method can provide better solutions than a high-performance commercial solver.

The remainder of this paper is organized as follows. A brief literature review is shown in Section II, while Section III presents the mathematical model for the problem and in Section IV the model-based solution methods are proposed. In Section V, a set of instances is proposed and the computational results are presented and discussed. Finally, the conclusions and future research indications are shown in Section VI.

II. Literature Review

In this Section a brief literature review is shown, taking into consideration some papers that tackle the PRP.

[17] studied a problem with multiple products, a single capacitated production plant, and homogeneous fleet. In this paper partial delivery is allowed, i.e., a single customer can receive more than one vehicle in the same period. Each vehicle is bound to a single route per period, i.e., the vehicle can not be refilled during the period. A Lagrangian relaxation over the coupling constraints was proposed and computational results shows that the integrated approach achieves solutions with lower cost.

[1], [2] [5], [7], [8], [9] and [10] addressed the PRP with a single product, a single capacitated production plant and homogeneous fleet with route length limit per vehicle in each period. Besides those characteristics, partial delivery is considered in [4].

A two-phase hierarchical heuristic is proposed for the PRP in [8]. In the first phase, the lot sizing problem of the PRP is solved by a dynamic programming algorithm (proposed by [28]). Next, the distribution problem is solved using the solution of the first phase as input. In the second phase, a procedure readjusts the production planning according to the route found, thus providing a small measure of integration between both problems.

In [9] a Greedy Randomized Adaptive Search Procedure - GRASP metaheuristic is proposed to deal with the PRP. Initially, the production planning and vehicle routing are separately tackled for each period of the planning horizon. In a second phase, local search procedures reschedule production, aiming to avoid unnecessary setup costs.

A memetic algorithm with dynamic population management is developed by [10]. In a memetic algorithm, an initial population (a set of solutions) is randomly generated, then they are modified by operators. Two incumbent solutions are iteratively generated by mixing they characteristics, forming new solutions. A local search procedure is called to improve the generated solutions. The incumbent population is subjected to proposed criteria to decide which solutions will compose the next population.

An Op-ALNS heuristic is introduced in [2], consisting in a variation of Adaptive Large Neigborhood Search - ALNS proposed in [26]. First, several initial solutions are generated by solving the production and distribution subproblems, then a local search procedure is applied on those initial solutions. This approach reached good quality solutions for the proposed instances of [5].

A two-phase iterative method is proposed in [1]. In the first phase, a production problem with visit costs is solved, which determines production scheduling and customer delivery. In the second phase, the available vehicles are optimized by solving a series of Travelling Salesman Problems. The visit costs are updated in this phase and, if needed, a new iteration is made. While the stop criteria (usually the maximum number of iterations or maximum number of iterations without improvement) is not met the procedure continues. A commercial solver was employed on the first phase problem, while the heuristics proposed in [22] was used in the second phase. Computational results show that this method finds good solutions for the instances proposed in [5].

Problems based on real data with multiple production plants are addressed in [21], [24] and [15]. In [21] and [24] the problem has heterogeneous vehicle fleet, while in [15] the vehicle fleet is outsourced.

A maritime distribution problem was tackled by [21], in which a two-phase method is proposed. In this method, initially, a commercial solver is used to solve the integrated problem with direct routes, i.e., vehicles can only go from the depot to the customer, with no route with more than one customer. In case the solution found is feasible, it provide a primal bound for the problem. In the second phase, two heuristics are employed to improve first phase's routing. The first heuristic swaps the vehicles for each customer, while the second one uses the closest neighbor method to rebuild the routes, starting from the "best" vehicle, until it is at its limit, moving to the next one. Two criteria are proposed to rank the vehicles, speed and cost over distance. Computational tests compare the performance of the proposed method with a commercial solver.

A problem from the furniture industry with time windows is addressed in [15]. In this problem backlogging is allowed, as well as the transfer of products from a plant to another, penalized by a dislocation cost. Customers are considered to be internet retail buyers, henceforth, and, there is no inventory at the customers. An Integer Programming commercial solver is employed in [15] to deal with the proposed model and computational results show that, although the method has not reached any optimal solution, it still have good results with a fixed one-hour running time. The problem of production and distribution of pork ration is addressed in [24] which developed a two-phase heuristic to deal with the problem.

The production routing problem with multiple products, single plant, homogeneous fleet and no partial delivery is addressed in [6] and [11]. A Tabu Search is proposed in [6] and results are compared with other papers ([10] and [7]). A hybrid relax-and-fix heuristic was proposed in [11] for the PRP. First, the lot sizing problem is solved by the relax-and-fix procedure, followed by a local search (proposed in [18]) in order to optimize routes and make adjustments to the lot sizes, if necessary.

Extensive reviews of PRP can be found in [3] and [23]. Table I summarizes the reported articles in this paper in regard of the following characteristics: i) number of production plants (single plant in table represented by s or multiple plants by m), ii) number of products (single product by s or multiple by m), iii) type of fleet (homogeneous fleet by hom, heterogeneous by het or outsourced by out), iv) partial delivery allowed (yes or no), and v) backlogging (yes or no).

TABLE I
SELECTED ARTICLES FROM THE LITERATURE OF THE PRP

Paper (Year)	Characteristics				
	i)	ii)	iii)	iv)	v)
Chandra and Fisher (1994)	s	m	hom	yes	no
Fumero and Vercellis (1999)	s	m	hom	yes	no
Boudia et al. (2005)	s	s	hom	no	no
Lei et al. (2006)	m	s	het	yes	no
Archetti et al. (2007)	s	s	hom	yes	no
Boudia et al. (2007)	s	s	hom	no	no
Boudia and Prins (2009)	s	s	hom	no	no
Bard and Nananukul (2009)	s	s	hom	no	no
Archetti et al. (2011)	s	s	hom	no	no
Armentano et al. (2011)	s	m	hom	no	no
Adulyasak et al. (2012)	s	s	hom	yes	no
Piewthongngam et al. (2013)	m	m	het	no	yes
Absi et al. (2014)	s	s	hom	no	no
Brahimi and Aouam (2015)	s	m	hom	no	yes
Darvish et al. (2016)	m	m	out	no	no
This paper (2017)	s	m	hom	no	no

III. Mathematical Model

In this paper we addressed the PRP with the features considered in [6], i.e., multiple products, a single production plant and a homogeneous fleet.

The aim of the problem is to minimize the total cost composed of production, inventory, and distribution costs. Inventory costs differ by location (customer or plant), however, they are period independent. A vehicle has a fixed cost if it is used, plus its routing cost, proportional to the route size. The production has an upper limit (capacity) and backlogging is not allowed. There is a minimum and maximum safety stock in each location. The vehicles have a maximum routing length and can only travel through one route per period, i.e., it is not possible to come back to the plant to refill and still delivery in the same period, and two or more vehicles cannot visit the same customer in the same period.

In order to model the problem we define its representation, parameters and variables. Let $G = (W, E)$ be a complete graph for the problem, where $W = \{0, 1, \ldots, N\}$ are the nodes set including the plant and the customers and $E = \{(k, l) \mid k, l \in W, k \neq l\}$ are the edges set connecting the plant and the customers. The plant, node $k = 0$, manufactures products $j = 1, \ldots, J$ which are delivered to the customers $k = 1, \ldots, N$ by the vehicles $v = 1, \ldots, V$. Indexes, parameters and variables are as follows:

Indexes

- k Index for the plant ($k = 0$) and for customers ($k = 1, \ldots, N$);
- v Index for vehicles ($v = 1, \ldots, V$).
- t Index for periods ($t = 1, \ldots, T$);
- j Index for products ($j = 1, \ldots, J$);

Parameters

- B Plant's capacity;
- b_j Production time, per unit, of product j;
- c_j^p Production cost, per unit, of product j;
- f_j^p Setup cost of product j;
- h_{jk} Unitary inventory cost of product j at customer k;
- L_{jk} Minimum inventory level of product j at customer k;
- U_{jk} Maximum inventory level of product j at customer k;
- C Capacity of each vehicle;
- L Maximum route lenght of each vehicle;
- f^v Usage cost of vehicle v;
- c_{kl}^v Transportation cost at edge (k, l);
- d_{jkt} Demand of customer k for product j in period t;
- M A big number.

Variables

- p_{jt} Amount of product j manufactured in period t;
- I_{jkt} Amount of product j in stock at customer k; in the end of period t;
- q_{jkt}^v Amount of product j delivered at customer k; by vehicle v in period t;
- x_{jklt}^v Amount of product j transported by vehicle v through edge (k, l) in period t;
- y_{jt} $= 1$, if product j is manufactured in period t, and $= 0$, otherwise;
- z_{klt}^v $= 1$, if the vehicle v travels through the edge (k, l) in period t and $= 0$, otherwise.

The mathematical model is given by (1) - (14).

- The objective function (1) represents the sum of the inventory, production, setup and distribution costs;
- Constraints (2) are the inventory balance between production, inventory, and delivery in the plant;
- Constraints (3) performs the balance between delivery, inventory and customer demand;
- Constraints (4) ensure that the plant capacity is respected;
- Constraints (5) ensure that the product can only be manufactured if the plant was set up to the production of this product;
- Constraints (6) and (7) represent the conservation of flow between the factory and customers;
- Constraints (8) ensure the vehicle capacity and (9)

guarantee the limit route length;
- Constraints (10) sets at most one route per vehicle at each period and constraints (11) assure each vehicle route should end at the plant;
- Constraints (12) ensure that each customer is visited at most by one vehicle in each period, not allowing partial delivery;
- Constraints (13) assure the safety stock at each customer. Constraints (14) define the domain of the decision variables;
- Constraints (6) and (7) eliminate sub-routing [17], which is when a vehicle passes in the same customer twice or more in the same period;
- From constraints (6) and the non-negativity of the variable q^v_{jkt}, the possibility of a vehicle picks up products from a customer and delivery to another is eliminated.

IV. Solution Methods

The PRP is a NP-Hard problem. Therefore, large instances, sometimes based on real world data, cannot be solved using exact algorithms within an acceptable computational time. Hence, many different heuristic approaches have been proposed in the literature to deal with this challenging problem. In this paper, we propose a relax-and-fix heuristic to find an initial solution to be used as a starting point of a fix-and-optimize procedure to enhance it. Both relax-and-fix and fix-and-optimize heuristics have been widely used in the literature to deal with production planning problems.

Those heuristics are based on a decomposition, $Q(1),\ldots,Q(R)$, of the set of all binary and integer decision variables (Q). Thereby, in each heuristic iteration, we solve a small mixed integer subproblem in which a single subset $Q(r)$, $r \in \{1,\ldots,R\}$ has its binary variables optimized and the variables of the other subsets are fixed in the incumbent value or linearly relaxed. As the number of binary variables in each subset $Q(r)$ is significantly smaller than the number of binary variables of the set Q, each subproblem can be solved in a fairly reduced time. In its simplest form (without overlapping), the relax-and-fix heuristic takes a partition of the set Q, i.e., we take subsets $Q(1),\ldots,Q(R)$ of Q that satisfy the conditions:

$$Q(1) \cup Q(2) \cup .. \cup Q(R) = Q \tag{15}$$

$$Q(i) \cap Q(j) = \emptyset \quad \forall i \neq j. \tag{16}$$

The number of iterations for the relax-and-fix procedure is equal to the number of subsets for the adopted partition. For each iteration $r = 1,\ldots,R$, the variables of the set $Q(i)$ with $i < r$, are fixed in their incumbent value (found from previous iterations), while variables of sets $Q(j)$ with $j > r$ are linearly relaxed and a subproblem is solved considering just the variables of the set $Q(r)$ as binary and/or integer. However, it should be noticed that the variables in the sets $Q(j)$ with $j > r$ are also optimized, but linearly relaxed. If a feasible sub-solution is found for each iteration, then the final solution is also feasible for the original problem.

There are also some relax-and-fix procedures which apply an overlapping technique in order to find better solutions. In those procedures, we consider some variables as binary and/or integer in a given iteration r, without fixing them in the next iteration $(r+1)$. Overlapping procedures tend to find a better solution at the end, as their subproblems are closer to the original problem. Nevertheless, they may consume much more computational time, since each subproblem has a greater number of binary and/or integer variables to be optimized.

Fix-and-optimize heuristic (proposed in [25]) is an improvement procedure that starts at a given initial feasible solution and, in each iteration, a small subproblem is solved, aiming to find a better solution. As the relax-and-fix, the fix-and-optimize heuristic takes a decomposition of the set of all binary and/or integer variables (Q) of the problem, and in each iteration, only the variables of a particular subset of Q are (re)optimized, while the others are fixed on its incumbent value.

When all variables that will be (re)optimized at each iteration are chosen randomly, it is said that the fix-and-optimize procedure is stochastic. However, when all variable subsets are previously known, it is said to be a deterministic procedure. The procedures developed by [19] and [13] show the advantage of using a stochastic approach in mip-based heuristics. For lot sizing problems, the decompositions based in periods or products are commonly found in the literature. The most common stop criterias are: reaching the maximum running time, reaching the maximum number of iterations, and the realization of some iterations without improvement in objective function value.

In this paper, a relax-and-fix heuristic is proposed to find an initial feasible solution, which will be used as input for a deterministic fix-and-optimize procedure. For both heuristics a decomposition over the period is used. More specifically, if, in a given iteration, the period t is selected to have its variables (re)optimized, then all variables $y_{jt}, \forall j$ and $z^v_{klt}, \forall v,k,l$ will be (re)optimized.

For each procedure, we introduce the parameters α and β to represent the number of consecutive periods in each decomposition, and the number of consecutive periods of overlapping, respectively. Preliminary tests were performed, for both heuristics, with α varying from 1 to 3 and β varying from 0 to 3 ($\beta = 0$ means the decomposition has no overlapping). We limited their values to 3 so the subproblems would stay small compared to the original problem.

V. Computational Results

In this Section, we present the computational results found by the proposed heuristics and compare with the results found by the commercial solver Cplex 12.6. In Subsection V-A, we presents the test instances used to perform the tests and in Subsection V-B, we present and discuss the computational results.

A. Instance Generation

Multiple products instances based on [6] were generated for computational tests. Instances have

$(N, T, J) \in (\{5, 10, 15\}, \{7, 14\}, \{3, 5\})$. For each combination (N, T, J), 10 instances were generated, in a total of 120 instances. Both instances and computational test results can be found in:

http://www.otm.icmc.usp.br/index.php/pt/pesquisa/instancias-de-teste.

Some instances proposed in [5] were also used to perform the tests. [5] addressed a PRP with a single product, a single plant with limited capacity and homogeneous fleet. The instance set with 14 customers and 6 periods was considered in this paper. The set is further divided into 4 classes with 120 instances each, with a different set of considered costs. Taking class 1 as the base, classes 2 and 3 have higher production and distribution costs, respectively, while class 4 has no inventory cost whatsoever. Both instances and results can be found in https://sites.google.com/site/ayossiri/publications .

The implementation for the model and heuristics were made in C++ with the solver IBM CPLEX 12.6. Computational tests were performed on a node of the Cluster Euler (CeMEAI-CEPID-FAPESP Process 132404/2014-1 and 476792/2013-4) equipped with 2 Intel Xeon E5-268v2 Processors and a 128GB RAM Memory. For all approaches the time limit to solve each test instance was arbitrarily set to one hour.

B. Computational Tests

Preliminary tests were performed, testing different decompositions for both heuristic procedures. Different combinations for α_{RF}, α_{FO}, β_{RF} and β_{FO} were tested in a small set of the generated instances and the results were evaluated with a performance profile based on [16], in order to set $\alpha_{RF} = 1$, $\beta_{RF} = 3$, $\alpha_{FO} = 3$ and $\beta_{FO} = 1$.

The performance profile of a given heuristic is a probability density function calculated over its performance in comparison with other heuristics. For each instance the best performance of all heuristics was found and a distance between performances defines a radius of performance for each heuristics. They defined the quotient $r_{p,h} = \frac{s_{p,h}}{min\{s_{p,h}|h \in H\}}$ to measure that distance, where p is the instance, h is the heuristic and H is the set of all heuristics. That quotient is then used to calculate the probability density function for all heuristics, with $\rho(h, \tau)$ being the percentage of instances that $r_{p,h} \leq \tau$, i.e. $\rho(h, 1)$ is the percentage of best solutions found by heuristic h.

For both the mathematical model and proposed heuristics with the given combination, results found for the generated instances are summarized in Table II. The first column shows the instance size, in the second column the average gap for the initial relax-and-fix solution is shown, the third column the average gap for the solution after the fix-and-optimize procedure is shown, the fourth column shows the average gap found by the Cplex solver and the fifth column shows average running time of the proposed method. The running time of the CPLEX for almost all instances achieved the time limit so, this value is not showing in Table II. The number shown between parenthesis in both second and third columns indicates the number of instances which no initial solution was found and the rows with * are averages ignoring the tests where the heuristic could not find an initial solution. It is noteworthy that CPLEX finds feasible solutions for all proposed instances. Over all 120 instances, the proposed heuristic finds feasible solutions for 90% of them.

As shown in Table II, the combined RF+FO found better solutions than the Cplex solver, although was less efficient than the Cplex to find feasible solutions for bigger instances. For the 10 instances with $(N/V, T, J) = (15, 14, 5)$, the RF+FO heuristic only found feasible solutions for 3 of them. However Cplex only found optimal solutions in the first and second instance groups, $(5, 7, 3)$ and $(5, 7, 5)$, in an average time of 2607 seconds and 3466 seconds, respectively. For the third group onwards the one-hour time limit was reached by the Cplex for all instances.

From the information between the second and third columns in Table II, it is noticeable the improvement provided by the fix-and-optimize procedure over the initial solution found by the relax-and-fix procedure, with bigger improvement for larger instances as expected, because the initial solution has a larger gap and therefore more potential to improve. To compare the performance of the methods the Figure 1 is shown, for the performance profile over the instances solved by both methods.

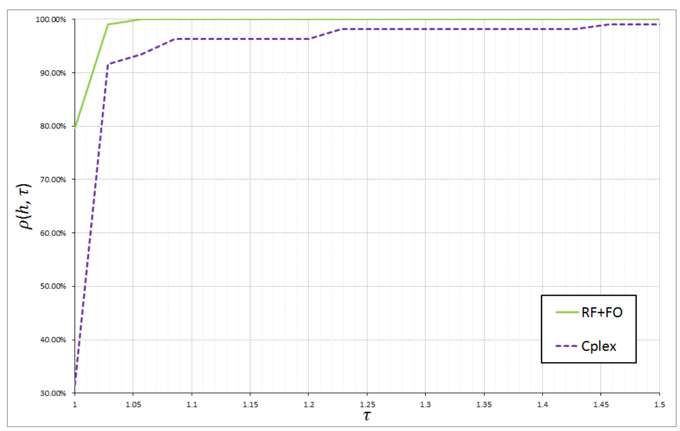

Fig. 1. Performance profiles (Dolan and Moré (2002)) - Solved generated instances.

In Figure 1, function $\rho(h, \tau)$ shows the percentile of instances for which the method h finds a solution less or equal to $\tau * S$, where S was the lowest value found by the compared methods. Therefore, when $\tau = 1$, $\rho(h, \tau)$ indicates the percentile of instances for which the method h finds the best solution.

It is noteworthy that $\sum_h \rho(h, \tau)$ can be greater than 1, as long as two or more methods find the best solution in at least one instance. The RF+FO performance curve reaches 100% at $\tau = 1.06$ approximately, which means that the proposed method is within at most at 6% of the best solution in all instances.

After the tests performed considering the generated instances, the instances in [5] were also tackled. Table

TABLE II
Heuristics and Cplex gaps - generated instances

Instances (N/V, T, J)	RF average gap	RF+FO average gap	Cplex average gap	RF+FO average time
(5, 7, 3)	3.45%	3.28%	3.04%	818.9
(5, 7, 5)	3.00%	2.87%	2.79%	888.1
(5, 14, 3)	4.96%	4.81%	4.86%	1586.2
(5, 14, 5)	4.61%	4.36%	4.81%	2505.9
(10, 7, 3)	7.79%	7.46%	8.19%	2552.7
(10, 7, 5)	5.76%	5.33%	6.60%	2594.1
(10, 14, 3)	6.03%	5.62%	6.99%	3600.0
(10, 14, 5)	6.57%	5.57%	7.96%	3600.0
(15, 7, 3)	8.77% (1)	6.35% (1)	7.21%	3442.6
(15, 7, 3)*	8.73%	6.35%	7.29%	3442.6
(15, 7, 5)	8.73%	6.75%	7.01%	3600.0
(15, 14, 3)	11.90% (4)	6.21% (4)	14.55%	3600.0
(15, 14, 3)*	11.90%	6.21%	19.59%	3600.0
(15, 14, 5)	27.35% (7)	0.66% (7)	25.85%	3600.0
(15, 14, 5)*	27.35%	0.66%	22.87%	3600.0

III summarizes the results found in the 14 customers instances. Gaps were calculated using the solution values given in [5]. T_{min} and T_{max} are both the smallest and biggest running time for each of the 120 instances of the correspondent class, and column Time > 300s indicates how many instances took more than 5 minutes to be solved. The average time was calculated excluding the instances that took more than 5 minutes, considering them outliers.

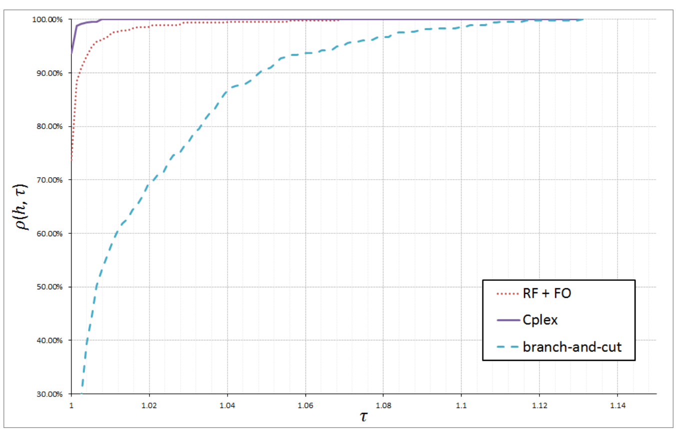

Fig. 2. Performance profile (Dolan and Moré (2002)) - Instances from (Archetti et al. (2011)) - 14 customers

As those instances have a small dimension, given the number of customers, Cplex could find the optimal solution in the majority of them, however as shown by column T_{max} it could not prove optimality in some cases and passed the time limit (see Table III). Branch-and-cut ([5]) was the fastest method, disregarding outliers, however with the biggest gap, worse than the initial solutions given by the relax-and-fix procedure. Although RF+FO is not as fast as the other two methods, it did not pass the time limit at any of the instances, while also finding solutions very close to the optimal values.

Figure 2 shows performance profile of the methods over the instances of [5] with 14 customers. The average gap difference is evident, with the curve for the branch-and-cut far below from the other two. While Cplex found the best solution in almost 95% instances, RF+FO finds success in approximately 75% instances, however it is at most 2% from the best solution in more than 97% of the tests.

VI. Conclusions and Future Works

In this paper we addressed the integrated Production Routing Problem. A mathematical model was presented and a solution method based on the mathematical formulation of the problem was proposed. Tests over generated instances with multiple products and instances from [5] were performed and showed that the proposed approach achieved better results than a commercial high-performance solver for medium size instances.

At a time limit of 1 hour, the proposed method is shown to be competitive in comparison to the Cplex solver, with a small disadvantage at small instances, but surpassing the Cplex in medium size instances. However for instances with 15 or more customers the proposed approach does not find good solutions.

Tests over instances from [5] confirms the tendencies observed in previous tests, RF+FO is faster than Cplex in smaller instances, however, it didnt́ find the optimal solutions to all considered instances. In comparison with the branch-and-cut algorithm ([5]), RF+FO found better solutions with more computational time. The heuristic was more consistent and only surpassed the time limit of 5 minutes in 10 tests (Table III), approximately 2.1%, with a maximum time of approximately 10 minutes, while branch-and-cut would take more than 2 hours for some instances in case a time limit was not set. In a real application, sometimes it is necessary to solve the problem several times, so, to find good solutions in less computational time is of the utmost importance.

For future works, we indicate further research in other methods to find initial solutions, given that the proposed method fails in finding initial solutions for some generated instances, and, for large size instances, the RF heuristic might take a high computational time. Also, other local search procedures, like ALNS or the fix-and-optimize with variable neighborhood search (VNS) proposed in [13] could lead to better solutions.

TABLE III
RESULTS FOR THE INSTANCES OF (ARCHETTI ET AL. (2011)) - 14 CUSTOMERS

	Class	Average Gap	Average Time	T_{min}	T_{max}	Time >300s
branch-and-cut [5]	1	2.131%	27.5	1	7200	12
	2	0.298%	11.3	0	115	0
	3	3.433%	35.5	0	1863	8
	4	0.923%	11.6	1	7201	7
Cplex	1	0.000%	44.1	4	3607	12
	2	0.000%	35.7	2	3604	7
	3	0.000%	81.4	12	3607	19
	4	0.017%	37.4	9	3607	24
RF	1	0.501%	—	—	—	—
	2	0.042%	—	—	—	—
	3	2.377%	—	—	—	—
	4	0.408%	—	—	—	—
RF+FO	1	0.054%	52.0	13	244	0
	2	0.009%	50.1	9	235	0
	3	0.343%	77.6	19	617	7
	4	0.032%	76.6	15	430	3

REFERENCES

[1] Absi, Nabil, et al. "A two-phase iterative heuristic approach for the production routing problem." Transportation Science 49.4 (2014): 784-795.

[2] Adulyasak, Yossiri, Jean-François Cordeau, and Raf Jans. "Optimization-based adaptive large neighborhood search for the production routing problem." Transportation Science 48.1 (2012): 20-45.

[3] Adulyasak, Yossiri, Jean-François Cordeau, and Raf Jans. "The production routing problem: A review of formulations and solution algorithms." Computers & Operations Research 55 (2015): 141-152.

[4] Archetti, Claudia, et al. "A branch-and-cut algorithm for a vendor-managed inventory-routing problem." Transportation Science 41.3 (2007): 382-391.

[5] Archetti, Claudia, et al. "Analysis of the maximum level policy in a production-distribution system." Computers & Operations Research 38.12 (2011): 1731-1746.

[6] Armentano, Vinícius Amaral, A. L. Shiguemoto, and Arne Løkketangen. "Tabu search with path relinking for an integrated production-distribution problem." Computers & Operations Research 38.8 (2011): 1199-1209.

[7] Bard, Jonathan F., and Narameth Nananukul. "The integrated production - inventory - distribution - routing problem." Journal of Scheduling 12.3 (2009): 257-280.

[8] Boudia, Mourad, Mohamed Aly Ould Louly, and Christian Prins. "Combined optimization of production and distribution." CD-ROM proceedings of the international conference on industrial engineering and systems management, IESM. Vol. 5. 2005.

[9] Boudia, Mourad, Mohamed Aly Ould Louly, and Christian Prins. "A reactive GRASP and path relinking for a combined production-distribution problem." Computers & Operations Research 34.11 (2007): 3402-3419.

[10] Boudia, Mourad, and Christian Prins. "A memetic algorithm with dynamic population management for an integrated production/distribution problem." European Journal of Operational Research 195.3 (2009): 703-715.

[11] Brahimi, Nadjib, and Tarik Aouam. "Multi-item production routing problem with backordering: a MILP approach." International Journal of Production Research (2015): 1-18.

[12] Chandra, Pankaj, and Marshall L. Fisher. "Coordination of production and distribution planning." European Journal of Operational Research 72.3 (1994): 503-517.

[13] Chen, Haoxun. "Fix-and-optimize and variable neighborhood search approaches for multi-level capacitated lot sizing problems." Omega 56 (2015): 25-36.

[14] Dantzig, George B., and John H. Ramser. "The truck dispatching problem." Management science 6.1 (1959): 80-91.

[15] Darvish, Maryam, Homero Larrain, and Leandro C. Coelho. "A dynamic multi-plant lot-sizing and distribution problem." International Journal of Production Research 54.22 (2016): 6707-6717.

[16] Dolan, Elizabeth D., and Jorge J. Moré. "Benchmarking optimization software with performance profiles." Mathematical programming 91.2 (2002): 201-213.

[17] Fumero, Francesca, and Carlo Vercellis. "Synchronized development of production, inventory, and distribution schedules." Transportation science 33.3 (1999): 330-340.

[18] Groër, Chris, Bruce Golden, and Edward Wasil. "A library of local search heuristics for the vehicle routing problem." Mathematical Programming Computation 2.2 (2010): 79-101.

[19] James, Ross JW and Almada-Lobo, Bernardo. Single and parallel machine capacitated lotsizing and scheduling: New iterative MIP-based neighborhood search heuristics. Computers & Operations Research 38.12 (2011): 1816-1825.

[20] Karimi, Behrooz, SMT Fatemi Ghomi, and J. M. Wilson. "The capacitated lot sizing problem: a review of models and algorithms." Omega 31.5 (2003): 365-378.

[21] Lei, Lei, et al. "On the integrated production, inventory, and distribution routing problem." IIE Transactions 38.11 (2006): 955-970.

[22] Lin, Shen, and Brian W. Kernighan. "An effective heuristic algorithm for the traveling-salesman problem." Operations research 21.2 (1973): 498-516.

[23] Moons, Stef, et al. "Integrating production scheduling and vehicle routing decisions at the operational decision level: a review and discussion." Computers & Industrial Engineering (2016).

[24] Piewthongngam, Kullapapruk, Supachai Pathumnakul, and Suphakan Homkhampad. "An interactive approach to optimize production-distribution planning for an integrated feed swinecompany." International Journal of Production Economics 142.2 (2013): 290-301.

[25] Pochet, Yves, and Laurence A. Wolsey. Production planning by mixed integer programming. Springer Science & Business Media, 2006.

[26] Ropke, Stefan, and David Pisinger. "An adaptive large neighborhood search heuristic for the pickup and delivery problem with time windows." Transportation science 40.4 (2006): 455-472.

[27] Sahling, Florian, et al. "Solving a multi-level capacitated lot sizing problem with multi-period setup carry-over via a fix-and-optimize heuristic." Computers & Operations Research 36.9 (2009): 2546-2553.

[28] Wagner, Harvey M., and Thomson M. Whitin. "Dynamic version of the economic lot size model." Management science 5.1 (1958): 89-96.

Henrique Hiroshi Motoyama Watanabe (correspondece author) received the M.Sc. degree in Computational Mathematics from the University of São Paulo. E-mail: *hhwatanabe@usp.br*.

Rafael Bernardo Zanetti Cirino received the M.Sc. degree in Computational Mathematics from the University of São Paulo and is enrolled as a Ph.D. student at University of São Paulo seeking a degree in Computational Mathematics.

Willy Alves de Oliveira Soler received the M.Sc. degree in Mathematics from Federal University of the Mato Grosso do Sul and is enrolled as a Ph.D. student at University of São Paulo seeking a degree in Computational Mathematics. He is also Professor at Federal University of Mato Grosso do Sul at the Mathematics Institute. E-mail: *willy.oliveira@ufms.br*.

Maristela Oliveira Santos received both the M.Sc. and Ph.D. degrees in Computational Mathematics from the University of São Paulo and is now a Professor at University of São Paulo. E-mail: *mari@usp.br*.

$$\text{Minimize} \quad \sum_{t=1}^{T}\{\sum_{j=1}^{J}[\sum_{k=0}^{N} h_{jk}I_{jkt} + c^{p}_{jt}p_{jt} + f^{p}_{j}y_{jt}] + \sum_{v=1}^{V}[\sum_{l=1}^{N} f^{v}z^{v}_{0lt} + \sum_{l=0,l\neq k}^{N} c^{v}_{kl}z^{v}_{klt}]\} \quad (1)$$

$$\text{subject to:} \quad p_{jt} + I_{j0,t-1} - I_{j0t} = \sum_{k=1}^{N}\sum_{v=1}^{V} q^{v}_{jkt} \quad 1 \leq t \leq T, 1 \leq j \leq J \quad (2)$$

$$\sum_{v=1}^{V} q^{v}_{jkt} + I_{jk,t-1} - I_{jkt} = d_{jkt} \quad 1 \leq t \leq T, 1 \leq j \leq J, 0 \leq k \leq N \quad (3)$$

$$\sum_{j=1}^{J} b_j p_{jt} \leq B \quad 1 \leq t \leq T \quad (4)$$

$$p_{jt} \leq M y_{jt} \quad 1 \leq t \leq T, 1 \leq j \leq J \quad (5)$$

$$\sum_{\substack{i=0\\ i\neq k}}^{N} x^{v}_{jikt} - \sum_{\substack{m=0\\ m\neq k}}^{N} x^{v}_{jkmt} = q^{v}_{jkt} \quad 1 \leq t \leq T, 1 \leq j \leq J, 1 \leq v \leq V, 0 \leq k \leq N \quad (6)$$

$$\sum_{i=1}^{N}\sum_{v=1}^{V} x^{v}_{ji0t} - \sum_{m=1}^{N}\sum_{v=1}^{V} x^{v}_{j0mt} = -\sum_{k=1}^{N}\sum_{v=1}^{V} q^{v}_{jkt} \quad 1 \leq t \leq T, 1 \leq j \leq J \quad (7)$$

$$\sum_{j=1}^{J} x^{v}_{jklt} \leq C z^{v}_{klt} \quad 1 \leq t \leq T, 1 \leq v \leq V, 0 \leq k, l \leq N \mid k \neq l \quad (8)$$

$$\sum_{k=0}^{N}\sum_{\substack{l=0\\ l\neq k}}^{N} c^{v}_{kl}z^{v}_{klt} \leq L \quad 1 \leq t \leq T, 1 \leq v \leq V \quad (9)$$

$$\sum_{k=1}^{N} z^{v}_{0kt} \leq 1 \quad 1 \leq t \leq T, 1 \leq v \leq V \quad (10)$$

$$\sum_{\substack{i=0\\ i\neq k}}^{N} z^{v}_{ikt} - \sum_{\substack{m=0\\ m\neq k}}^{N} z^{v}_{kmt} = 0 \quad 1 \leq t \leq T, 1 \leq v \leq V, 0 \leq k \leq N \quad (11)$$

$$\sum_{k=0}^{N}\sum_{v=1}^{V} z^{v}_{klt} \leq 1 \quad 1 \leq t \leq T, 0 \leq l \leq N \quad (12)$$

$$L_{jk} \leq I_{jkt} \leq U_{jk} \quad 1 \leq t \leq T, 1 \leq j \leq J, 0 \leq k \leq N \quad (13)$$

$$p_{jt}, q^{v}_{jkt}, x^{v}_{jklt} \geq 0, y_{jt}, z^{v}_{klt} \in \{0,1\}, 1 \leq t \leq T, 1 \leq j \leq J, 0 \leq k, l \leq N \quad (14)$$