TRENDS AND CHALLENGES OF MATHEMATICS EDUCATION IN MOZAMBIQUE (1975-2016)

Luca Bussotti
International Studies Centre of ISCTE-IUL, Portugal
Technical University of Mozambique, Mozambique
E-mail: Luca.Bussotti@iscte-iul.pt

Paolo Bussotti
University of Udine, Italy
E-mail: paolobussotti66@gmail.com

Abstract

Mathematics has always been a difficult issue, especially in the African countries. Mozambique is not an exception. This country had been colonized by Portugal until 1975. When the independence was obtained, a socialist regime was adopted (1977). The learning of mathematics entered the struggle against colonial and imperialistic ideas. Its best ally was Paulus Gerdes, one of the most relevant ethnomatematicians of the world, who carried out an intense promotion of this approach to mathematics in Mozambican school system. Albeit the great international impact of Gerdes' ideas, Mozambique never implemented his methodology. When, at the end of the 80s, the country changed from socialism to liberalism, voting a democratic Constitution in 1990, its school system was aligned to the measures of International Monetary Fund (IMF) and World Bank (WB). The most recent ones are represented by the Millennium Development Goals. Despite the various reforms of Mozambican school system, the results of Mozambican children in mathematics are among the worst in Africa. The reasons of such a failure are here explained, through a historical approach based on national documents. The most recent experiences of school reform carried out by international agencies together with national institutions are stressed. The negative results obtained by the Mozambican learners as to mathematics are due to several reasons: 1) a lack of consideration of the Mozambican cultural substrate; 2) an improper massification of the school system, where the quality of instruction has been neglected; 3) the specific choice to marginalize mathematics education.
Keywords: *ethnomatematics, international agencies, mathematics education, Mozambique, school reforms, teaching methods.*

Initial Considerations on Mathematics Education

In the history of didactics, educational methods and science education, no subject has been so profoundly studied as mathematics. There is no doubt: mathematics is difficult and for most of students the learning of mathematics is a problematic question. Furthermore, mathematics is strictly connected with the development of sciences as physics and engineering which have a profound influence on the daily life, so that mathematics education has also, at least in part, to be connected with the changing necessities of society. In the Western world, starting at least from the 19th century, the debate on mathematics education has been huge and has involved some of the most important mathematicians. Such a debate has concerned any age range of the learners: from basic education until mathematics education at the university. Only to give some aspects of a complicated fresco and without any claim to be exhaustive: until the beginning of the 19th century geometry was basically taught by means of a synthetical

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

approach deriving from Euclid. In 1794 the famous French mathematician Adrien Marie Legendre wrote a very successful handbook (Legendre, 1794) for gymnasia and high schools in which the synthetic approach and the theory of proportions were replaced by an arithmetic-algebraic approach. The book was not completely rigorous, but was easier to teach and to learn. Because of Legendre's success, a series of handbooks were published. However, the level of these handbooks was progressively decreasing: there were several mistakes, a complete lack of rigour and the concrete risk to compromise geometry education. Also, as a consequence of this situation, starting from the half of 19th century, a profound debate on the best way to teach geometry was born (Bass, 2005; Bolondi, 2005; Giacardi, 2003; Klein, 1924, 1953; Pepe, 2006; Viola, 1964; Wilson, 1868). There were several tendencies: mathematicians who devised a return to Euclid with some necessary modifications, others who, in contrast to this, judged that Euclid's *Elements* was not a suitable text to be proposed as the basis for geometry education. Furthermore, the second half of the 19th century was the golden age of projective geometry. Stimulated by the success of this branch of mathematics, some mathematicians proposed to introduce projective geometry within the curriculum of gymnasia and high schools. A quite important educational movement was born: the fusionism (Arana, Mancosu, 2012; Borgato, 2006; De Paolis, 1884; Karp, Schubring, 2014). The fusionists believed that plane and spatial geometries had to be taught jointly and not separately. Therefore, the debate on mathematics education was extremely lively. It is enough to think that, thanks to an idea of the important mathematician David Eugen Smith, at the Fourth International Congress of Mathematicians, Rome 6-11 April 1908, the CIEM (International Commission on Mathematical Instruction) was founded. The President was Felix Klein, one of the most important mathematicians ever. Mathematicians as Federigo Enriques and Guido Castelnuovo adhered to the CIEM. Around that period, almost all the main European countries as well as USA founded national commissions to study mathematics education.

After the second world war the "Commission internationale pour l'étude et l'amélioration de l'einseignement des mathématiques" was founded in 1950. Starting from 1955 the Commission began to publish a series of books on mathematics education. The first of these books, whose original French title is *L'einsegnement des mathématiques* (1955) (Piaget, Dieudonné, Lichnerowicz, Choquet, Gattegno, 1955), is particularly significant. The Commission was influenced by Piaget's (1896-1980) idea that the evolutive phases of the person, starting from her/his birth until 15/16 years are modelled by mental structures, acquired in the personal evolution, which are similar to the structures of abstract algebra from a formal and operational point of view. A famous mathematician Jean Dieudonné tried to show that the results obtained in the course of history become clear, from a conceptual and operational point of view, only if they are framed inside groups and rings theory, which, in its turn, is inscribed into the evolutive structure of our mind. A famous pedagogue Caleb Gattegno (1911-1988) tried to connect the use and the functioning of the calculating machines with the elementary arithmetical operations. He used particular configurations as the *tangram*. Furthermore, he also proposed the recourse to audio-visual media in order to clarify some situations and insisted on the fact that the kids must be able to use instruments as their own hands, papers and scissors to construct geometrical figures. This manual activity was considered by him significant for the learners to become confident with the world of geometry. The whole of these ideas gave rise to a series of didactical experiments aimed at founding the best way to teach mathematics at the elementary schools, gymnasia and high school. The idea of these experiments, carried out basically between the '60 and the '70 of the 20th century was to prove that some aspects of modern mathematics, as abstract algebra and topology are based on intuitive knowledge and that an intuitive, manual and visual approach based on some aspects of these disciplines might be, from a didactical point of view, more fruitful than other approaches.

It is also appropriate to remind the reader *The School Mathematics Project*, a remarkable educational contribution – a very encyclopaedia - published in the 60s of the 20th century by the Cambridge University Press, where all sections of mathematics teaching were addressed, for elementary, middle and high schools.

The chosen examples have been selected only to give an idea of a debate on mathematics education which, nowadays is livelier than ever. It is enough to remember that a very recent article on *The New York Times*, 5 August 2017 (Amy Quin, 2017) informs us that in England it is possible to adopt Chinese textbooks of mathematics translated into English because, at all appearances, the way in which mathematics is taught in China is fruitful, as proved by the excellent results of the Chinese children in international mathematical competitions.

The number of international journals specifically dedicated to mathematics education is simply huge and many journals devoted to science education accept papers concerning mathematics education. This means that the problem is on agenda. In principle: mathematics is a discipline founded on abstract and formal bases. This character of mathematics is rapidly increasing. In many contexts, the didactical approach is formal, too. This is particularly true at the university, but this tendency is conspicuous at the level of gymnasia and high schools, too. Basically, an algebraic and analytical approach is preferred to a geometrical, intuitive and visual one, even in teaching geometry. The problem is that, in many countries, the teaching and the learning of mathematics is difficult, and the scholastic achievements of the students are not good. Therefore, there is the attempt to reintroduce visual, manual and intuitive issues to teach mathematics. There are many ideas, albeit it seems difficult to identify precise and full convincing educational lines.

The methodology adopted is based upon a qualitative approach: a historical analysis of public policy related to mathematics education in Mozambique is proposed. The starring point is the contribution of the Holland-Mozambican ethno-mathematician Gerdes. The proposed analysis also offers a panorama of the mathematics educations' existing state in Mozambique based on official documents and reports as well as on few scientific articles which are available. Mathematics education in Mozambique is also compared with that of other African countries, in order to have a clearer idea of the specific problems faced by Mozambique as to this specific subject.

The African Way to Mathematics Education: A Difficult Road

While dealing with mathematics education in Mozambique, the basic data is that – as it will be clarified in what follows – the results as to mathematics in Mozambican schools are, on average, very bad. The teaching of mathematics in Mozambique is based on a single handbook which is similar to the less innovative handbooks used in the Western schools (the prototype of this text are the handbooks used in in Portugal). They are, in many cases, not suitable to get a good mathematics education in the Western countries. Is it thinkable they are for mathematics education in Mozambique? The answer is "no, they are not". Before this phase there was one, in which a new idea of mathematics teaching/learning was proposed; it was based on ethnomathematics. This means – in synthesis, because we will see the thing in some details in the last section of this paper -, that, once conquered the independence from Portugal, the idea was born that a Western and colonial way to teach mathematics existed. This way had the only aim to maintain the local population subjugated to the colonialists. Therefore, it was felt the need to think of a new, ethnic way to conceive and to teach mathematics, in which great part of the art, architecture and games used by the communities in different areas of Mozambique (and in general of Africa) were employed to induce mathematical concepts in the pupils. This set of ideas was particularly lively in the period between the 70s and the 90s of the 20th century. The approach was extremely ideological (Benot, 1972). If one tries to analyse the question in retrospect: from a mere didactical point of view, which were the ideas of the ethnomathematicians? Within which degree their proposals are rationally usable in a well- balanced mathematics education? Basically, in contrast to what the ethnomathematicians thought of, we are convinced that the difficulties met by the Mozambican learners represent – in a more serious way – the difficulties they meet everywhere as to mathematics learning. Therefore, in the last but one section of this research, we will present some proposals of Gerdes

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

(Gerdes, 1991, 2012), an important expert of ethnomathematics who studied the situation in Mozambique, to point out the strong and the week points of his ideas and to show that some proposals to teach mathematics in the Western world, returning to a more visual and intuitive approach, have some points of contact with those of the ethnomathematicians. On the other hand, Gerdes' concept to counterpose ethnomathematics to a Western mathematics seems to us unacceptable and independent from some of Gerdes' didactical ideas which might have some positive aspects.

The main hypothesis to explain the lack of success of mathematics teaching in Mozambican schools depends on the difficult economic and social situation of the whole country. However, to grasp the problems of mathematics education in Mozambique, internal factors have to be considered too.

A Hint to History of Mathematics Education in the Post-Colonial Mozambique

After having obtained their independences, from the end of the 1950s, the African countries began to reflect on the role of education, including mathematics. Mathematics had to be inserted in a general project of political, intellectual and cultural autonomy. The colonial system of education was directed "to prepare the ´natives´ to be used in the administration of the colony" (Mohammed, Halai, Karuku, 2016: 3). In Mozambique, too, mathematics was applied in favour of colonial power (in this case Portugal): for instance, it was used to calculate the "imposto de palhota" (the "hut tax"), the main instrument to withdraw money through a mechanism of taxation that Mozambican peasants were to be able to calculate (Gerdes, 1991, 2012).

In the aftermath of African independences, great efforts were carried out to make the educational system compatible with the traditional African cultures. In 1960, Addis Ababa hosted an important meeting on education in Africa, under the aegis of UNESCO (The Addis Ababa Conference for Education for Africa). Its most significant result was the recommendation that African school system had to be deeply reformed in the areas of curricula, textbooks and teaching methods (Mohammed, Halai, Karuku, 2016: 3). The nature of the colonial education system, in which no attention had been posed on the socio-cultural African context, was evaluated as the most serious obstacle for the progress of education in Africa.

Both teaching and learning mathematics became one of the most difficult challenges for the new African independent States. This was due to two reasons: 1) mathematics had always been interpreted by local communities as an instrument of the colonial regimes' domination; 2) its innate epistemological characteristic (abstraction), which is, in general, extraneous to the African way of thinking. A seminal study at the end of the 1960s pointed out that the principles of mathematics were completely alien to the culture of Kpelle people, a community living in Liberia (Gay, Cole, 1967). According to Gay and Cole, Kpelle children had a clear idea of methods of measuring thanks to the fact that they accompanied their mothers to the market place to buy rice. Nevertheless, when these children entered a formal school, they forgot completely their out-of-school experiences, so that "they neither understood, nor properly used, the system of measurement taught in school because this formal way of measuring was not linked with their informal understandings" (Masingila, De Silva, 2010).

Another research revealed that several populations did not have the idea of number. For instance, a local population of Kalahari Desert only knew the first two numbers, 1 and 2; after 2, they used to say "many". In Changana, the local language of the South of Mozambique, very similar to Zulu language of South Africa, only the first three numbers are expressed through specific words; starting from number 4, they say "mune wa vanhu" ("a quartet of people"). Ancient Bantu people could only express the ideas of 1, 2 and 3 (Gerdes, 1991, 2012). In addition, various numbers, in many African populations, express symbolic meanings and are characterized by systems of taboo (Zalavsky, 1976).

Starting from these assumptions, mathematics education in Africa appeared a difficult enterprise. To reduce the distance between abstract principles of mathematics and local cultures, many countries inserted mathematics into specific political and cultural frameworks. They also reflected on the best way to insert mathematics into the new school systems.

The most serious attempt was the launch of the "New Mathematics" approach. This ambitious program – named African Mathematics Program – had its official start in 1962, in Entebbe (Uganda), under the coordination of Prof. Martin, of MIT. It was funded by Ford Foundation and USAID. Until 1970, the "Entebbe Mathematics Series" was composed of 60 books, which were extensively used spread in many African schools, where, in fact, more than 25.000 textbooks were distributed (AMP, 1969).

In parallel to the attempt promoted by some African countries in cooperation with the USA, UK and France carried out similar initiatives. The UK concentred its efforts around the group of the "School Mathematics Project", which joined Anglophone West African countries, and the group of the "School Mathematics Project for East Africa"; whilst France created the "Institut de Recherche de l'Enseignment de la Mathématique" (IREM) (Gerdes, 1991, 2012).

In 1976 the First Pan-African Congress of Mathematics, held in Rabat, stood for a mathematics able to help for the solution of concrete problems. Nevertheless, at the turn of the 1970s and 1980s the basic ideas of a New Mathematics entered a final crisis. Important countries, as Nigeria (1977) and Kenya (1982) decided to exit from this program, due to its abstraction.

Probably, the attempt carried out by the New Mathematics in Africa was not as genuine as it might appear. The fact that the USA, the UK and France were at the head of this movement let guess that cultural local factors were not duly valued. The failure of this movement favoured alternative models, first of all ethnomathematics.

This last approach found its major success at the end of the 1970s, in occasion of the International Conference of Khartoum, held in 1978. Here, D'Ambrosio launched a challenge to learn mathematics according to local mental and cultural structures, especially in Third World (D'Ambrosio, 2000).

A few years later, in 1983, different positions of African mathematicians emerged clearly. There was a group of scholars who considered mathematics as a universal science, whose practical applications had to be classified as "trivial". On the other hand, other experts continued to think of the relations between mathematics education and process of political emancipation of African countries, as D'Ambrosio (Brazil), El Tom (Sudan) and Ashour (Egypt). Mathematics education became characterized from a political point of view, so that only some "progressive" countries, including Mozambique, tried to elaborate different ways to learn mathematics.

In this context Mozambique – which obtained its political independence from Portugal only in 1975 – tried to establish a learning of mathematics consistent with its revolutionary political programs and objectives.

From a cultural point of view, the legacy left by Portuguese colonialism was poor. Local people had been systematically excluded from school, so that education was considered a priority before the political independence, too. In the "liberated zones" of the North of Mozambique (namely Cabo Delgado and Niassa) a new idea of school, embedded on the principles of Marxism, was developed, despite enormous difficulties: in Cabo Delgado there were about 100 schools for 100.000 students, meanwhile in Niassa the ratio between teachers and students was about 10:2000. Frelimo (the movement which fought against Portugal to free the country, and which has been the ruling party since 1975) adopted a strategy which aimed at creating Zones of Pedagogic Influence (ZIP), whose aim was to give shared guidelines to the teachers of the liberated zones' popular schools.

The new formal education system had to find an integration with other two visions: from one side, the idea of a "New Man" which constituted the basis of the ideology of social redemption that Samora Machel, the leader of Frelimo and the first President of Mozambique,

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

who wanted to implement such an idea in the new, free country. It meant to cut all the links with the colonial and humiliating past as to give value to local cultural traditions. Formal and informal cultures, theory and practice had to cohabit, finding a necessary integration in the new, popular school system (Mazula, 1975).

In the aftermath of independence, Mozambican government tried to spread the philosophy of education. According to UNESCO, 90% of the 11 million people living in Mozambique in 1975 were illiterate, ignoring the official language, Portuguese (UNESCO, 2015). In 1975 the first general reform was realized: Mozambican school system included eleven classes, from 1st to 11th. New disciplines were introduced, such as Political Education, History and Geography of Mozambique, Cultural Activities. Scientific education and Humanities were considered the pillar of new education by Samora Machel himself (Machel, 1980).

The politics of education carried out by Mozambican Government in the first years of independence led to a diminution of illiteracy, which fell from 97% in 1974 to 72% in 1982 (Mario & Nandja, 2005).

From this first period of independence, mathematics was considered as one of the priorities by Samora Machel to develop a new education in Mozambique, inside a socialist society (Machel, 1977). In this regard, Paulus Gerdes – with the support of Samora Machel - exerted a decisive influence on the methodology of mathematics education in the new Mozambique. In Gerdes' opinion, a possible answer to overcome the difficulties connected to mathematics education might be found in the principles of ethnomathematics. He proposed a "complementary" methodology of teaching, based on traditional object (see next point in the paper).

Gerdes tried to implement a philosophy and a practice of teaching mathematics "implanting it in the cultural context of students and professors" (Castello Branco Fantinato, 2004: 216).

In 1983 a new reform (L. 4/83) originated the National System of Education (SNE), but it was not sufficient to guarantee the link theory-practice, school-community. So, school continued to be characterized as a tool for the urban elite. The rate of illiteracy remained extremely high (SNE, 1985).

Furthermore, Mozambique was dilacerated by a civil war from 1977 to 1992. Thence, only in 1992, with the law number 6, the government could modify the L. 4/83, introducing a new school system, and opening the third phase of Mozambican education politics (Mario & Nandja, 2005). The transition from a centralized and Marxist system to a different one, which gathered its inspiration from liberal principles (according to the Constitution of 1990) determined the openness of the market of education to private schools and universities (Rosário, 2013; Bussotti L., 2014). These latter became prevalent in comparison to the public schools, both at high school education level and at the academic level. SNE assumed the current configuration, with the following subdivisions:

Table 1. A hint of Mozambican scholastic system.

Cycle	**School Level**	**Age of the Learners**
First	Pre-scholar level (optional)	From 3 to 6
Second or general	Primary school of first and second degree. Secondary school of first and second degree	From 6 to 14 From 14 to 18
Third	University or Academies	18-23

The end of the second cycle of secondary general school previews to pass the exams of all the disciplines. Mathematics is generally the discipline with the major rate of failures.

To avoid that the education offered in the schools were not extraneous to the local cultures and communities, Mozambican government experienced some attempts, starting from the 1990s. In 1993 the first experience of bi-lingual teaching was launched in Gaza and Tete Provinces (called PEBIMO), thus extended, in 2004, to all the other regions of Mozambique. In 2004 a new curriculum in primary school was adopted. Among its features, it is worth remembering the introduction of at least 20% of a local curriculum in relation to the national curriculum. The privileged areas of this reform were local languages, arts, cultures. Scientific disciplines were excluded from this courageous attempt. In 2007 a new curriculum was introduced for the secondary general school, as shown in the next section.

The Reform of General High School Curriculum (2007-2008) and Its Effects on Mathematics Education

Mozambican politics of education have always been embedded, since the end of the 1980s – when the country decided to adhere to the programs of structural adjustment of FMI and WB – into the great strategies of international subjects, in particular the UN system, namely UNESCO and UNDP.

Regarding the most recent period, a turning-point for Mozambique's educational policy was the decision taken in Paris in 1997 for a bilateral support by UNESCO. Primary education was the focus of this agreement, known as a "country-driven strategy as a basis for donor assistance" (Africa Recovery, 1998). It meant that Mozambique – as well as other African countries, as Ethiopia and Ghana - would have received UN funds, if the general strategy of education proposed by UN system would have been accepted.

So, in 2000 Mozambique adhered to the Millennium Development Goals (MDG) defined by the United Nations, and valid until 2015. The second, and most significant goal, of MGD was to extend primary education. Within 2015 all the children should have been entered an elementary school. A few years later, a group of Mozambican think-tankers proposed a sort of national plan for the implementation of the MDG, known as "Agenda 2025". Mozambican Government, at that time led by Joaquim Chissano, adopted this strategy. Starting from the ascertainment that Mozambique was, in 2002, the 170th country (out of 175) in the world for level of development, the national strategy for Agenda 2025 gave importance to education as a fundamental tool to develop the human capital (AGENDA 2025, 2003). The document pointed out that in 2001 the illiteracy rate was 56,7% of the whole population, although admitting meaningful progresses in the last few years: as a matter of fact, the level of schooling increased, passing from 43,6% in 1999 to 62,6% in 2002 (AGENDA 2025, 2002: 29).

The philosophy of MDG as of Agenda 2025 focused on the massification of education, considering primary school as a human right, with a particular emphasis on females (Lewin, 2009; Kite, Roche, Wise, 2014). As shown in the described picture, most attention was focused on primary school. On the other hand, secondary school faced, in these last few years, tremendous challenges in Africa. This is due to the quickly increasing number of pupils as well as to the inadequate skill of many teachers. The issue relative to the quality of secondary school in Africa has become a serious object of investigation only at the end of the validity of the MDG, 2015. For instance, in 2016 the Master Card Foundation, the University of Bristol and the University of Rwanda organized an important Meeting in Kigali, whose slogan was "Enabling Teachers, Enabling Youth". One of the paper presented emphasized the need of "Improving the professional capabilities of educators to deliver quality teaching in challenging delivery contexts is key for raising the quality of education for all including the most marginalised" (Bainton, Mbogo Barrett, Tickly, 2016: 3).

In Mozambique, the adoption of the MDG strategy brought to a significant expansion of schooling. Nonetheless, this happened without taking into account the issue of quality in secondary school. It is worth remembering that in 2015, Mozambican Parliament accepted to recruit only 8000 teachers for public schools, instead of the 15.000 proposed by the sector of education. This circumstance had, as a consequence, a structural lack of teachers and very numerous classes, with 60-70 students (especially in the primary school). The teachers forced to give lessons up to 12 classes. In addition, most of public schools' directors do not have the skill to manage such structures, and their only objective is to be in step with the statistics, avoiding of carrying out deep work on the quality of the teachers (Bonde, 2016: 81).

Both the system of education under colonialism and that developed during the first years of independence were based on humanities rather than on scientific and technical disciplines. This way of conceiving and developing education was overcome by the National Government Program, approved when Guebuza was president. For the first time, the Mozambican Government gave the first insight of the reform which had to be applied to the Secondary General School. In 2004, Primary School had been submitted to a similar transformation.
The main axes, which had to be the basis of the new secondary school's reform, were:

1. New programs, emphasizing an approach based on disciplines belonging to the technical and professionalizing sphere;
2. Expansion of e-learning;
3. Reduction of school failure under 15%.

Therefore, under the aegis of UNESCO, Mozambican Government prepared, in 2007, a general reform of its education system. The document which introduced this important reform aimed at reflecting on the state of art of the secondary general education system, preparing a new curriculum. In this document (INDE, 2007) some relevant points are highlighted:

1. A low rate of conclusion of primary and secondary education;
2. A pyramidal educational structure, with a wide base and a narrow top;
3. A substantial increasing of the number of enrolments at a national scale (between 1999 and 2003 +36% in the primary school and +89% in the secondary school).

In parallel, according to the Strategic Plan of Education 2005-2009 regarding the secondary school (MEC, 2005) the objectives were the following:

1. Expansion of enrolments, with a particular emphasis on the girls;
2. Building of new schools;
3. Improvement of the quality of teaching.

Regarding the last issue, the Strategic Plan considered, as its priority, a transformation of the curriculum of General High School oriented towards the insertion of the new graduates into the labour market rather than into an academic system.

To reach these objectives, the Strategic Plan intended to overcome a teaching considered "too encyclopaedic" and, thence, judged unable to respond to the new challenges of modern society. A new curriculum based on the following items was introduced:

1. Integrate the various disciplines of the secondary school;
2. Integrate theory and practice;
3. Improve the knowledge of Portuguese language, which constituted the biggest obstacle for the new graduates.

A curriculum oriented towards new technologies in detriment of Mathematics or Natural Sciences, and a methodology of teaching, which had to develop technical and practical skills of the children were privileged.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

According to a series of researches developed by Mozambican institutions in collaboration with international donors, two great difficulties characterizing Mozambican secondary school's learners emerged: a) the correct use of Portuguese; b) the skill to understand and use many aspects of Mathematics, as "geometry, percentage calculation, work with radicals and exercises with the radicals and exercises which need calculation with decimal numbers or decimal annotation" (INDE, 2007: 6).

Nevertheless, in the new teaching philosophy as well as in the objectives of educational policy, no space is reserved to skills related to Mathematics or to scientific knowledge. It is clearly explained that the reform of the new curriculum for the General Secondary Teaching was conceived to look for a logic coherence with the new curriculum of Primary school in 2004. Thus, Secondary School programs, as they have been defined in 2007, proposed the following innovations:

1. Focus on professionalization, which corresponds to a practical "know-how". It means a new approach for the resolution of practical problems, the development of skills useful in the daily, professional life (as working-group capability, a critical approach and strategies for a continuing training), with the introduction of professionalizing disciplines, also through specific, short modules;
2. A new approach regarding the cycles of learning: in concrete, it means that the mechanisms of evaluations have to be emphasized at the end of each cycle. Although the possibility of intermediate, annual evaluations is not excluded;
3. An integration of the teaching-learning, in the effort to link the different disciplines in a logical framework;
4. An integration of contents coming from local cultures within the new curriculum, pointing out the importance of activities to be realized together with local communities;
5. The introduction of local languages, coherently with the Primary Level Program;
6. The introduction of transversal topics, as Human Rights and Democracy, Gender, Health and Nutrition and so forth.

The specific field of Mathematics (treated together with Natural Sciences) aimed at developing:

1. The logic reasoning, using appropriated methods;
2. The skill to transform mathematical symbols and formulas into concepts used in the daily life;
3. The development of skills to classify, to correlate, to deduce, etc.

These documents pointed out at least two responses given by Mozambican institutions to the problems of secondary school: a) massification is the key-word of the new educational politics; b) secondary school has to fit with the labour market needs, so that "abstract" disciplines as mathematics will have a marginal role in relation to more "practical" ones, as technology, IT's and so on.

The Programs of Mathematics in the Last Cycle of General School

According to the Curricular Plan of the General Secondary teaching (INDE, 2007), among the various general objectives, scientific education is under-represented, since it is mentioned just at the point e) in a very generic form ("To develop the scientific spirit and the reflexive thought"). Coherently, the specific objectives of the 2nd cycle are concentrated on linguistic skills, tolerance towards cultural and sexual diversity and the know-how to carry forward autonomous business.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

The part of the Plan which deals with scientific skills reflects the marginalization of mathematics in Mozambican General School System. For, just two pages are dedicated to the teaching of *Area of Mathematics and Natural Sciences* (INDE, 2007: 57-59), with the following subdivision: Biology, Chemistry, Physics. Mathematics is not mentioned in this classification, and geometry is put together into the *Area of Visual and Scenic Arts.* Here, at the point b), *Descriptive Design and Geometry,* "Orthogonal projections and Axonometric and Rigorous Perspective" are mentioned" (INDE, 2007: 60). The child has to acquire the following skills: to apply the specific vocabulary of descriptive geometry; to represent with exactness, on two-dimensional pictures, objects which in truth hold three dimensions and which have to be rigorously defined; to demonstrate viewing capability in the space and graphic representation of real or imagined forms.

Despite this evident lack of specific references to Mathematics and the subsequent skills the children should acquire, in the last two years of General School System (11^{th} and 12^{th}) the discipline of Mathematics is present in the common trunk. Nevertheless, geometry (which is mentioned in the Curricular Plan) is absent, with the only exception of Option C (Visual and Scenic Arts).

In terms of workload, in the last two years of General School, the learners have to attend three hours per week of Mathematics and, in the Option C, three hours of Geometry in the 11^{th} year and two in the 12^{th}.

As we will show later, this system failed. Before explaining the probable reasons of this unsuccess, it is worth remembering the attempt carried out by Gerdes, with its approach based on ethnomathematics, which aimed at making the learners confident with this complex discipline in Mozambique.

Mathematics Education: Analysis of Ethnomathematics as a Possible Strategy for Mozambique

This research concerns only an aspect of ethnomathematics, that is, its possible didactical value, according to the vision offered by Gerdes.

A premise is necessary: albeit, probably, no one has been able to offer a generally shared definition of mathematics, it seems to us desirable that the pupils, at the end of the high school, can grasp two fundamental features of mathematics:

1) *the concept of demonstration*, which is something completely different from having verified the truth of a proposition, without exception, for a certain number of cases. Rather, the demonstration is a general argument, which holds for every case included in the proposition;
2) *the concept of generality*. This is connected with that of demonstration. The assertion that a proposition is valid for an entire series of forms or numbers needs a completely different approach than that necessary to verify such a proposition for any single form or number.

The idea to introduce the students to the abstract and general concepts and procedures of mathematics by means of educational approaches, which exploit some aspects of their daily experience, is absolutely agreeable. Nonetheless, such an idea has to be a key to introduce the learners to those abstract concepts and procedures in the most plain and simple possible manner, not a way to hide the existence of the abstract nature of mathematics.

Let us see how Gerdes suggests teaching some classical theorems and problems, starting from the Pythagorean theorem. Likely this approach is thought for 12-14 years old girls and boys.

He considers the "star design", that he also calls "toothed square" (Gerdes, 1991, 2012: 88) which is a configuration like this:

ISSN 1822-7864 (Print) ISSN 2538-7111 (Online)

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

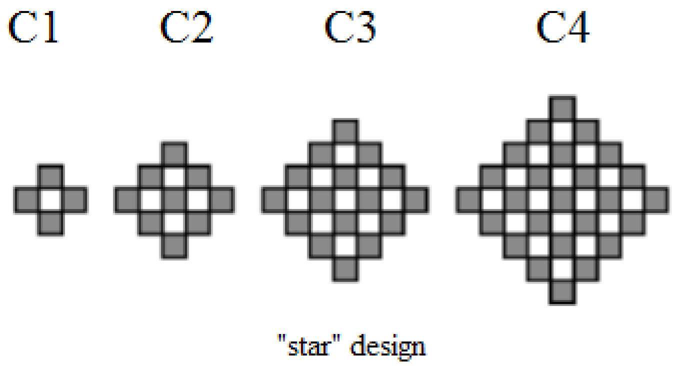

Figure 1: The toothed squares. Gerdes, *Etnomatemática*, 1991, 2012, p. 86.

Gerdes claims that this configuration is very common in the artistical representations and in the manufacts of several indigenous populations, not only in Africa, but in Europe and in America, too (Gerdes, 1991, 2012: 86-88). As to Africa, the configurations have been discovered in textiles deriving from Nigeria and Algeria, in baskets from Lesotho and Mozambique, and in a game board from Liberia. Gerdes also refers to a design from Angola (Gerdes, 1991, 2012: 87). The author states to be favourable to the idea that such a configuration might be useful to introduce the Pythagorean theorem. He describes a series of steps: first of all, referring to Figure 1, C3, it is easy to show that the entire area of the figure is equal to the 4x4=16, area of the shaded squares, plus the 3x3=9, area of the unshaded squares, which Gerdes represents by the following image.

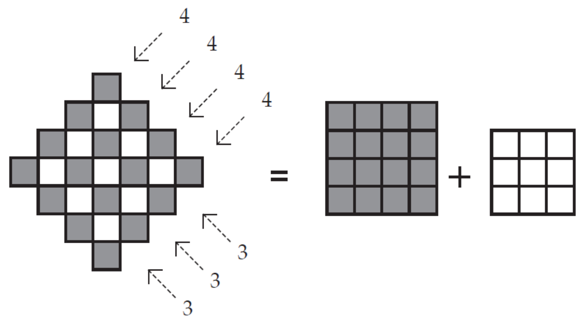

Figure 2: A toothed square decomposed into two squares. Gerdes, *Etnomatemática*, 1991, 2012, p. 88.

After that, he explains a technique, at which the learners might arrive through a series of experiments (Gerdes 1991, 2012, p. 89), to transform the toothed square into a square having the same surface. The reasoning is easily expressible through the figure adopted by Gerdes.

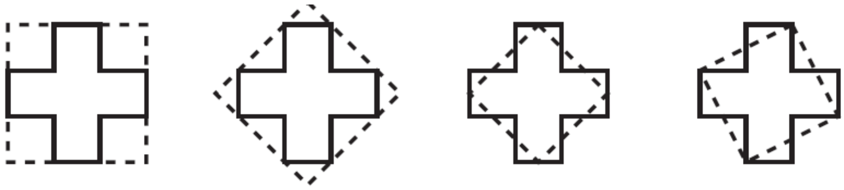

Figure 3: Relations between toothed squares and squares. Gerdes, *Etnomatemática*, 1991, 2012, p. 89.

Now, Gerdes unifies the train of thought summarized in the figures 2 and 3 in this manner:

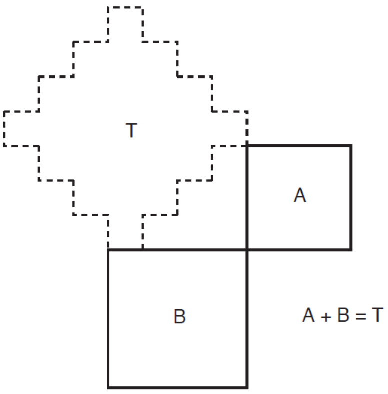

Figure 4.1. An intuitive approach to Pythagoras theorem 1. Gerdes, *Etnomatemática*, 1991, 2012, p. 90.

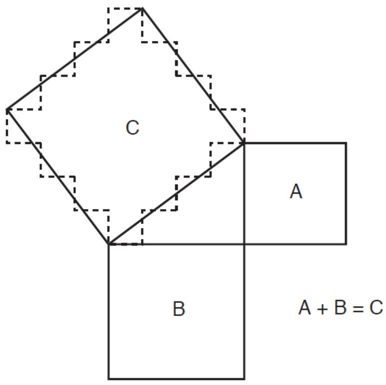

Figure 4.2. An intuitive approach to Pythagoras theorem 2. Gerdes, *Etnomatemática*, 1991, 2012, p. 90.

He claims: "When we now draw the last real square (area *C*) on the same figure, we arrive at the Pythagorean Theorem for the case of (a, b, c) right triangles with $a : b = n : (n + 1)$, where the initial toothed square has $(n + 1)$ teeth on each side. case of the (3, 4, 5) right triangle. On the basis of these experiences, the learners may be led to *conjecture* the Pythagorean theorem in general" (Gerdes, 1991, 2012: 90-91).

Before explaining the conclusion of Gerdes, a commentary is necessary: that the square *C* has the same area of the toothed square *T* is out of discussion and this is the reason why the toothed side of Figure 4,1 can be replaced by the side of the square represented in Figure 4,2, which is considered as the hypotenuse of the right triangle of Figure 4,2. The reason why this happens is conspicuous, albeit a rigorous proof is not completely trivial. However, the construction of Gerdes' reasoning is partially misleading: the relation which ties the right triangles with small sides equal to n and n+1 becomes significant only if the hypotenuse, too, is a whole number. Therefore, what should the kids conjecture in general? 1) That when the small sides are two whole numbers of the form n and n+1, the hypotenuse is also a whole number? In general, this is obviously false. As well known, it is true for some particular Pythagorean triples of which (3,4,5) is the "smallest" one; 2) That when the sides are n and n+1, then the Pythagorean theorem is valid? This is true, but it is reasonable to have the doubt that the learners thought the Pythagorean theorem true only in this case, that is they might reach a very misleading and wrong conclusion; 3) The truth of Pythagorean theorem in general? Well, if a learner, by means of this reasoning, were able to guess the general validity of the Pythagorean theorem, he/she would be e very brilliant mathematician!

Therefore, the heuristic value of these argumentations by Gerdes seems, at least, debatable.

Let us now analyse the proof proposed by Gerdes: be given two squares A' and B'. It is always possible to decompose A' into 9 squares and B' into 16 squares, so to obtain a toothed square whose area is 25. Since a toothed square can be transformed into a square (Figure 4,2) T', whose area is the same as the sum of the two squares and since the configuration can be arranged as in Fig 4,2, then we should conclude that the area of the square constructed on the hypotenuse of any right triangle is equal to the sum of the areas constructed on the small sides. This proof, based anyway on the problematic heuristic analysed some lines above, has, nonetheless, a merit: one is free to divide the square A' into 9 squares and B' into 16 squares, but it is manifest that, in general, the area of any sub-square in which A' is divided is not equal to the area of any sub-square in which B' is divided. Therefore, the construction does not depend on the surfaces of the single sub-squares. It is appropriate that this element is pointed out, which is missing in Gerdes' book, because, otherwise, the learners might think of something arbitrary in the construction. In fact, the construction by Gerdes does not depend either on the areas of the single sub-squares, or on any decision regarding the square to divide into 9 sub-squares and that to divide into 16 sub-squares. In principle, one is completely free to divide the smaller square into 16 sub-squares and the bigger one into 9. The proof would work equally.

Thence, this feature of the construction, which has valuable aspects, has also problematic aspects from a didactical point of view, because the abstraction from the values of the areas of the single sub-squares is, in itself, something not completely trivial. Furthermore, there is another question: the construction has been obtained without posing any constructive rule. To use another expression, without stating any axiom. Is this a good approach? This is debatable, because geometry is exactly a discipline based on axioms. It is agreeable that, when the pupils are 12-14 old, an explicit axiomatic structure is not introduced, but, at least, it is necessary to explain in advance which operations are allowed and which are not. This might introduce to the concept of rigour without any explicit formalism. We mean: an intuitive and visual approach is desirable, but it has to be used to pave the way for the learners to get a more rigorous and general vision of geometry, which is typical of Euclid and which we consider extremely formative for the learners.

Of course, in Euclidean geometry the rigour is present, but intuition as well as the skill to imagine the constructions remain a fundamental datum to solve the problems by synthetic, rather than by analytical, methods. It is true: the real challenge is to develop the geometrical intuition and vision in the learners, but the other aspect is that such an intuition has to converge towards the idea of a theory in which the propositions are interconnected in a rigorous manner. It seems that Gerdes's approach, starting from some right presuppositions, is coherent in this respect. However, it seems that the best way is to start from Euclid's text and from Euclid's axioms, in particular. To each axiom or proposition it might be useful to associate an intuitive explanation and to make it clear to the learners that Euclid's axioms are anything but natural, unavoidable and necessary truths, rather they construct a space with some features.

A trivial example: the famous fifth postulate, the parallel axiom. It might seem natural that two parallel lines never mutually cut and that two lines maintaining the same distance always exist. But this is not the case, for example, in the visual space. If we consider two lines such that the distance between two their points A-A' and B-B' is equal when both points are near (Figure 5 A) to us, who measure the distance, we see that, when the points B-B' go to infinity, the lines seem to converge (Figure 5B). This is typical when we observe two long parallel lines of street lamps. When the lines are far from our visual point, they seem to converge. Therefore, in the visual space no parallel line exists. Hence, the fifth axiom of Euclid is absolutely justified. It is not something "natural". Critical and visual considerations of this kind are necessary to grasp the true structure of Euclid's geometry.

PROBLEMS
OF EDUCATION
IN THE 21st CENTURY
Vol. 75, No. 5, 2017

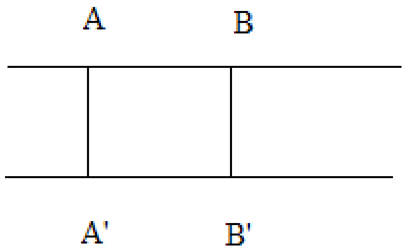

Figure 5A: Two parallels.
Picture drawn by the authors

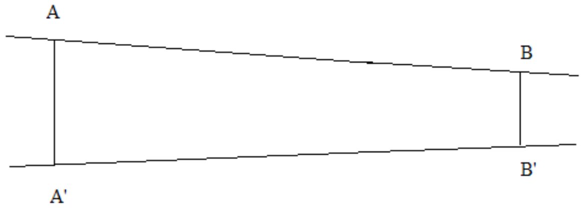

Figure 5B: Apparent convergence of two parallels.
Picture drawn by the authors

Gerdes also analyses some aspects of Euclid's axiomatic. His analysis starts following Alexandrov's idea to replace Euclid's fifth axiom with the "rectangle axiom" (Gerdes, 1991, 2012: 61): if the opposite sides of a quadrilateral are equal and if the angles at the basis are right, then the quadrilateral is a rectangle. After that, Gerdes shows that a rectangle can be constructed basing upon some procedures used by the Mozambican peasants to edify some parts of their houses. A brief discussion (Gerdes, 1991, 2012: 63-66) follows starting from such constructions. This discussion has some interesting aspects: for example, the relations posed by the learners between the rectangles and the other parallelograms or between axioms and theorems.

However, beyond any single example dealt with by Gerdes, what is missing in Gerdes' approach is the development of an initial set of considerations towards a global educational perspective: he seems so interested in showing the existence of mathematics in the pre-colonial African cultural heritage that his proposals appear a set of ideas where, every time, he begins once again from the level 0, so to say. It is difficult to grasp how these ideas might be fruitfully interrelated, albeit some of his proposals could be interesting. We add that, in many of his contributions Gerdes deals with the construction of polyhedron and, more in general, with several aspects of space geometry starting from some constructions developed by African populations in their daily life. Relying upon them, the pupils, guided by the teachers, should be able to identity some regularities in these constructions, to repeat them using, for example, paper, and to guess general geometrical propositions (see, for example, chapter 5, pp. 57-84).

In history of mathematical education, the idea to use concrete objects as paper and scissors – but also other objects - to introduce the learners in a practical manner to geometry and, above all, to space geometry and to the axiomatic aspects of this discipline, but also to other branches of mathematics, is not new (Cundy, Rollett, 1952, Read 1965, this text is on the tangram. The literature on this subject is huge, but we restrict to mention this important text; Checcucci 1971a, Checcucci 1971b, Bussotti P, 2013). Many of these attempts have several contact points with those of the ethnomathematicians, but their foundation appears more solid.

Mathematics education is a great problem everywhere; it is comprehensible that in countries as Mozambique this problem is even more serious than in Western countries; synthetic geometry should be the basis of mathematics education, whereas this discipline is becoming more and more neglected; the idea to resort to practical constructions is good, but such constructions have to be inserted into a clear educational plane; Euclid's *Elements* are the texts from which to start. Obviously, it is necessary to work on the text and to modify some difficult aspects by means of didactical ideas and devices, but the old Euclid continues to be a good guide for the future.

Conclusions

This research faces the failure of mathematics education in Mozambique, especially insofar as secondary school is concerned. This is connected with several aspects of Mozambican situation: the history of this country, the political choices, the current strategy of education, particularly in regard to mathematics education.

One might think that Mozambique, which is one of the poorest countries of the world, does not have sufficient cultural and educational conditions to support the teaching of mathematics in its schools. This is only a part of the truth. In fact, this research showed that important factors which contributed to determine the current situation of mathematics education in Mozambique are represented by specific elements: among them it is worth remembering here at least three. Firs of all, the interruption of an original methodology of teaching related to ethnomathematics and applied to Mozambican context. It was experimented by Paulus Gerdes. This method is problematic, both in general terms and, specifically, in regard to its applications to Mozambique mathematics education. Nevertheless, it represented an important attempt which could be discussed and modified, but, in any case, nourished and supported by national and international authorities. However, ethnomathematics was abandoned, and methods to teach mathematics similar to those used in the Western world were imposed by the new, general policies of education carried out by international agencies, as World Bank, UNDP and so on. Secondly, and consequently, the reforms of Mozambican school curricula penalized the role of mathematics, with the substantial disappearance, at the level of the last two years of General school, of geometry. Finally, specific factors related to the modest commitment of Mozambican teachers in relation to their profession is another remarkable aspect which contributed to the failure of mathematics education in Mozambique. Empirical research carried out by Mozambican scholars showed that this specific element plays an important role to explain the very bad performance of Mozambican school system in the teaching and learning of mathematics not only at international level, but also inside the African context.

References

AMP (African mathematics program) (1969). *Executive meeting.* London (Technical Reference: 1656 Ng). Retrieved from: http://pdf.usaid.gov/pdf_docs/PDAAA580A1.pdf.

AGENDA 2025 (2003). *Estratégia de Desenvolvimento Nacional* [*Strategy of national development*]. Maputo: República de Moçambique.

Amy Quin (2017). Britain turns to Chinese textbooks to improve its maths scores. *The New York Times*, August 5.

Arana, A., Mancosu, P. (2012). On the relationship between plane and solid geometry. *The Review of Symbolic Logic, 5* (2), 294-353. doi: https://doi.org/10.1017/S1755020312000020.

Bainton, D., Mbogo B., A., Tikly, L. (2016). *Improving secondary school teacher quality in Sub-Saharan Africa.* Bristol: Bristol Working Papers in Education Series 3.

Bass, H. (2005), Mathematics, mathematicians, and mathematics education. *Bulletin of the American Mathematical Society* (New Series), *42* (4), 417–430.

Benot, Y. (1972). *Idéologie des Indépendances Africaines* [*Ideology of the African independences*]. Paris: Maspero.

Baptista, J. (2012). *Moçambique: casamentos prematuros causam aumento de desistências escolares* [Mozambique: Premature marriages cause an increasing of school dropouts]. Retrieved from: http://genderlinks.org.za/programme-web-menu/moambique-casamentos-prematuros-causam-aumento-de-desistncias-escolares-2012-09-18/.

Bolondi, G. (2005). Geometria proiettiva, geometria descrittiva, e geometria dello spazio nella scuola italiana [Projective geometry, descriptive geometry and space geometry in the Italian school]. In M. Franciosi (Ed.), *Prospettiva e geometria dello spazio* [*Prospective and space geometry*] (pp. 145-176). Sarzana: Agorà.

Bonde, R. A. (2016). *Políticas públicas e qualidade de ensino em Moçambique* [Public policies and teaching quality in Mozambique]. Master Thesis in Public Politics, Strategy and Development. Rio de Janeiro: Federal University of Rio de Janeiro. Retrieved from http://www.ie.ufrj.br/images/pos-graducao/pped/dissertacoes_e_teses/RUI_BONDE.pdf.

Borgato, M. T. (2006). Il fusionismo e i fondamenti della geometria [Fusionsim and foundations of geometry]. In L. Giacardi (Ed.), *Da Casati a Gentile* [From Casati to Gentile] (pp. 125-157). La Spezia-Lugano: Agorà-Lumières Internationales.

Bussotti, L. (2014). La democrazia in Mozambico: concettualizzazione e periodizzazione [Democracy in Mozambique: Conceptualization and periodization]. *La Critica Sociologica* [*The Sociological Critic*], *47* (190), 55-77. doi: 10.1400/222523.

Bussotti, P. (2013). Vittorio Checcucci and his contributions to mathematics education: A historical overview. *Problems of Education in the 21th Century*, *53*, 22-39.

Castello Branco Fantinato, M. C. (2004). A construção de saberes matemáticos entre jovens e adultos do Morro de São Carlos [The construction of mathematical knowledge among young people and adults]. *Revista Brasileira de Educação* [*Brazilian Journal of Education*], 27, 109-124.

Checcucci, V. (1971a). *Creatività e matematica*. *Quaderni di Corea* [*Creativity and mathematics. Notebooks of Corea*], third series, 5. Monograph. Italy.

Checcucci, V. (1971b). *Alla conquista di un contenuto. La geometria delle trasformazioni, ambiente di base per l'apprendimento della matematica*. *Quaderni di Corea* [Towards the conquest of a content. The geometry of transformations, a basic environment to learn mathematics. Notebooks of Corea], third series, 8. Monograph.

Cundy, H. M., & Rollett, A. P. (1952). *Mathematical models*. Oxford: Clarendon University Press.

D'Ambrosio, U. (2000). Adequate mathematics for third world countries (1st ed. 1979). In Elton, M. (Ed.), *Developing mathematics in third world countries, Vol. 33.* Netherland: North Holland, pp. 33-46.

De Paolis, R. (1884). *Elementi di geometria* [*Elements of geometry*]. Torino: Loescher.

Francisco, J. (2016). *A massificação do ensino em Moçambique sob a égide da internacionalização das políticas educacionais: implicações para a gestão escolar* [The massification of education in Mozambique under the auspicies of the internationalization of educational policies: Implications for school management]. Master Thesis. Corumbá: Universidade Federal do Mato Grosso do Sul. Retrieved from http://ppgecpan.sites.ufms.br/files/2016/04/JONAS-DISSERTACAO-MASSIFICA%C3%87%C3%83O-DO-ENSINO-EM-MO%C3%87AMBIQUE-2016.pdf.

Gay, J., & Cole, M. (1967). *The new mathematics and an old culture: A study of learning among the Kpelle of Liberia.* New York: Holt, Rinehart and Winston.

Gerdes, P. (2012). *Etnomatemática* [*Ethnomathematics*]. Maputo: ISTEG (1st Ed. 1991).

Giacardi, L. (2003). L. Cremona, G. Vailati, C. Segre. Tre diversi approcci al problema dell'insegnamento della matematica tra '800 e '900 [L. Cremona, G. Vailati, C. Segre. Three different approaches to the problem of mathematica teaching between 19th and 20th century]. In *Atti del XXIII Congresso UMI-CIIM* [Proceedings of the 23th UMI-CIIM Conference] (pp. 63-75). Bologna: UMI.

INDE (2007). *Plano Curricular do Ensino Secundário Geral (PCESG)* [Curricular plan for the secondary school]. Maputo: Imprensa Universitária.

Karp, A., Schrubing, G. (Eds.) (2014). *Handbook on the history of mathematics education*. New York-Heidelberg-Dordrecht-London: Springer.

Kite, G., Roche, J. M., Wise, L. (2014). Building a post-2015 framework that leaves no one behind: A proposal to incentivise inclusive progress. Paper presented at International Sustainable Development Conference. Available from http://ic-sd.org/2015/04/19/ proceedings-from-icsd-2014/.

Klein, F. (1924, 1953). *Elementary mathematics from an advanced standpoint. Arithmetic-Algebra-Analysis*, translated from the 3rd German edition by E. R. Hedrick and C. A. Noble, New York: Dover Publications.

Legendre, A. M. (1794). *Éléments de Géométrie avec des Notes* [Elements of geometry with notes]. Paris: Firmin Didot.

Lewin, K. (2009). Access to education in sub-Saharan Africa: Patterns, problems and possibilities. *Comparative Education*, *42* (2), 151-174. doi: 10.1080/03050060902920518.

Macamo, E. M. (2015). *Insucesso escolar em Moçambique. Estudo de caso na Escola Secundária Graça Machel* [School failure in Mozambique. A case-study on the secondary school Graça Machel]. Master Thesis in Education Management. Lisboa: Universidade Aberta.

Machel, S. (1977). *Le processus de la Révolution Démocratique Populaire au Mozambique* [The process of democratic, popular revolution in Mozambique]. Paris: L'Harmattan.

Mario, M., Nandja, D. (2005). *Literacy in Mozambique: Education for all challenges.* Maputo: UNESCO.

Masingila, J., De Silva, R. (2001). Teaching and learning school mathematics by building on students' out-of-school mathematics practice. In Atweh, B., Forgasz, H., & Nebres, B. (Eds.), *Sociocultural research on mathematics education* (pp. 329-344). Mahwah (New Jersey)-London: Lawrence Erlbaum Associates.

Matope J. (2012). Exames do secundário tiveram "resultados desastrosos" em Maputo [Exams of the secondary school had "disastrous results" in Maputo]. *Jornal Notícias* [*Newspaper Notícias*]. Retrieved from https://noticias.mmo.co.mz/2012/08/exames-do-secundario-tiveram-resultados.html.

Mazula, B. (1995). *Educação, cultura e ideologia em Moçambique: 1975-1985* [Education, culture and ideology in Mozambique: 1975-1985]. São Paulo: Edições Afrontamento para Fundo Bibliográfico de Língua Portuguesa.

MCM/DE (2015). 80% estudantes chumba em exames extraordinários da 7ª, 10ª e 12ª classe [80% of the students fails at extraordinary exams of the 7th, 10th and 12th class]. *Folha de Maputo*. Retrieved from http://www.folhademaputo.co.mz/pt/noticias/nacional/80-estudantes-chumba-em-exames-extraordinarios-da-7-10-e-12-classes/.

MEC (2005). *Plano Estratégico da Educação II. PEE II – 2005-2009* [Strategic plan of education II. PEE II – 2005-2009]. Maputo: MINED.

Mohammed, M., Halai, A., Karuko, S. (2016). Issues for quality enhancement and harmonization of education in East Africa. In Halai, A., Tennant, G. (Eds), *Mathematics education in East Africa* (pp. 1-8). Dar es Salaam: Aga Khan University,

Osório, C. (2007). A socialização escolar: educação familiar e escolar e violência de género nas escolas [School socialization: Familiar and school education and gender violence in the schools]. *Outras Vozes* [*Other Voices*], *19*, 1-9.

Pepe, L. (2006). Insegnamenti matematici e libri elementari nella prima metà dell'Ottocento. Modelli francesi ed esperienze italiane [Mathematics teaching and elementary books in the first half of the 19th century. French modeals and Italian experiences]. In: L. Giacardi (Ed.), *Da Casati a Gentile* [*From Casati to Gentile*] (pp. 65-98). La Spezia-Lugano: Agorà-Lumières Internationales.

Piaget, J., Dieudonné, J., Lichnerowicz, A., Choquet, G., Gattegno, C. (1955). *L'einsegnement des mathématiques* [*The teaching of mathematics*]. Neuchatel and Paris: Delachaux and Niestlé.

Read, R. C., (1965). *Tangram. 330 Puzzles*. New York: Dover.

RM (República de Moçambique) (2005). *Programa Quinquenal do Governo 2005-2009* [Five-year plan of the Government 2005-2009]. Maputo: RM.

Rosário da Costa, L. J. (2013). Universidades Moçambicanas e o futuro de Moçambique [Mozambican universities and the future of Mozambique]. *Ensino Superior* [*Academic Education*], *10*, 46-55.

UNESCO (2015). *Mozambique. Annual Report 2015.* Maputo: UNESCO. Retrieved from http://unesdoc.unesco.org/images/0024/002461/246143e.pdf.

UNITED NATIONS (1998). Special initiative: Highlights of progress. *Africa Recovery, 11* (4). Retrieved from https://www.un.org/ecosocdev/geninfo/afrec/vol11no4/march98.htm.

Viola, T. (1964). Didactique sans Euclide et pédagogie euclidienne [Didactics without Euclid and Euclidea pedagogy]. *L'Enseignement mathématique* [*The teaching of mathematics*], 2 (10), 5-27.

WB (World Bank) (2015). *Moçambique. Indicadores de Prestação de Serviços* [Mozambique. service delivery indicators]. Retrieved from: http://documents.worldbank.org/curated/en/442101468187768256/pdf/95999-PORTUGUESE-WP-PUBLIC-Box391432B-ADD-SERIES-WB-Mozambique-SDI-Brief-PUBLIC.pdf.

Wilson, J. M. (1868). Euclid as a text-book of elementary geometry. *Educational Times,* 21, 125-128.
Zalavsky, C. (1976). African numbers. *Teacher*, *94*, 91-96.

Received: *August 25, 2017*   Accepted: *October 10, 2017*

Luca Bussotti	PhD, Researcher at the International Studies Center, ISCTE-IUL, Lisbon, Avenida das Forças Armadas, Lisboa, Portugal. E-mail: Luca.Bussotti@iscte-iul.pt
Paolo Bussotti	PhD, Researcher and Lecturer of History of Science and History of Mathematics, via Petracco and via delle scienze, University of Udine, Italy. E-mail: paolobussotti66@gmail.com Website: https://www.researchgate.net/profile/Paolo_Bussotti