

**PREDICTION OF VIBRATION RESPONSE CAUSED BY  
ROTATING MACHINERY**

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**ABSTRACT**

*Excitation energy of rotating machinery propagates through the connection paths resulting in vibration responses at the structures such as machine foundations. Accurate prediction of these vibration responses at a point of a complex structure, where the operational behavior cannot be measured directly, is an important engineering problem for design optimization, component selection and condition monitoring. The main step of the prediction is to identify the internal forces acting on the connection paths of the machinery. At the circumstances where direct measurement is impossible or impractical due to physical constraints, an indirect approach is to identify the exciting forces based on multiplication of an inverted frequency response functions (FRFs) matrix and a vector of vibration responses measured at the points where the exciting forces are transmitted through. The aim of this paper is to present matrix inversion method to identify the exciting internal forces and hence predict the vibration response at the point of interest.*

**Keywords:** *Vibration Response, Matrix Inversion, Cross-Coupling, Rotating Machinery.*

## **DÖNEL MAKİNE KAYNAKLI TİTREŞİM CEVABININ ÖNGÖRÜSÜ**

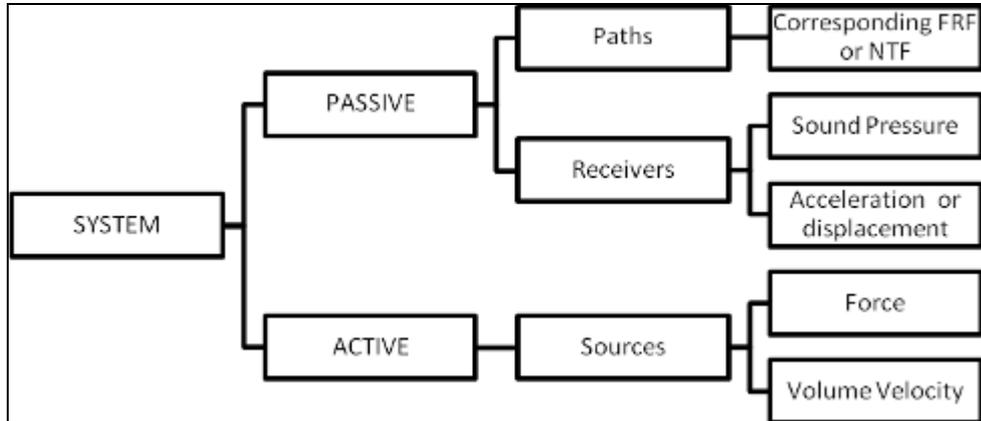
### **ÖZ**

*Dönel makinelerin tahrik enerjisi makinenin bağlantı patikalarından yayılarak makine kaidesi gibi yapılar üzerinde titreşim cevabı ortaya çıkarmaktadır. Kompleks bir yapının titreşim cevaplarının doğru olarak öngörülmesi dizayn optimizasyonu, ekipman seçimi ve durum izlemesi açısından önemli bir mühendislik problemidir. Öngörünün ana aşaması makinenin bağlantı patikalarına etkiyen dahili kuvvetleri tespit etmektir. Doğrudan kuvvet ölçümünün mümkün veya pratik olmadığı durumlarda, frekans cevap fonksiyonları (FCF) matrisinin tersi ile tahrik kuvvetlerinin yayıldığı noktalarda ölçülen titreşim cevapları vektörünün çarpılması ile tahrik kuvvetlerinin tespit edilmesi endirekt bir yaklaşım olarak uygulanabilir. Bu makalenin amacı dahili kuvvetleri tespit etmek ve böylelikle ilgi noktasının titreşim cevabını öngörmek amacıyla kullanılacak matris evirme metodunu sunmaktır.*

**Anahtar Kelimeler:** *Titreşim Cevabı, Matris Tersine Çevirme, Bağlı Etki, Dönel Makineler*

### **1. INTRODUCTION**

A mechanical vibrating system can be divided into two main groups; active and passive. As shown in Figure 1, active part is the source creating the vibrational energy which can be defined as force or volume velocity. On the other hand, passive part consists of the receiver and the paths through which the vibrational energy is transferred. This part can be defined as acceleration/sound pressure and transfer functions, respectively.



**Figure 1.** Description of a Mechanical Vibrating System

The major step of the vibration response prediction (VRP) is to identify the exciting internal forces acting on the structure through the connection paths. The most straightforward solution is to measure the forces directly. However, this may not be possible due to the complexity of the structure and the challenges of load cell applications. Consequently, indirect methods have been widely studied in the literature [1-20]. Dynamic stiffness method, introduced by Verheij [1], seems to be the most simple way, especially for the rubber linked structures. However, accurate complex dynamic stiffness data of the rubber mounts is seldom available and even if there is, it is only valid for a given load condition. Another approach, called transmissibility method, can be implemented to predict the vibration response at a point of interest. In this method, the forces are replaced by the measured responses at the force identification points and the propagation paths are represented by the transmissibilities. This approach is much simpler and faster but unconsidered potential cross-coupling between the paths lead to incorrect predictions [2, 5].

In the early 1980s, the matrix inversion method has been developed [6, 7]. This method basically involves multiplication of a vector of vibration responses with an inverted matrix constituted by the frequency response functions (FRFs).

In this study, prediction of the vibration response will be discussed for a structure excited by an electric driven pump. Dynamic stiffness method is commonly used for the rotating machinery. However, the machinery may be mounted without resilient elements. Even if it is connected with resilient mounts, the stiffness data of the mounts may not be available. This study investigates the application of direct inversion method for these cases and the effect of the cross-coupling terms.

## **2. MATRIX INVERSION METHOD**

The vibration response of any point is related to the acting force and corresponding point transfer function. However, the response at a particular point is not only related to the force acting on the point but also the other forces acting on the structure. This contribution from the other forces is called cross-couplings. If the system consists of more than one source or path, the cross-couplings should be considered. Cross-coupling effects are focused on considering all transfer functions between the path inputs, as illustrated in Equation (1). It can be stated that a high dynamic response at any point does not indicate that the corresponding point force is the only exciter. Due to this reason, prediction of vibration response at a point results in errors if the cross-couplings are not taken into account. Biermayer et al. reported that the error can be up to 10 dB in case of not considering the cross-coupling terms [21].

$$\vec{X}_1 = F_1 H_{11} + F_2 H_{21} + \dots + F_n H_{n1} \quad (1)$$

Where  $n$  is the number of paths,  $F$  is the exciting force and  $H$  is FRF between the force and the path inputs.

In order to identify the exciting forces, the inverse procedure should be applied. In this presented approach, a square FRF matrix,  $n \times n$ , is constituted by measured FRFs between the path inputs to take the cross couplings into account. A vector of vibration responses is created by using the responses measured at the points. Thus, internal forces propagating through each path can be defined in matrix notation by applying matrix inversion as shown in Equation (2).

$$\{F_i(\omega)\} = [H_{ij}(\omega)]^{-1} \{\bar{X}_i(\omega)\} \quad (2)$$

Where  $i$  and  $j$  denotes the number of paths and forces, respectively.

Once the exciting internal forces are calculated, the vibration response at the point of interest,  $k$  can be predicted assuming that the system is linear. The contribution of each path to the total response can be found by multiplying the identified internal forces with the corresponding transfer functions or FRFs. Since the system is assumed to be linear, the response of the point of interest can be identified by adding up the partial contributions, as illustrated in Equation (3). Besides, the dominant transfer paths can be determined according to the partial contributions.

$$\bar{X}_k(\omega) = \sum_{i=1}^n \bar{X}_{ki}(\omega) = \sum_{i=1}^n F_i(\omega) H_{ki}(\omega) \quad (3)$$

Due to the defined partial contributions with Equation (3), main reasons for the high contributions of a transfer path can be determined such as high transfer function, high point mobility etc. Then, some remedial measures can be applied by means of reducing transfer function or increasing local stiffness.

### 3. CASE STUDY

A set-up composed of an electric motor driven pump and its foundation was created in order to predict the vibration response at the defined target on the foundation, as shown in Figure 2.



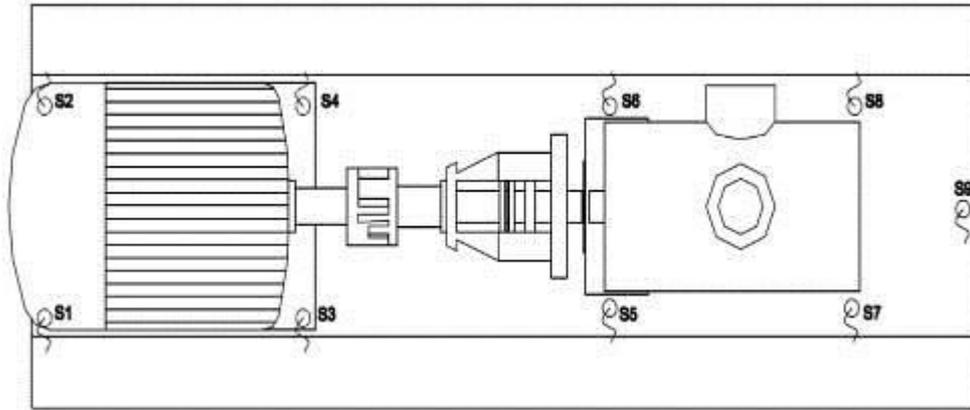
**Figure 2.** Electric Motor Driven Pump Set-up

Experimental study was composed of three parts. First, the pump ran at 3000 rpm and the acceleration was measured as the vibration response by means of accelerometers at the connection paths, S1 to S8, as shown in Figure 3. The measured responses should include complex quantities having magnitudes and phase information. They can be derived for each frequency by using auto- and cross-spectra as;

$$a_i = \sqrt{G_{ii}} e^{-j\angle G_{i1}} \quad (4)$$

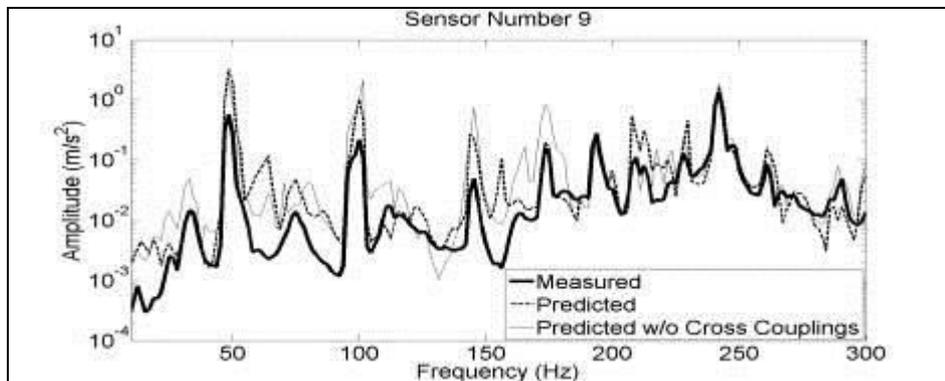
where  $G_{ii}$  is the auto-spectrum of the response at point  $i$  and  $G_{i1}$  is the cross-spectrum between the reference position, 1 and point  $i$ . Equation (4) should only be implemented if the coherence function between  $a_1$  and  $a_i$  is about one. Besides, the reference position should be one of the points at where the response is measured [16, 22].

Second part was measuring the transfer functions of the structure by means of an impact hammer. FRFs were measured from the path locations to the target location, S9. H1 was considered as the FRF and frequency range was selected as 0-300 Hz. Finally, in the third part, the mentioned methodology was implemented; the results were presented and compared with the measured ones.



**Figure 3.** Schematic View of the Set Up

It is really challenging to perform a pure one dimensional excitation at the path connections because moments and lateral forces may be arisen. However, moments and rotations were neglected in the study since they are very hard to measure [23]. In this study, the vertical exciting forces were aimed to be identified and then the vibration responses were predicted assuming that the system is linear and time-invariant. By applying the matrix inversion method along with the cross-coupling terms (Equation (2) and (3)), the response at the target, S9 was identified.



**Figure 4.** Vibration Response of S9

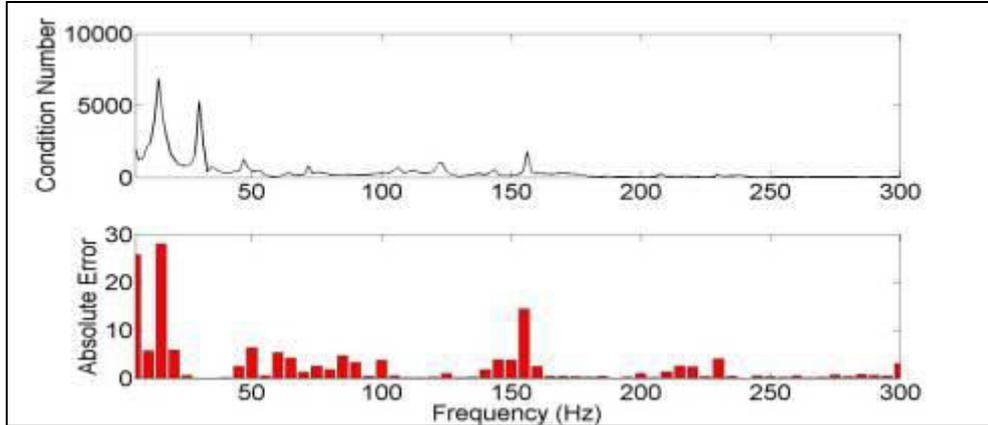
As shown in Figure 4, the predicted response catches the vibrational character of the measured response with admissible amplitude differences

and it can be stated that there is an acceptable consistence between the measured and predicted result with 30 % average error in the frequency band. The internal forces were also identified such that cross-coupling terms were not included. Internal point force propagating through each path was determined by using its point FRF and the path input. As illustrated in Figure 4, it was identified that the prediction without considering cross-coupling terms increased the overall error with about a rate of 40 % in the frequency band.

In this methodology, there are some inevitable discrepancies, which can be occurred due to some assumptions and measurement errors. Neglected moment and lateral forces as well as the energy transmission loss through the paths result in error in the force identification. Besides, measurement errors are almost unavoidable in all experimental studies. The process of measuring vibration is prone to errors introduced by various factors based on the excitation, sensor, structure and environment [24-26]. Furthermore, measured vibration is usually contaminated with other unfavorable effects such as noise. These assumptions and measurement errors result in high condition number of the matrix to be inverted. Thus, the condition number of the accelerance matrix, which is basically the ratio of the largest singular value to the smallest one, can be used as a quality indicator for the response prediction based on the matrix inversion method. It is calculated by using Equation (5) and compared with the absolute error of prediction in Figure 5.

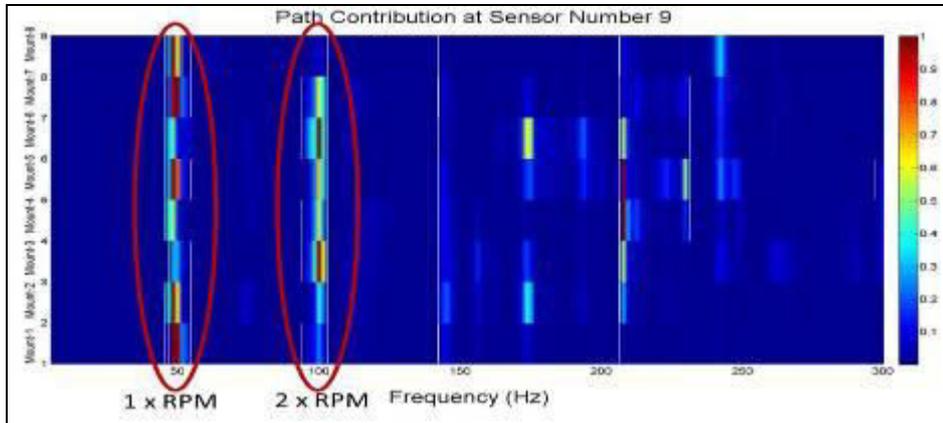
$$K_2(H) = \frac{\sigma_{max}}{\sigma_{min}} \quad (5)$$

As shown in Figure 5, the absolute error increases with increasing condition number. Measurement errors, which usually occur in any FRF measurement, amplify the prediction errors due to the inversion of the FRF matrix. Thus, it can be stated that the condition numbers serve as the parameters influencing the error.



**Figure 5.** Condition Number of the FRF Matrix vs. Absolute Error

In addition to the vibration response prediction, the dominant paths with the corresponding frequency are the focus of interest. For that purpose, path contribution plot (PCP) was generated by using partial contributions for the paths as stated in Equation (3). According to the Figure 6, 50 and 100 Hz dominate the response since they correspond to the first and second order of the pump, respectively.



**Figure 6.** Path Contribution Plot (PCP) for S9

#### **4. SUMMARY AND CONCLUSIONS**

In this study, the prediction of vibration response caused by rotating machinery was discussed with the methodology of matrix inversion. An experimental case study composed of an electric motor driven pump and its foundation was performed. One dimensional and vertical exciting force was considered and assumed to be the sum of the vertical internal path forces by ignoring the lateral forces and moments. The vibration response of the target located on the foundation was predicted by including the cross-couplings between the paths and compared with the actual measured response. The results show that the predicted response fits well with the measured one in terms of character and magnitude. However, there are some acceptable discrepancies occurred due to the combination of assumptions and measurement errors. On the other hand, it can be determined that the including cross-coupling terms improves the results with a rate of about 40 %. In conclusion, the matrix inversion method including cross-couplings can be implemented for the structures excited by a rotating machinery, when the primary concern is predicting the vibration response and determining the dominant paths for condition monitoring and design optimization.

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