Journal of Naval Sciences and Engineering
2018, Vol.14, No.1, pp.23-31
Physics/Fizik

RESEARCH ARTICLE

AN ALTERNATIVE APPROACH TO GENERATE MAXWELL ALGEBRAS

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Date of Receive: 02.04.2018 **Date of Acceptance: 18.04.2018**

ABSTRACT

Symmetries are so important for explaining our universe. From this idea, one can get more knowledge about the universe using wider symmetries. Last century shows us that the Group theoretical methods provide very effective mathematical background to construct extended symmetries. Recent years there are a lot of studies on the Maxwell symmetry which is obtained by extension of Poincare symmetry. In this paper, we present an alternative method to produce the Maxwell algebras. We show that $D = 4$ *Maxwell algebras can be obtained by inducing from* $D = 6$ *Maxwell-Lorentz algebra. From this method, some Maxwell algebras are constructed.*

Keywords: *Lie Algebras, Tensors, Group Extension, Maxwell Algebra.*

MAXWELL CEBİRLERİNİN OLUŞTURULMASI İÇİN ALTERNATİF BİR YAKLAŞIM

ÖZ

Simetrilerin kullanımı kainatın açıklanmasında büyük bir önem arzetmektedir. Bu fikirden hareketle denilebilir ki, daha geniş simetrilerin kullanılmasıyla kainatın işleyişi hakkında daha fazla bilgi edinebiliriz. Geçtiğimiz asır bize gösterdi ki Grup teorik metotlar, simetrilerin genişletilmesinde çok kullanışlı bir matematiksel temel hazırlamaktadır. Son yıllarda, Poincare simetrisinin genişletilmiş hali olan Maxwell simetrileri hakkında pek çok çalışma yapılmıştır. Biz bu çalışmada Maxwell cebirlerinin oluşturulması için alternatif bir yöntem sunduk. $D = 6$ *Maxwell-Lorentz cebrinin indirgenmesi ile* $D = 4$ *Maxwell cebirlerinin elde edilebileceğini gösterdik. Bu yöntemi kullanarak bazı Maxwell cebirlerini oluşturduk.*

Anahtar Kelimeler: *Lie Cebirleri, Tensörler, Grupların Genişletilmesi, Maxwell Cebri.*

1. INTRODUCTION

The main role of Lie algebras in constructing of Supersymmetry and Gauge theory shows us that the Group theory is very important in physics. Lie algebras represent symmetries in physics, and so one can say if we extend given Lie algebra, we obtain extended symmetry, in other words new interactions or new interacting terms. Supersymmetry is a clear example of this motivation which is obtained from extension of Poincare symmetry.

Maxwell algebra is a non-central extension of Poincare algebra [1-6] in which momentum operators are no longer abelian and it satisfies the relation $[P_a, P_b] = iF_{ab}$, where F_{ab} is antisymmetric tensor under Lorentz transformations and $a, b = 0,1,2,3$. Gauging this symmetry and supposing F_{ab} is electromagnetic stress tensor, one can get the equation of motion of charged particle which is moving along constant electromagnetic field [1].

In 2005 Soroka [7] derived new kind of Maxwell algebra by adding new six degrees of freedom into Poincare algebra. Soroka used the generator Z_{ab} instead of F_{ab} and new generator depends on new coordinate parameter

θ^{ab}. Thus, the space-time is extended as $\Phi(x^a) \to \Phi(x^a, \theta^{ab})$. This algebra is used to produce new symmetries [7-12], to obtain generalization of (super)gravity theories [13-25] and for applying higher spin field and Landau dynamics [26-28].

There are some methods about extension of Lie algebras in literature. The expansion [29-35], S-expansion [36-37], and Chevalley-Eilenberg (CE) cohomology [38-40] methods are used to obtain new algebras with changing group dimensions. If we don't need to change group dimensions it is enough to use contraction, deformation and extension methods [32-33].

In this paper, we present $D = 4$ Maxwell algebras can be obtained from $D = 6$ Maxwell-Lorentz algebra by dimensional reduction. After these notional preparations some Maxwell algebras are obtained.

2. GENERATING D = 4 MAXWELL ALGEBRAS FROM D = 6 MAXWELL-LORENTZ ALGEBRA

Commutation relationships of Maxwell-Lorentz algebra can be shown as,

$$\begin{aligned} [J_{AB}, J_{CD}] &= i(\eta_{AD}J_{BC} + \eta_{BC}J_{AD} - \eta_{AC}J_{BD} - \eta_{BD}J_{AC}), \\ [J_{AB}, Z_{CD}] &= i(\eta_{AD}Z_{BC} + \eta_{BC}Z_{AD} - \eta_{AC}Z_{BD} - \eta_{BD}Z_{AC}), \\ [Z_{AB}, Z_{CD}] &= i(\eta_{AD}Z_{BC} + \eta_{BC}Z_{AD} - \eta_{AC}Z_{BD} - \eta_{BD}Z_{AC}), \end{aligned} \tag{1}$$

where J_{AB} is the Lorentz generator, Z_{AB} is Maxwell generator in $D = 6$, the Minkowski metric is $\eta_{AB} = (+,-,-,-,-,+)$ and capital Latin indices $A, B, \ldots = 0,1,2,3,5,6$. Then if we choose following relations,

$$M_{ab} = J_{ab}, \quad P_a = \alpha J_{5a} + \beta J_{6a} + \rho Z_{5a} + \sigma Z_{6a}, \quad Z_{ab} = Z_{ab}, \tag{2}$$

we get commutation of momentum generator as follows,

$$[P_a, P_b] = i(\alpha M_{ab} - \beta M_{ab} + \alpha\rho Z_{ab} - \beta\sigma Z_{ab}). \tag{3}$$

where M_{ab}, P_a, Z_{ab} are Lorentz, momentum and Maxwell generators respectively and $\alpha, \beta, \rho, \sigma$ are arbitrary constants. After that, choosing the constants as $\alpha = 1, \beta = 0, \rho = -1, \sigma = 0$ then the AdS-Maxwell algebra [17] is obtained as given below,

$$\begin{aligned}
&[M_{ab}, M_{cd}] = i(\eta_{ad}M_{bc} + \eta_{bc}M_{ad} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac}),\\
&[P_a, P_b] = i(M_{ab} - Z_{ab}),\\
&[M_{ab}, P_c] = i(\eta_{bc}P_a - \eta_{ac}P_b),\\
&[M_{ab}, Z_{cd}] = i(\eta_{ad}Z_{bc} + \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac}),\\
&[Z_{ab}, Z_{cd}] = i(\eta_{ad}Z_{bc} + \eta_{bc}Z_{ad} - \eta_{ac}Z_{bd} - \eta_{bd}Z_{ac}),
\end{aligned} \tag{4}$$

Also, if one changes the selection with $\alpha = 0, \beta = 1, \rho = 0, \sigma = -1$ then commutation relationship of momentum generators takes following form,

$$[P_a, P_b] = -i(M_{ab} - Z_{ab}), \tag{5}$$

thus, we get dS-Maxwell algebra. Lastly, taking the constants as $\alpha = 1, \beta = 1, \rho = \frac{1}{2}, \sigma = -\frac{1}{2}$ the semi simple Maxwell algebra presented in [21] is obtained by following relation,

$$[P_a, P_b] = iZ_{ab}. \tag{6}$$

One can generate different kind of Maxwell algebras by changing the choices given in Eq. (2) and also using simple Maxwell-Lorentz algebra,

$$\begin{aligned}
&[J_{AB}, J_{CD}] = i(\eta_{AD}J_{BC} + \eta_{BC}J_{AD} - \eta_{AC}J_{BD} - \eta_{BD}J_{AC}),\\
&[J_{AB}, Z_{CD}] = i(\eta_{AD}Z_{BC} + \eta_{BC}Z_{AD} - \eta_{AC}Z_{BD} - \eta_{BD}Z_{AC}),\\
&[Z_{AB}, Z_{CD}] = 0,
\end{aligned} \tag{7}$$

or Maxwell-General-Linear group [16] given below,

$$\begin{aligned}
&[L^A{}_B, L^C{}_D] = i(\delta^C{}_B L^A{}_D - \delta^A{}_D L^C{}_B),\\
&[L^A{}_B, Z_{CD}] = i(\delta^A{}_D Z_{BC} - \delta^A{}_C Z_{BD}).
\end{aligned} \tag{8}$$

4. CONCLUSION

We showed that $D = 4$ Maxwell algebras can be obtained by dimensional reduction from $D = 6$ Maxwell-Lorentz algebras. The (A)ds-Maxwell and semi simple Maxwell algebras were constructed by using presented method. For generating different Maxwell algebras two possible $D = 6$ algebras were given.

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