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Concavity and point of inflection of a streamline function and potential function

Souradeep Hazra
Aerospace Engineering, Chandigarh University, Gharuan

Abstract:

This paper is based on the concavity andpoint of inflection of a streamline function and potential function. Though the streamline and potential function is orthogonal to each other but they must have concavity and must be having a point of inflection.

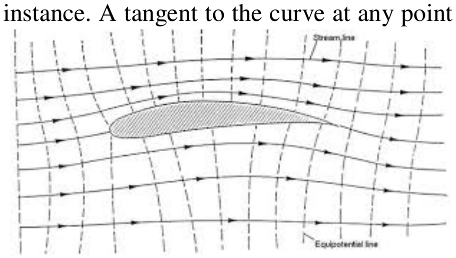
The inequalities and their proof will give the concavity and the point of inflection of the streamline function and potential function. By this research work we can determine the concavity of the imaginary streamline and potential line of a given velocity vector and can relate it to the propagation of aerofoil in air.

Keywords — **Concavity, point of inflection streamline function and potential function.**

I. INTRODUCTION

The study of the streamline and potential flow is very important in fluid mechanics.by the following equation we can determine the concavity and point of inflection in any point of the fluid flow.

Stream line is an imaginary line drawn through the flow field in such a manner that the velocity vector of the fluid at each and every point on the streamline is tangent to the streamline at that instance. A tangent to the curve at any point gives the direction of the velocity vector at that point.



Whereas a steady, irrotational flow is classified as a potential flow.

In the given figure of the aerofoil we can see that at some place the equipotential line and the stream line there is some concave portion so as a result there is a possibility of having a point of inflection. So, this can be determined by the following inequalities.

II. RELATED MATHEMATICS

CONCAVITY OF THE POTENTIAL FUNCATION: -

Where, u is the velocity component on x-direction
And v is the velocity component on y-direction
For potential function,

$$\frac{\partial \phi}{\partial x} = u$$

$$\frac{\partial \phi}{\partial y} = v$$

We know,

$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy$$

$$\Rightarrow udx + vdy = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{u}{v}$$

Let,

$$\frac{\partial \phi}{\partial x} = u \qquad \frac{\partial^2 \phi}{\partial x^2} = r$$

$$\frac{\partial \phi}{\partial y} = v \qquad \frac{\partial^2 \phi}{\partial x \partial y} = s$$

$$\frac{\partial^2 \phi}{\partial y^2} = t$$

The equation of the potential function,

$$\frac{dy}{dx} = -\frac{u}{v}$$

Then, if we find out the double derivative then we can determine the concavity.

$$\frac{\partial^2 y}{\partial x^2} = -\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \qquad \text{.........1}$$

Where,

$$\frac{du}{dx} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y \partial x}\frac{dy}{dx}$$

$$\Rightarrow \frac{du}{dx} = \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y \partial x}\left(-\frac{\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial y}}\right) = r - s\frac{u}{v}$$

$$\Rightarrow \frac{du}{dx} = \frac{vr - us}{v}$$

Similarly,

$$\frac{dv}{dx} = \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y^2}\frac{dy}{dx}$$

$$\Rightarrow \frac{dv}{dx} = \frac{\partial^2 \phi}{\partial x \partial y} + \frac{\partial^2 \phi}{\partial y^2}\left(-\frac{\frac{\partial \phi}{\partial x}}{\frac{\partial \phi}{\partial y}}\right) = s - t\frac{u}{v}$$

$$\Rightarrow \frac{dv}{dx} = \frac{vs - tu}{v}$$

On substituting the value in equation 1, we get,

$$\frac{d^2 y}{dx^2} = -\frac{v\left(\frac{vr - us}{v}\right) - u\left(\frac{vs - tu}{v}\right)}{v^2}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = \frac{-\left(v^2 r - uvs - uvs + u^2 t\right)}{v^3}$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -\frac{\left(v^2 r - 2uvs + u^2 t\right)}{v^3}$$

If,

$$\frac{d^2 y}{dx^2} > 0\left(concave\ downwards\right)$$

$$\frac{d^2 y}{dx^2} < 0\left(concave\ \text{upwards}\right)$$

And, $\frac{d^2 y}{dx^2} = 0\left(At\ point\ of\ inflection\right)$

CONCAVITY OF THE STREAMLINE FUNCATION: -

Where, u is the velocity component on x-direction

And v is the velocity component on y-direction

$$\frac{\partial \psi}{\partial x} = -v$$

$$\frac{\partial \psi}{\partial y} = u$$

We know,

$$d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy$$

$$\Rightarrow -vdx + udy = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{v}{u}$$

Let,

$$\frac{\partial \psi}{\partial x} = -v; \quad \frac{\partial^2 \psi}{\partial x^2} = r$$

$$\frac{\partial \psi}{\partial y} = u; \quad \frac{\partial^2 \psi}{\partial x \partial y} = s$$

$$\frac{\partial^2 \psi}{\partial y^2} = t$$

The equation of the streamline function,

$$\frac{dy}{dx} = \frac{v}{u}$$

Then, if we find out the double derivative then we can determine the concavity.

$$\frac{\partial^2 y}{\partial x^2} = \frac{u\frac{dv}{dx} - v\frac{du}{dx}}{u^2} \qquad \text{......2}$$

Where,

$$\frac{du}{dx}=\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\frac{dy}{dx}$$
$$\Rightarrow \frac{du}{dx}=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y\partial x}\frac{dy}{dx}$$
$$\Rightarrow \frac{du}{dx}=\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y\partial x}\left(-\frac{\frac{\partial\psi}{\partial x}}{-\frac{\partial\psi}{\partial y}}\right)=r+s\frac{v}{u}$$
$$\Rightarrow \frac{du}{dx}=\frac{ur+vs}{u}$$

Similarly,

$$\frac{dv}{dx}=\frac{\partial v}{\partial x}+\frac{\partial v}{\partial y}\frac{dy}{dx}$$
$$\Rightarrow \frac{dv}{dx}=\frac{\partial^2\psi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}\frac{dy}{dx}$$
$$\Rightarrow \frac{dv}{dx}=\frac{\partial^2\psi}{\partial x\partial y}+\frac{\partial^2\psi}{\partial y^2}\left(-\frac{-\frac{\partial\psi}{\partial x}}{\frac{\partial\psi}{\partial y}}\right)=s+t\frac{v}{u}$$
$$\Rightarrow \frac{dv}{dx}=\frac{us+tv}{u}$$

On substituting the value in equation 1, we get,

$$\frac{d^2y}{dx^2}=\frac{v\left(\frac{ur+vs}{u}\right)-u\left(\frac{us+tv}{u}\right)}{u^2}$$
$$\Rightarrow \frac{d^2y}{dx^2}=\frac{\left(v^2s+uvr-uvt-u^2s\right)}{u^3}$$
$$\Rightarrow \frac{d^2y}{dx^2}=\frac{\left(v^2s+uvr-uvt-u^2s\right)}{u^3}$$

If,

$$\frac{d^2y}{dx^2}\prec 0\left(concave\ downwards\right)$$
$$\frac{d^2y}{dx^2}\succ 0\left(concave\ \text{upwards}\right)$$

And, $\frac{d^2y}{dx^2}=0\left(At\ point\ of\ inflection\right)$

RESULT

For equipotential line:

Concavity inequation is: -

$$\frac{-\left(v^2r-2uvs-u^2t\right)}{v^3}<0\left(i.e.,\ \frac{d^2y}{dx^2}<0\right)$$
$$\Rightarrow \frac{\left(v^2r-2uvs-u^2t\right)}{v^3}>0\left(concave\ downwards\right)$$

and $-\frac{\left(v^2r-2uvs-u^2t\right)}{v^3}>0$

$$\Rightarrow \frac{\left(v^2r-2uvs-u^2t\right)}{v^3}<0\left(concave\ \text{upwards}\right)$$
$$\frac{\left(v^2r-2uvs-u^2t\right)}{v^3}=0\left(At\ point\ of\ inflection\right)$$

For streamline:

Concavity inequation is: -

$$\frac{\left(v^2s+uvr-uvt-u^2s\right)}{u^3}<0\left(i.e.,\ \frac{d^2y}{dx^2}<0\right)\left(concave\ downwards\right)$$
$$\frac{\left(v^2s+uvr-uvt-u^2s\right)}{u^3}>0\left(concave\ \text{upwards}\right)$$
$$\frac{\left(v^2s+uvr-uvt-u^2s\right)}{u^3}=0\left(At\ point\ of\ inflection\right)$$

Note: - These inequation can be called as sdh's inequation.

CONCLUSIONS

By the given derivation and the result, the potential line and the stream line is having some concavity and point of inflection at some place in the flow of fluid. In case of different flow net the concavity and point of inflection can be determined by this inequality and which can provide a different dimension in the stability of an aerofoil in air.

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Souradeep Hazra is born on 08.08.1999 on Monteswar, a village in Purba Burdhwan, West Bengal. He is currently studying Aerospace Engineering in Chandigarh University. Presently he is in Second year of his course. He is also having a research interest in the field of cosmology and space science. He has also worked in small projects based on computer programming.