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Residual Stress analysis of Equilateral Triangular Sectioned bar of Non-Linear Work- Hardening Materials

Dinkar Lal [1], Lokendra Pal Singh[2]

Department of Mechanical Engineering, Sam Higginbottom University of Agriculture, Technology and Sciences, Allahabad- 211007
Uttar Pradesh, INDIA

Abstract:

The aim of the paper is to present a theoretical analysis of growth of the elastic-plastic boundary and the resulting stresses while loading, and the torsional springback and the residual stresses after unloading are calculated.

1. Introduction

In sheet metal forming operations, springback is a decisive parameter in designing the appropriate tooling. Final part shape depends on the springback which occurs after the removal of applied loads from the deformed sheet and results in the deviation of the product from the applied tooling shape. One of the major technical issues associated with these bars is elastic recovery of material (springback) after completion of forming process. In forming operation, springback is an important consideration in designing the punch and die set. During the forming process, when the load is applied, the sheet is deformed plastically and the contour of the sheet section matches that of the die. On removal of the applied load, the sheet section takes up a different shape due to elastic recovery on removal of the applied load is commonly known as springback. Torsional spring is the measure of angle of untwist on removal of the torque after twisting the section beyond the elastic limit.

Initially springback studies were limited to sheet bending operations only. Sachs[1], Schroeder[2], Gardiner[3], Singh and Johnson[4] and others studied the springback considering bending of sheets of different shapes, and depicted springback as a function of material thickness, length and width of the sheets taken. Their studies were limited to V and U-shaped dies for applying bending loads and they predicted the springback as a measure of change in the curvature distribution.Huch[5], Nadai[6] and Upadhyay[7], have all done a number of excellent works on the elasto plastic torsion of bars with rectangular sections, but their interest has been limited to monotonically increasing loads only. Dwivedi *et al* [8,9] analytically predicted the residual angle of twist and torque relation etc. for bars of elastic strain-hardening materials with narrow rectangular sections. This works, however, has the limitation that it is valid for thin rectangular strips only. Dwivedi *et al* [10,11] dealt with the torsional springback of square-section bars of linear and non linear work-hardening materials. Dwivedi *et al* [12]also dealt with the torsional springback of L-shaped section bars of non linear work-hardening materials.

An accurate analysis of springback has been made in the past on sheet bending and tube bending operations through experiment [13-18].Torsional springback in thin tubes with non-linear work hardening analysis by Choubey *et al*[19-21]. Dwivedi *et. al.* [23] Study of Residual Stresses in I Sectioned Bars of Non-Linear work-hardening materials under torsion. Lal *et al* [24-33] Analysis the Springback of different cross sectioned bar of linear and non linear work-hardening materials under torsional loading.

In the following, a numerical scheme has been prepared for analysing the problem of torsional springback and elastic-plastic boundary in equilateral triangular cross sectioned bars of non-linear work-hardening materials.

2. Basic Theory

2.1 Elastic Torsion

Consider a prismatic bar under elastic torsion [22]. Let u , v and w be the small displacements of a point (*x*, *y*, *z*) , relative to its initial position, in the *X*-, *Y*-, *Z*-directions respectively. At a section *z* = constant, the cross-section rotates about the *Z* axis, and so

$$u = -yz\theta \quad , \quad v = xz\theta \quad \text{and} \quad w = \theta f(x, y) \qquad (1)$$

where θ is the angle of twist per unit length. For elastic deformation, θ is small and is constant along the length of bar; $\theta f(x, y)$ is called the warping function and is assumed to be independent of *z*.

If a stress function ψ is taken such that

$$\tau_{xz} = \frac{\partial\psi}{\partial y} \quad \text{and} \quad \tau_{yz} = -\frac{\partial\psi}{\partial x}, \qquad (2)$$

then the elastic torsion equation is given as

$$\nabla^2\psi = \left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}\right) = -2G\theta \qquad (3)$$

with $\psi = a$ constant (taken to be zero) along the boundary of the cross-section.

The torque T is given by

$$T = 2\iint_A \psi \, dx \, dy, \qquad (4)$$

Where A is the cross-section of the bar.

2.2 *Plastic Torsion*

A somewhat less realistic representation of the tensile stress-strain behaviour of compressible metals, other than the Ramberg-Osgood relation, is the piecewise linear relation (elastic-linearly strainharding materials). However, this approximation does model certain features of plastic flow. In particular, the behavior of aluminium alloys is very closely approximated by this type of an idealization.

If the stresses are non-dimensionalized by the yield stress, σ_Y and the strain by the corresponding yield strain

$\varepsilon_y = \sigma_y/E$, then a generalized form of the constitutive equation is given by

$$\bar{\varepsilon}_{ij} = (1+\nu)\bar{S}_{ij} + \frac{3}{2}\alpha(\bar{\sigma}_e)^{n-1}\bar{S}_{ij} + \left(\frac{1-2\nu}{3}\right)\bar{\sigma}_{kk}\delta_{ij} \qquad (5)$$

where $\bar{\varepsilon}_{ij}$ and $\bar{S}_{ij}$ are respectively the normalized strain and non-dimensional deviatoric stress components, $\bar{\sigma}_{ij} = \sigma_{ij}/\sigma_y$ and α=0.02 is the permanent plastic strain corresponding to the usual engineering definition of yield. Using the von Mises criterion, the yielding of the material is characterized by $\bar{\sigma}_e = 1$

For uniaxial tension, equation (7) is reduces to

$$\varepsilon = \frac{\sigma}{E}[1+(\frac{\sigma}{\sigma_y})^{n-1}] \qquad (6)$$

From Fig. 1, the amount of springback twist is

$$\theta_s = \theta_P - \theta_R \qquad (7)$$

Since the slope of the elastic loading line (AB) and that of the unloading line (XY) are the same, hence

$$\theta_s = \left(\frac{T_P}{T_0}\right)\theta_0 \qquad (8)$$

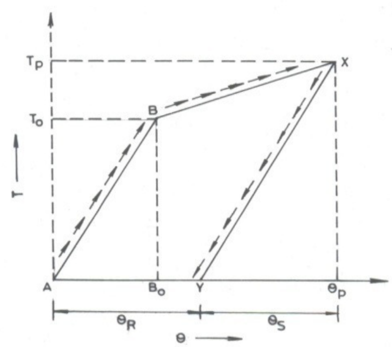

Fig. 1 Loading-Unloading curve

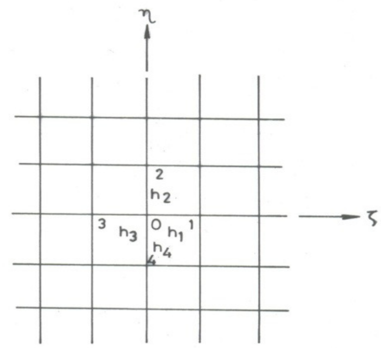

Fig. 2 Definition of grid system

Corresponding to this recovered angle of twist, the unloading problem is solved by seeking the solution of corresponding elastic torsion equation

$$\frac{\partial^2\psi_s}{\partial x^2} + \frac{\partial^2\psi_s}{\partial y^2} = -2G\theta_s \qquad (9)$$

If the stress function ψ_P corresponding to the plastic twisting θ_p at point X (Figure 1) be known by solving equation (7), then after unloading, the resulting ψ surface on the cross-section is given by

$$\psi_R = \psi_P - \psi_S \qquad (10)$$

and residual shear stress, $(\tau_{ij})_R$, are given by

$$(\tau_{xz})_R = \frac{\partial \psi_R}{\partial y}, (\tau_{yz})_R = -\frac{\partial \psi_R}{\partial x}$$

(11)

3. Scheme of numerical solution

Before coming to a numerical solution, coordinates are non-dimensionalized as

$$\xi = \frac{x}{L}, \eta = \frac{y}{L} \quad (12)$$

where L is a characteristic length of the prismatic bar. Further, a new stress function ϕ defined by

$$\phi(\xi,\eta) = 1/L^2(\psi(x,y)) \quad (13)$$

On replacing the coordinates and stress function by non-dimensionalized coordinate and ϕ, elastic torsion equation and plastic torsion equation equation reduces to

$$\nabla^2\phi = -2G\theta \quad \text{(elastic deformation)} \quad (14)$$

$$-\theta E = \{(1+\nu) + \frac{3}{2}\alpha(2-n)(\bar{\sigma}_e)^{n-1}\}\nabla^2\phi$$

$$+\frac{3}{2}\alpha(n-1)(\bar{\sigma}_e)^{n-2}\{(\bar{\sigma}_e\phi_\xi)_\xi + (\bar{\sigma}_e\phi_\eta)_\eta\}$$

(Plastic deformation) (15)

Stress components are given by

$$\tau_{xz} = L\phi_\eta, \tau_{yz} = L\phi_\xi \quad (16)$$

and therefore

$$\bar{\sigma}_e = \frac{\sqrt{3}}{\sigma_y} L[(\phi_\xi)^2 + (\phi_\eta)^2]^{1/2} \quad (17)$$

Equation (4), which gives the value of the torque becomes

$$T = 2L^4 \iint_A (\phi) d\xi\, d\eta \quad (18)$$

4. Results and Discussion

Since the numerical technique described is based on finite difference approximations, it becomes necessary to decide upon an approximate mesh size which will give a solution converging to the actual one. By carrying out actual computations, it was found that the size of suitable mesh depends on the shape of the cross-section. If the gradient of the stress function is expected to change rapidly in a particular direction then a finer mesh must be used in that direction.

For the equilateral triangular section a solution was obtained using different mesh sizes. It was seen that if we keep reducing the mesh size (h) (Fig. 2), then after a certain value of h the solution converge to a value which does not change appreciably with any further decrease in the mesh size. The solution was attempted by taking 10, 12, 14 and 15 meshes along the vertical edge of the section; a mesh size of 1/30 (i.e. 16 meshes along horizontal edge) was found to be an optimum choice from the point of the view of accuracy and the computational time, and hence, all the results in the following correspond to h=1/16.

The effect of n on the elasto-plastic boundary and on σ¯_e has been shown. The difference between the initial guess (obtained from the elastic solution) of the elasto-plastic boundary and the elasto-plastic boundary obtained from elasto-plastic solution. Some of the conclusions which can be drawn from these graphs are as follows; Elasto-plastic boundaries, for different values of θ¯_p and N for an equilateral triangular section are shown in Fig. 3 and 4, respectively. It is clear from Fig. 3 that the elasto-plastic boundary starts developing at first from the re-entrant corner, where stress concentration is very high, and with increasing values of $\bar{\theta}_p$, the plastic zone moves inward encompassing more and more of outer zones. It is also seen from Fig. 4, that work-hardening index (N) has little effect on the elasto-plastic boundary. It means that even if one commits some error in determining the value of n the elasto-plastic boundaries are hardly affected.

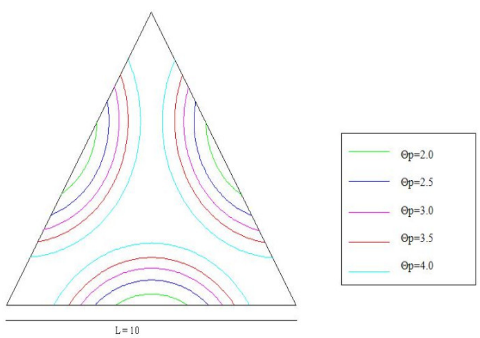

Fig 3 Elasto-Plastic boundaries of a equilateral triangular section for N=5

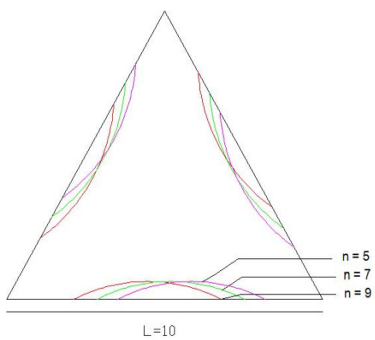

Fig.4 Elasto-Plastic boundaries of a equilateral triangular section for $\bar{\theta}_p$=4.0.

Fig.5 and Fig. 6 shows the variation of equivalent stress ($\bar{\sigma}_e$) along the line of symmetry SS' for different values of $\bar{\theta}_p$for N=5 & N=9. From the figures we come to know that as the strain hardening parameter is increased variation in stress is redused. In Fig. 7& 8 variations of this stress are shown for different values of N for $\bar{\theta}_p$ =3.0 and $\bar{\theta}_p$ =4.0. It is clear from Fig. 7 and 8 that N has a little effect on $\bar{\sigma}_e$ and $\bar{\sigma}_e$ decreases with increase in N but it is opposite in case of residual $\bar{\sigma}_e$ where it increases with increasing value of N. Figures 9 and 10 shows the variation of stress function Φ along the line of symmetry SS' for N=5 and N=9 taking $\bar{\theta}_p$ as a parameter. In fig11 and 6.0 the same is shown for $\bar{\theta}_p$=3.0and $\bar{\theta}_p$=4.0 taking N as a parameter.

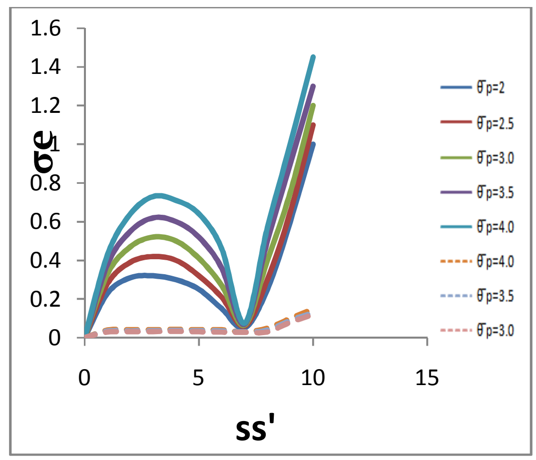

Fig.5 Equivalent stress $\bar{\sigma}_e$ along SS' for *N*=5

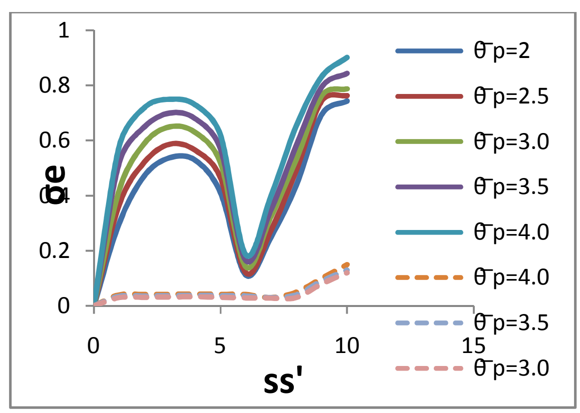

Fig. 6 Equivalent stress $\bar{\sigma}_e$ along SS' for *N*=9

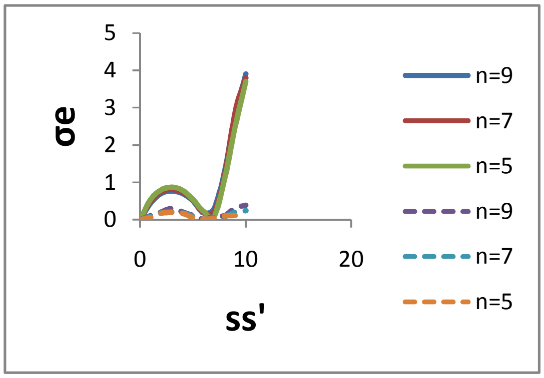

Fig7 Equivalent stress $\overline{\sigma}_e$along the line of symmetry SS' for $\overline{\theta}_p$ =3.0

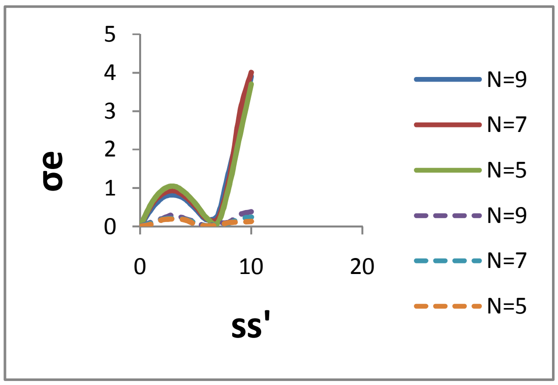

Fig8 Equivalent stress $\overline{\sigma}_e$along the line of symmetry SS' for $\overline{\theta}_p$ =4.0

4. Conclusion

The proposed numerical scheme is found to predict the torsional springback quite successfully. The accuracy of the theoretical results, of course, depends on the mesh size. The non linear strain hardening index (*n*) has little effect on the elasto-plastic boundary.

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