

# Novel PD-I Controller for Underdamped Second Order-Like Processes

Galal Ali Hassaan

Emeritus Professor, Department of Mechanical Design & Production,  
Faculty of Engineering, Cairo University, Egypt

## Abstract:

This research paper aims at investigating the use of a novel PD-I controller to control second order underdamped processes. Controller tuning based on using MATLAB control and optimization toolboxes. Using the suggested tuning technique, it is possible to eliminate completely the kick characteristic associated with PID controllers and some other tuning techniques. The tuning results of the proposed PD-I controller are tabulated for process damping ratio from 0.1 to 0.6 and from 0.7 to 0.95 while the natural frequency of the second order process covers a range from 5 to 15 rad/s

**Keywords** — Underdamped second order processes, novel PD-I controller, controller tuning, MATLAB optimization toolbox, Control system performance.

## I. INTRODUCTION

The author investigated the application of a new series of controllers and compensators for control of second and third order processes. This research aimed at replacing the first generation of the PID controllers by new ones (the second generation) providing better performance of the control systems and providing better robustness and better capability of disturbance rejection. The proposed controller is entirely new and it is expected to solve some of the disadvantages of PID controllers and opens new prospective for its application in control engineering.

Hassaan, Al-Gamil and Lashin (2013) used a PIDF controller to control a highly oscillating second order process. They tuned the controller for a second order process having 85.4 % maximum percentage overshoot Through the tuned PIDF controller, they could eliminate completely the process overshoot and reduce the settling time from 8 to 0.62 s (for the closed-loop control systems) [1]. Hassaan (2014,2015) presented a number of new controllers to control a highly oscillating second order process including an I-PD controller [2], PPI controller [3], PI-P controller [4], 2DOF controller [5], PI-PD controller [6] and a PD-PI controller [7].

Hassaan (2015) studied the rejection of disturbance associated with highly oscillating second order processes using PI-PD controller [8], PPI controller [9], PIP controller [10], cascade controller with PI and P sub-controllers [11], cascade controller with PI and PID sub-controllers [12] and cascade controller with PID plus first order compensator and P sub-controller [13].

Hassaan (2014) studied the robustness of some new controllers used to control second order processes including: PDF, PDFF, PIDF, PID plus first order controllers [14] and I-PD, PD-PI and PI-PD controllers [15]. Hassaan (2014) investigated the use and tuning of a PD-PI controller to control integrating plus delay time and first order delayed processes [16,17]. He investigated controlling a very slow second order process using a PDF controller [18] and a PDFF controller [19]. Furthermore, he investigated the disturbance rejection associated with a delayed double integrating process using a PD-PI controller [20], an I-PD controller [21], a PI-PD controller [22], a PID plus first order lag controller [23], a 2DOF controller [24] and a PI-P controller [25].

## PROCESS

The controlled process is second-order-like process having the transfer function,  $G_p(s)$ :

$$G_p(s) = (\omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2)) \quad (1)$$

Where:

$\omega_n$  = process natural frequency

$\zeta$  = process damping ratio

The damping ratio of the process is less than a unit value where the process is classified as an underdamped one.

## II. CONTROLLER

The proposed controller has a novel structure and never used before. It is a PD-I controller in which the proportional part (P-action) is connected in parallel with the derivative part (D-action) with their outputs added together through a summer circuit having an output fed to the integral part (I-action). The block diagram of the proposed controller is shown in Fig.1.

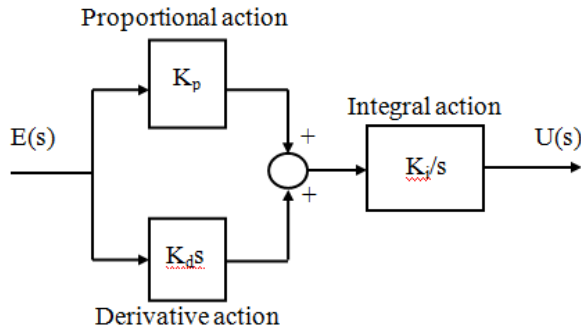


Fig.1 PD-I controller block diagram.

The proportional gain of the PD-I controller is  $K_{pc}$ , its derivative gain is  $K_d$  and its integral gain is  $K_i$ . The transfer function of the PD-I controller using the block diagram of Fig.1,  $G_c(s)$  is:

$$G_c(s) = [K_d K_i s + K_{pc} K_i] / s \quad (2)$$

## III. CONTROL SYSTEM TRANSFER FUNCTION

The block diagram of the closed-loop control system comprising the PD-I controller and the second order process is shown Fig.2 with  $R(s)$  as the reference input,  $E(s)$  is the error,  $U(s)$  is the control signal and  $C(s)$  is the process output.

Using the block diagram of Fig.2 and the process and controller transfer functions given

by Eqs.1 and 2, the closed loop transfer function of the control system to a reference input,  $C(s)/R(s)$  is given by:

$$C(s)/D(s) = (b_0 s + b_1) / [s^3 + a_1 s^2 + a_2 s + a_3] \quad (3)$$

Where:

$$b_0 = K_d K_i \omega_n^2$$

$$b_1 = K_{pc} K_i \omega_n^2$$

$$a_1 = 2\zeta\omega_n$$

$$a_2 = \omega_n^2 + K_d K_i \omega_n^2$$

$$a_3 = K_{pc} K_i \omega_n^2$$

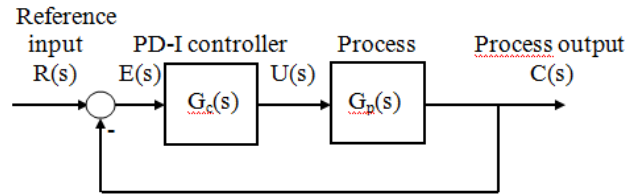


Fig.2 Control system block diagram.

## IV. CONTROLLER TUNING AND SYSTEM TIME RESPONSE

The controller has three parameters:  $K_{pc}$ ,  $K_d$  and  $K_i$ . The controller parameters are tuned as follows:

- The control and optimization toolboxes of MATLAB are used to assign the three parameters of the controller ( $K_{pc}$ ,  $K_d$  and  $K_i$ ) [26].
- The integral of absolute error (IAE) is chosen after a number of trials as an objective function for the optimization process.
- The step response of the closed-loop control system is plotted using the command 'step' of MATLAB [27].
- The time-based specifications of the control system are extracted using the MATLAB command 'stepinfo' [28].

A sample of the tuning results is shown in Table 1 for a process damping ratio in the range  $0.1 \leq \zeta \leq 0.6$  and a natural frequency in the range  $5 \leq \omega_n \leq 15$  rad/s.

The tuning process revealed very close values for the three controller parameters for the damping ratio range used in Table 1. Thus mean values were taken which are tabulated in Table 1.

**Application 1:**

Consider a second order process having a 10 rad/s natural frequency and an 0.4 damping ratio. A PD-I control is to be used to control this process. Tune the controller using Table 1, plot its unit step response and compare with the unit step response using a tuned PID controller.

TABLE 1  
PD-I CONTROLLER TUNING FOR  $0.1 \leq \zeta \leq 0.6$

Parameter $\zeta$	$K_{pc}$	$K_d$	$K_i$
0.1	1.4995	0.4287	0.2080
0.2	0.8424	0.2450	0.3983
0.3	0.8695	0.3042	0.3219
0.4	0.7989	0.3140	0.3110
0.5	0.7988	0.3111	0.3078
0.6	0.7656	0.3098	0.2972

- The tuned parameters of the PD-I controllers are given from Table 1 corresponding to  $\zeta = 0.4$ . They are:

$$K_{pc} = 0.7989$$

$$K_d = 0.3140$$

And  $K_i = 0.3110$

- Comparison with a control system comprising a tuned PID controller: Hassaan (2014) published a research work simplifying the tuning of the PID controller used with underdamped second order processes to a single set of controller parameters given by [29]:

$$K_{pc} = 10.01023$$

$$K_i = 9.00696$$

And  $K_d = 0.66373$

- The unit step response of the control system using a PD-I and PID tuned controllers for the same process is shown in Fig.3.

- The time based specifications of the control system are obtained by the 'stepinfo' command and given as:

- Maximum percentage overshoot,  $OS_{max}$ :
- $OS_{max} = 0$  with PD-I controller
- $OS_{max} = 0.9553\%$  with PID controller
- Settling time,  $T_s$ :

- $T_s = 16.66$  s with PD-I controller
- $T_s = 1.8099$  s with PID controller
- Response kick:
- Kick free with PD-I controller
- Kick at 0.06 s with PID controller.

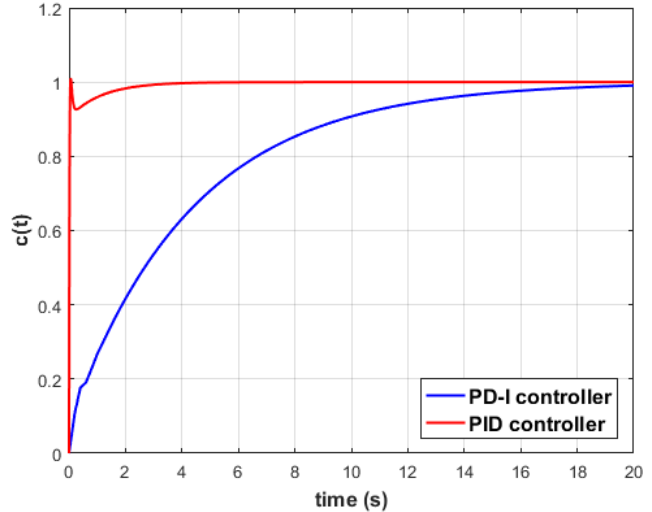


Fig.3 Step response using for  $\omega_n = 10$  rad/s and  $\zeta = 0.4$  using PD-I and PID controllers.

The second range of the process damping ratio is:  $0.7 \leq \zeta \leq 0.95$ . The PD-I controller is tuned for the specific values: 0.7, 0.8, 0.9 and 0.95 of the damping ratio. The tuning results are presented in Tables 2, 3, 4 and 5.

TABLE 2  
PD-I CONTROLLER TUNING FOR  $\zeta = 0.7$

Parameter $\omega_n$ (rad/s)	$K_{pc}$	$K_d$	$K_i$
5	1.1200	0.0871	0.9629
7	1.2991	0.0786	1.1749
9	1.4591	0.0763	1.3652
11	1.6063	0.0760	1.5405
13	1.0494	0.1069	0.8810
15	1.0494	0.1084	0.8812

TABLE 3  
PD-I CONTROLLER TUNING FOR  $\zeta = 0.8$

Parameter $\omega_n$ (rad/s)	$K_{pc}$	$K_d$	$K_i$
5	1.1175	0.0832	0.9595
7	1.2977	0.0779	1.1754
9	1.4571	0.0727	1.3627
11	1.0499	0.1010	0.8810
13	1.0494	0.1066	0.8810
15	1.0494	0.1062	0.8810

TABLE 4  
PD-I CONTROLLER TUNING FOR  $\zeta = 0.9$

Parameter $\omega_n$ (rad/s)	$K_{pc}$	$K_d$	$K_i$
1	0.6476	0.8777	0.5228
3	1.0843	0.5013	0.9581
5	1.0968	0.0844	0.9176
7	1.2694	0.0763	1.1268
9	1.6297	0.0797	1.5639
11	1.0490	0.1067	0.8621
13	1.0493	0.1060	0.8623
15	1.0498	0.1021	0.8625

TABLE 5  
PD-I CONTROLLER TUNING FOR  $\zeta = 0.95$

Parameter $\omega_n$ (rad/s)	$K_{pc}$	$K_d$	$K_i$
5	1.4700	0.4722	1.4011
7	1.7342	0.4465	2.0561
9	2.1726	0.4548	2.3410
11	2.5701	0.4475	2.5956
13	2.5547	0.3575	2.6460
15	2.6432	0.3086	2.7236

**Application 2:**

Consider a second order process having a 11 rad/s natural frequency and an 0.7 damping ratio. A PD-I control is to be used to control this process. Tune the controller using Table 2, plot its unit step response and compare with the unit step response using another tuning technique. Compare with the time response of the control system if a PID controller is used instead of the PD-I one.

- Tuned PD-I controller parameters (Table 2):
  - $K_{pc} = 1.6063$
  - $K_d = 0.0760$
  - $K_i = 1.5405$
- Using the ITAE standard form [29] provides two equations in  $K_{pc}$ ,  $K_d$  and  $K_i$ . One of the controller parameters has to be assumed. Therefore,  $K_{pc}$  is assumed having the value of the tuned PD-I controller using the present technique and the other two unknowns are evaluated from the attained two equations. The result is:
  - $K_{pc} = 1.6063$
  - $K_d = 0.3080$
  - $K_i = 3.5062$
- Using Eq.3 and the 'step' command of the MATLAB, the unit step response of the control system incorporatind a PD-I

controller and the second order process under study is shown in Fig.4 using the two tuning techniques.

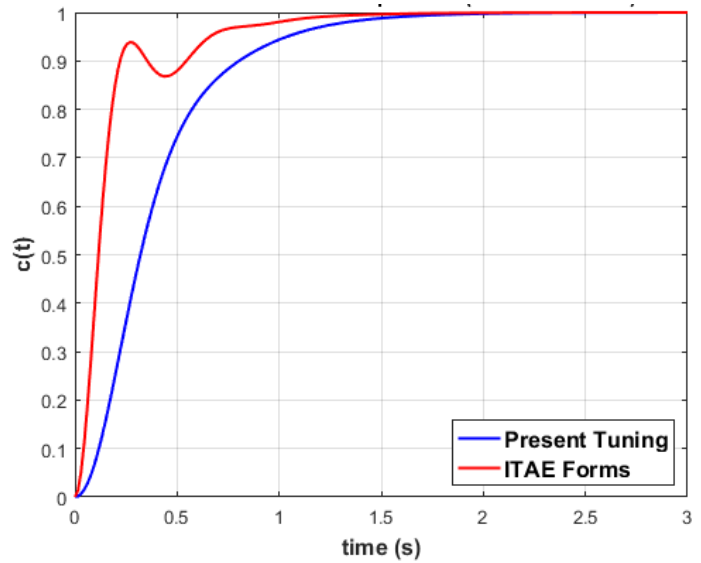


Fig.4 Step response using Table 2 for  $\omega_n = 11$  rad/s and  $\zeta = 0.7$ .

- The time based specifications of the control system are obtained by the 'stepinfo' command and given as:
  - Maximum percentage overshoot,  $OS_{max}$ :
    - $OS_{max} = 0$  with present tuning
    - $OS_{max} = 0$  with ITAE forms
  - Settling time,  $T_s$ :
    - $T_s = 1.3312$  s with present tuning
    - $T_s = 0.9934$  s with ITAE forms
  - Response kick:
    - Kick free with present tuning
    - Kick at 0.25 s with ITAE forms.

**Comparison with using a PID controller:**

- PID controller parameters: as in Application 1 using Hassaan's simplified tuning technique for PID controllers [29].
- The unit time response of the controlled process is shown in Fig.5 for both controllers.

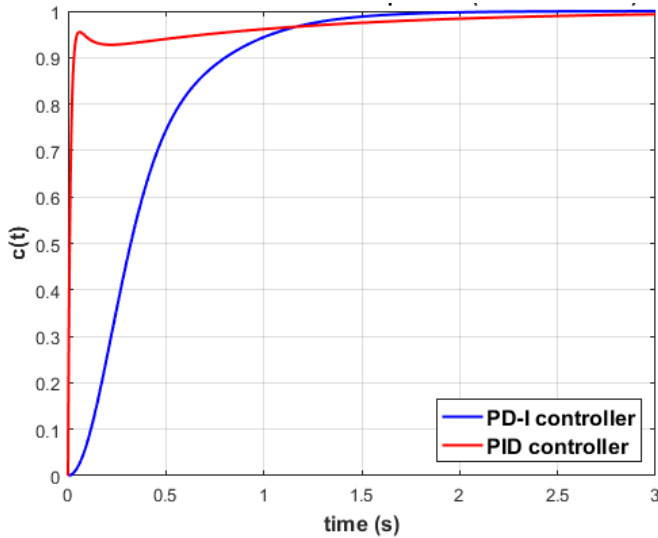


Fig.5 Step response using for  $\omega_n = 11$  rad/s and  $\zeta = 0.7$  using PD-I and PID controllers.

- The time based specifications of the control system are obtained by the 'stepinfo' command and given as:
  - Maximum percentage overshoot,  $OS_{max}$ :
    - $OS_{max} = 0$  with PD-I controller
    - $OS_{max} = 0$  with PID controller
  - Settling time,  $T_s$ :
    - $T_s = 1.3312$  s with PD-I controller
    - $T_s = 1.7594$  s with PID controller
  - Response kick:
    - Kick free with PD-I controller
    - Kick at 0.06 s with PID controller.

### V. CONCLUSION

- The dynamic problem of tuning a novel PD-I controller for use with second-order-like underdamped processes was investigated.
- The PD-I controller was tuned using control and optimization toolboxes with an integral of the absolute error as an objective function.
- The tuning results were presented for two ranges of damping ratio: from 0.1 to 0.6 and from 0.7 to 0.95 and natural frequency of the second order system from 5 to 15 rad/s.

- The performance of the control system using the proposed controller was compared with that using a classical PID controller.
- The used tuning technique was compared with another technique based on the ITAE standard forms.
- In the two applications studied in this research work, the tuned PD-I controller could eliminate completely the kick characteristics of the time response appeared with PID controllers and the ITAE standard forms tuning technique.
- It was succeeded to produced a zero maximum overshoot for most of the process parameters under study.
- With process damping ratio  $\geq 0.7$ , the PD-I controller could produce a faster step time response (compared with using a PID controller).

### REFERENCES

1. G. Hassaan, M. Al-Gamil and M. Lashin, "Tuning a PIDF controller used with a highly oscillating second order process", *International Journal of Emerging Technology and Advanced Engineering*, vol.3, issue 3, pp.943-945, 2013..
2. G. Hassaan, "Tuning of an I-PD controller used with a highly oscillating second order process", *International Journal of Mechanical Engineering and Technology*, vol.5, issue 5, pp.115-121, 2014.
3. G. Hassaan, "PPI controller tuning for use with a highly oscillating second order process", *International Journal of Science and Engineering*, vol.1, issue 3, pp.32-37, 2015.
4. G. Hassaan, "Tuning of an PI-P controller used with a highly oscillating second order process", *International Journal of Research in Information Technology*, vol.3, issue 7, pp.206-231, 2015.
5. G. Hassaan, "Tuning of a 2DOF controller used with a highly oscillating second order process", *International Journal of Modern Trend in Engineering and Research*, vol.2, issue 8, pp.292-298, 2015.
6. G. Hassaan, "Tuning of an PI-PD controller used with a highly oscillating second order process", *International Journal of Innovation Technology*, vol.1, issue 5, pp.67-69, 2014.
7. G. Hassaan, "Tuning of an PD-PI controller used with a highly oscillating second order process", *International*



- Journal of Scientific and Technology Research*, vol.3, issue 7, pp.145-147, 2014.
8. G. Hassaan, "Disturbance rejection with highly oscillating second order-like process, Part III: PI-PD controller", *Journal of Research in Science, Technology, Engineering and Management*, vol.1, issue 1, pp.53-58, 2015.
  9. G. Hassaan, "Disturbance rejection with highly oscillating second order-like process, Part VI: PPI controller", *International Journal of Science and Engineering*, vol.1, issue 9, pp.20-27, 2015.
  10. G. Hassaan, "Disturbance rejection with highly oscillating second order-like process, Part VII: PIP controller", *World Journal of Engineering Research and Technology*, vol.1, issue 2, pp.1-10, 2015.
  11. G. Hassaan, "Disturbance rejection with highly oscillating second order-like process, Part VIII: Cascade controller with PI and P sub-controllers", *International Journal of Research Engineering Science*, vol.15, pp.1-5, 2015.
  12. G. Hassaan, "Disturbance rejection with highly oscillating second order-like process, Part IX: Cascade controller with PI and PID sub-controllers", *International Journal of Advancement in Engineering Technology, Management and Applied Science*, vol.2, issue 9, pp.6-25, 2015.
  13. G. Hassaan, "Disturbance rejection with highly oscillating second order-like process, Part XI: Cascade controller with PID plus first order compensator and P-sub-controller", *International Journal of Advancement in Engineering Technology, Management and Applied Science*, vol.2, issue 9, pp.67-76, 2015.
  14. G. Hassaan, "Robustness of PDF, PDFF, PIDF and PID plus first order controllers used with second order processes", *International Journal of Research Innovative Technology*, vol.1, issue 4, pp.56-61, 2014.
  15. G. Hassaan, "Robustness of I-PD, PD-PI and PI-PD controllers used with second order processes", *International Journal of Scientific and Technology Research*, vol.3, issue 10, pp.27-31, 2014.
  16. G. Hassaan, "Tuning of a PD-PI controller used with an integrating plus delay time process", *International Journal of Scientific and Technology Research*, vol.3, issue 9, pp.309-313, 2014.
  17. G. Hassaan, "Tuning of a PD-PI controller for first order delayed processes", *International Journal of Engineering Research and Technology*, vol.3, issue 4, pp.2751-2755, 2014.
  18. G. Hassaan, "Tuning of a PDF controller for a very slow second order process", *Elixir Mechanical Engineering Journal*, vol.75, pp.27501-27503, 2014.
  19. G. Hassaan, "Tuning of a PDFF controller for a very slow second order process", *International Journal of Advanced Research in Computer Science and Technology*, vol.2, issue 3, V2, pp.175-178, 2014.
  20. G. Hassaan, "Controller tuning for disturbance rejection associated with a delayed double integrating process, Part I: PD-PI controller", *International Journal of Computer Techniques*, vol.2, issue 3, pp.110-115, 2015.
  21. G. Hassaan, "Controller tuning for disturbance rejection associated with a delayed double integrating process, Part II: I-PD controller", *International Journal of Science and Engineering*, vol.1, issue 3, pp.1-8, 2015.
  22. G. Hassaan, "Controller tuning for disturbance rejection associated with a delayed double integrating process, Part III: PI-PD controller", *International Journal of Recent Engineering Science*, vol.13, pp.1-8, 2015.
  23. G. Hassaan, "Controller tuning for disturbance rejection associated with a delayed double integrating process, Part IV: PID plus first order lag controller", *International Journal of Advanced Research in Computer Science and Technology*, vol.3, issue 3, pp.67-71, 2015.
  24. G. Hassaan, "Controller tuning for disturbance rejection associated with a delayed double integrating process, Part V: 2DOF controller", *International Journal of Engineering and Technology*, vol.1, issue 4, pp.26-31, 2015.
  25. G. Hassaan, "Controller tuning for disturbance rejection associated with a delayed double integrating process, Part VI: PI-P controller", *International Journal of Advanced Research in Management, Architecture, Technology and Engineering*, vol.1, issue 1, pp.1-5, 2015.
  26. C. Lopez, "MATLAB optimization techniques", Springer, 2014.
  27. L. Moysis and T. Azar, "Introduction to control system design using MATLAB", *International Journal of System Dynamics Applications*, vol.6, issue 3, pp.130-170, 2017.
  28. Mathworks, "stepinfo", <https://www.mathworks.com/help/control/ref/stepinfo.html>, 2018.
  29. G. Hassaan, "On simple tuning of PID controllers for underdamped second order processes", *International Journals of Mechanical and Production Engineering Research and development*, vol.4, issue 3, pp.61-68, 2014.
  30. D. Graham and R. Lathrop, "The synthesis of optimal response, criteria and standard forms", *Transactions AIEE*, vol.72, issue 5, pp.273-288, 1953.

## **BIOGRAPHY**



### **Galal Ali Hassaan**

- Emeritus Professor of System Dynamics and Automatic Control.
- Has got his B.Sc. and M.Sc. from Cairo University in 1970 and 1974.
- Has got his Ph.D. in 1979 from Bradford University, UK under the supervision of Late Prof. John Parnaby.
- Now with the Faculty of Engineering, Cairo University, EGYPT.
- Research on Automatic Control, Mechanical Vibrations, Mechanism Synthesis and History of Mechanical Engineering.
- Published 10's of research papers in international journals and conferences.
- Author of books on Experimental Systems Control, Experimental Vibrations and Evolution of Mechanical Engineering.
- Chief Justice of International Journal of Computer Techniques.
- Member of the Editorial Board of some International Journals including the International Journal of Engineering and Techniques.
- Reviewer in some international journals.
- Scholars interested in the author's publications can visit:

<http://scholar.cu.edu.eg/galal>