Characterization of Simulator for Water Level Control in the Tank – Single Loop
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Abstract: The simulator industrial process has been created with one of its aims to simulate the water level control in the tank-single loop, to obtain the characteristics of the simulator, then created of modeling mathematical and simulation using MATLAB/Simulink, the results of modeling and simulation showed that, gain of the process tank is 2654 m/s² and the time constant is 124 seconds. Furthermore, through the Root Locus stability method and step input test, it was known that the simulator is stable.

Keywords — Characteristic, simulator, modeling and simulation, water level control in the tank.

I. INTRODUCTION
Liquid level control in the tank is one of the most important aspects applied to many plants in the process industry [1], such as petroleum and gas industries, chemical industry, environmental protection, water treatment, metallurgy and other industries. For the examples of level control applications in the plant of the process industry are control levels in the boiler or in the pressurized tank. The purpose of the level control is to ensure the quality and quantity of raw materials, semi-finished products, or finished products in a container by checking and adjusting the balance between the input and the material output of the container [2].

In the Research Center for Metrology – Indonesia Institute of Sciences has been created a simulator to simulate industrial process measurement and control system such as level control-single loop and cascade method using PID controller. The simulator consists of a mini plant which a used to measure and control the water level in the tank, flow rate in the pipe and it used for the industry technician training and research [3].

In this paper, a model and simulation of water level control in the tank-single loop is designed using MATLAB/Simulink to obtain the characteristics of the simulator.

II. SIMULATOR OF WATER LEVEL CONTROL
The process industry simulator that has been made is shown in Fig. 1.

The water level control system in the simulator is a water level control system using PID controllers in open tanks, where the water flows out from the tank by gravity (gravity flow tank) [4] as shown in piping and instrumentation diagram in Fig. 2.

The work process in Fig. 2 is described as follows: Water is sucked from the reservoir by the pump, and then flows into the tank, the water flow rate that flows into the tank is regulated by valve 2 manually in the tank, there is a valve 1 which has a function to regulate the flow rate of water flowing out from the tank.
As for the working process of automatic control of the system, the level transmitter (LT) type of differential pressure measures water level in the tank. The measurement results of LT is electrical current output signal with the range is from 4 mA to 20 mA, it is become input to the level controller (LC) using PID Controller. Then LC calculates the difference between signal input value of LT with set point on LC, that is water level desired. If there is an error then the LC ordered the control valve (CV) through current to pressure (I/P) to open or close the valve. Consequently, the flow of water into the tank becomes increased or decreased. The process continues until error is zero. The work process of level control is also shown through the block diagram in Fig. 3.

![Fig. 2. P&ID of water level control-single loop on simulator](image)

**III. METHODOLOGY**

The model of the components diagram block of in Fig. 3 is based on the basic parameter of the simulator design and the equipment specifications as well as the instruments used in the system as shown in Table I.

<table>
<thead>
<tr>
<th>Components</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tank height</td>
<td>1.25 meter</td>
</tr>
<tr>
<td>Level maximum height</td>
<td>0.8 meter</td>
</tr>
<tr>
<td>Level minimum height</td>
<td>0.2 meter</td>
</tr>
<tr>
<td>Tank diameter</td>
<td>0.25 meter</td>
</tr>
<tr>
<td>Maximum flow rate</td>
<td>19 liter/minute</td>
</tr>
<tr>
<td>Pipe diameter for water out of the tank</td>
<td>0.5 inch</td>
</tr>
<tr>
<td>Differential Pressure Transmitter</td>
<td>Input 0 to 100 IN H2O Output 4 to 20 mA</td>
</tr>
<tr>
<td>Current to pressure (I/P)</td>
<td>Input 4 to 20 mA Output 0.2 to 1 bar</td>
</tr>
<tr>
<td>Controller</td>
<td>PID Controller</td>
</tr>
<tr>
<td>Control valve</td>
<td>0.5 inch</td>
</tr>
</tbody>
</table>

**A. Process Tank**

Process tank is representation of the water level changes in the tank. The input signal of the tank is the water flow rate $q_{in}$ (m$^3$/s), while the output signal is the water level in the tank, $h$ (m). The relationship between $q_{in}$ and $h$ is shown in the block diagram in Fig. 4.

![Fig. 3. Block diagram of level control](image)

The water level in the tank $h$ is determined to be a maximum of 0.8 m, when the maximum flow rate of water entering the tank $q_{in}$ 19 liters/minute = 0.0003154 m$^3$/s, obtained by adjusting the flow rate of water flowing out from the tank $q_{out}$ through the regulated valve K1 as shown in Fig. 2.

The model of the relationship between $q_{in}$ and the valve opening K1, $q_{out}$ and water level in the tank $h$ in an equilibrium condition is modeled by the gravity flow tank model as shown in Fig 5.

The water level $h$ (m) which occurs in the tank is obtained by the following equation [5]:

$$\frac{dh}{dt} = q_{in}(t) - q_{out}(t)$$  \hspace{1cm} (1)
Where $A$ is the area of the tank (m$^2$), $q_{in}$ is water flow rate filling the tank (m$^3$/s) and $q_{out}$ is water flow rate out of the tank (m$^3$/s).

By using the Bernoulli equation, the flow rate of water flowing out from the tank bottom hole $q_{out}$ is obtained by the following equation,

$$q_{out} = a \sqrt{2gh}$$

(2)

Where $g$ is gravity (m/s$^2$) and $a$ is area of opening valve $K_1$ (m$^2$) as shown in Fig.2.

By substituting equation (2) to equation (1), the formula of $A \frac{dh}{dt}$ is obtained as follows:

$$A \frac{dh}{dt} = q_{in} - a \sqrt{2gh}$$

(3)

Then, on the steady state $dh/dt = 0$, the equation (3) becomes:

$$0 = q_{in} - a \sqrt{2gh}$$

(4)

From equation (4) in the steady state $q_{in} = q_{out}$, in the water level $h$ is also in the steady state, then to obtain the opening area of the valve $a$ in order to obtain the desired water level, equation (5) is obtained as follows:

$$a = \frac{q_{in}}{\sqrt{2gh_{ss}}}$$

(5)

where $h_{ss}$ is height of water level in steady state (m).

The level $h(t)$ that occurs due to the flow rate of water filling the tank $q_{in}$, can be obtained from equation (3) as shown as by equation (6):

$$\frac{dh}{dt} = \frac{q_{in}}{A} - \frac{a}{A} \sqrt{2gh}$$

(6)

The deviation for the equilibrium point $h$ is obtained by following equation:

$$h = h_{ss} + \Delta h$$

(7)

where $h_{ss}$ is water level in steady state (m). By modifying equation (5), the $h_{ss}$ is obtained by following equation:

$$h_{ss} = \frac{q_{in,ss}^2}{2ga^2}$$

(8)

where $q_{in,ss}$ is water flow rate flowing to the tank in the steady state condition. Then, the combination of equation (6) and equation (7) is embedded by equation as follows:

$$\frac{d}{dt}(h_{ss} + \Delta h) = \frac{q_{in}}{A} - \frac{a}{A} \sqrt{2gh}$$

(9)

Because $h_{ss}$ is constant, equation (9) becomes:

$$\frac{d\Delta h}{dt} = f(q_{in}, h) = \frac{q_{in}}{A} - \frac{a}{A} \sqrt{2gh}$$

(10)

Using first order Tylor series, the equation (10) can be written as follows:

$$\frac{d\Delta h}{dt} = \frac{\Delta q_{in}}{A} - \frac{a}{2A\sqrt{h_{ss}}} \Delta h$$

(11)

So equation (11) is the model of the water level system in the tank with the gravity flow tank model where the system is linear.

Laplace transform of equation (11) with assuming $\Delta h(0) = 0$ can be obtained from the equation as follows:

$$s \Delta h = \frac{\Delta q_{in}}{A} - \frac{a}{2A\sqrt{h_{ss}}} \Delta h$$

(12)

Then, the algebraic equation for $\Delta h(s)$ can be written as follow:
\[ \Delta h = \frac{1}{A} \frac{a}{\sqrt{2} g} \Delta q_{in} \]  \hspace{1cm} (13)

Because transfer function for \( G(s) \) is,
\[ G(s) = \frac{\Delta h}{\Delta q_{in}} = \frac{1}{s + \frac{a\sqrt{2} g}{2 A} \frac{h_{ss}}{a}} \]  \hspace{1cm} (14)

Then equation (14) is solved to obtain gain and time constant, now, equation (14) can be written as follow:
\[ G(s) = \frac{\sqrt{2} h_{ss}}{A \sqrt{2} h_{ss} g + 1} \]  \hspace{1cm} (15)

Or by using equation (15) can be written in another form as written by equation (16) as follows:
\[ G(s) = \frac{\Delta q_{in, ss}}{A \Delta q_{in, ss} g + 1} \]  \hspace{1cm} (16)

Equation (16) is a form of a first-order transfer function commonly shown by equation (17).
\[ G(s) = \frac{K}{T s + 1} \]  \hspace{1cm} (17)

where \( K \) is gain and \( T \) is time constant (seconds) so from equation (15), \( K \) and \( T \) is obtained as follow:
\[ K = \frac{\sqrt{2} h_{ss}}{a / g} \]  \hspace{1cm} (18)

and,
\[ T = \frac{A \sqrt{2} h_{ss}}{a / g} \]  \hspace{1cm} (19)

Then, \( K \) and \( T \) from equation (16) is obtained as follow:
\[ K = \frac{\Delta q_{in, ss}}{a^2 g} \]  \hspace{1cm} (20)

and,
\[ T = \frac{A \Delta q_{in, ss}}{a^2 g} \]  \hspace{1cm} (21)

The equations 18, 19, 20 and 21 are linear model equations for obtaining \( K \) and \( T \). Then for validation, the equations are compared with the calculation result. The simulation result with the non linear model equation, equation (6) and the gain \( K \) from non-linear model simulation results is obtained through equation (22), as follows: [6]
\[ K = \frac{\text{output on steady state}}{\text{input on steady state}} = \frac{\Delta y}{\Delta x} \]  \hspace{1cm} (22)

As for obtaining the time constant, the equation (23) is used: [8]
\[ \tau = K A \]  \hspace{1cm} (23)

where \( A \) is tank area (m²)

**B. Level Transmitter**

Level Transmitter (LT) is an instrument to measure water level in the tank. The type of LT which is used in this experiment is the differential pressure transmitter. The LT measures the level by measuring the pressure difference \( P \) (Pa) between the pressure inside the tank caused by the water level with the relative pressure to the atmosphere, as shown in Fig. 6 [6.7],

\[ h_m \] is level measuring span (m), \( L_H \) is maximum water level (m) measured from the bottom of the tank, \( L_L \) is minimum level of measurement (m) and \( P_m \) is pressure of measurement (Pa).

The level measurement using differential pressure transmitter is represented in the block diagram as shown in Fig. 7. [4]
From Figure 6, the span level measurement on the simulator uses the following equation:

$$h_m = L_H - L_L$$  \hspace{1cm} (28)

The equation for the maximum measurement pressure signal $P_m$ of the differential pressure transmitter is obtained from equation (29):

$$P_m = \frac{\Delta P}{\Delta h} h_m$$  \hspace{1cm} (29)

The differential pressure transmitter converts the pressure input signal, into an output current signal with a range of 4 to 20 mA. Thus, $G_2$ differential pressure transmitter gain is obtained by the following equation:

$$G_2 = \frac{I(t)}{P(t)} = \frac{I_{\text{max}} - I_{\text{min}}}{P_m}$$  \hspace{1cm} (30)

Where $I_{\text{max}} = 20 \text{ mA}$ and $I_{\text{min}} = 4 \text{ mA}$

Then, $G_{LT}$ in Fig. 7 is obtained by the following equation:

$$G_{LT} = G_1 G_2$$  \hspace{1cm} (31)

C. Final Control Element (FCE)

The final control element using control valve as shown in Fig. 8, the control valve as an actuator to control the flow rate of water filling in the tank. The valve is mounted to the pipe where water flows through the valve body. The size of opening that liquid flows through is given by the position of the valve stem. This is controlled by changing the pressure on one side of the diaphragm which causes a change in the position of the plug. It can be done because it is controlled by the pressure changes that occur in the diaphragm, it causes the position of the plug changed. The pressure signal to the diaphragm is obtained from the current to pressure converter (I/P), it is a device that converts an electrical signal (4 mA to 20 mA) to a proportional pressure signal output (0.2 bar to 1 bar or 20 kPa to 100 kPa) [4].
The control valve is illustrated in the block diagram as shown in Fig. 9.

\[
\text{Current, mA} \quad \rightarrow \quad \text{Pressure, Pa} \quad \rightarrow \quad \text{Stem position, m} \quad \rightarrow \quad \text{Flow rate, m}^3/\text{s}
\]

\[
G_3 \quad \rightarrow \quad G_4 \quad \rightarrow \quad G_5
\]

\[
\text{Current, mA} \quad \rightarrow \quad \text{Flow rate, m}^3/\text{s}
\]

\[
G_4 \quad \rightarrow \quad G_5
\]

Fig. 9. Block diagram of control valve [4]

The G3 block diagram is the gain of current to pressure converter (I/P) which is obtained by equation as written as follows [4]:

\[
G_3 = \frac{P_c(t)}{I(t)} = \frac{P_{\text{max}} - P_{\text{min}}}{I_{\text{max}} - I_{\text{min}}}
\]  

(32)

Where \( P_c \) is output pressure signal (kPa) from I/P, and \( I \) is input signal to I/P (mA) as shown in tabel I.

In the G4 block diagram is the relation between the pressure acting on the diaphragm to the valve stem position is obtained using the following equation:

\[
P_c(t) A_d = K_s x(t)
\]  

(33)

Where \( A_d \) is diaphragm area (m\(^2\)), \( K_s \) is spring stiffness constant (N/m) and \( x \) is change in stem position (m).

Equation (33) is solved to obtain \( x(t) \) so that the equation for \( x(t) \) can be written as follows:

\[
x(t) = \frac{A_d}{K_s} P_c(t)
\]  

(34)

Then G4 gain is obtained by using equation as follows:

\[
G_4 = \frac{x(t)}{P_c(t)}
\]  

(35)

The G5 diagram block is relationship between valve stem position with flow rate of water flowing out of valve.

The flow rate of water \( q(t) \) (m\(^3\)/s) that passes through the valve is given by the following equation [4]:

\[
q(t) = \alpha(t) C_v \sqrt{\frac{\Delta P(t)}{\rho}}
\]  

(36)

Where \( \alpha(t) \) is the fractional opening of the valve (m), \( C_v \) is the flow coefficient of the valve, \( \Delta P(t) \) is the is pressure drop across the valve (Pa) and \( \rho \) is the density of the liquid (kg/m\(^3\)). Using linear valve type, the value of \( \alpha \) is the stem position \( x(t) \) of the valve. Assume that pressure drop across the valve is constant, the water flow through the valve is linearly related to the valve stem position. Hence the process valve model is given by the following equation [4]:

\[
q(t) = G_5 x(t)
\]  

(37)

From equation (37) G5 is obtained by equation (38):

\[
G_5 = \frac{q(t)}{x(t)} = \frac{\text{m}^3/\text{s}}{\text{m}}
\]  

(38)

Since the used valve has linear characteristics, the G5 gain is calculated as follows:

\[
G_5 = \frac{q_{\text{max}}}{\text{change in stem position 100%}}
\]  

(39)

Where \( q_{\text{max}} \) is flow rate maximum of water (m\(^3\)/s).

Then \( G_V \) in Fig. 9 is obtained by following equation:

\[
G_V = G_3 G_4 G_5
\]  

(40)
IV. RESULT AND DISCUSSION

A. Process Tank

For the water level h in the tank will reach a maximum height of 0.8 m with the flow rate of water filling to a tank \( q_{in} \) set to a maximum of 19 liters/min or 0.0003154 \( m^3/s \) and \( g = 9.8 \) \( m/s^2 \) obtained by adjusting the opening of the valve K1 as shown in Figure 2. Then by using equation (5), it is found that the opening area of the valve K1 is 7.97 x 10\(^{-5}\) \( m^2 \), as shown in Fig. 5.

Further, based on equation (6) a simulation for obtaining process dynamics at the level \( h(t) \) that occurs when the water flow rate \( q_{in} = 0.0003154 \) \( m^3/s \) fills in the tank and \( a = 7.97 \times 10^{-5} \) \( m^2 \), using Simulink as shown in Fig. 10, the simulation results as shown in Fig. 11.

Based on the response curve of Fig. 11 and the equation (22) the gain \( K = 2536.46 \) \( s/m^2 \) can be obtained. Then using equation (23), where the tank area \( A = 0.05 \) m\(^2\), the time constant is \( \tau = 124 \) seconds.

The values of \( K \) and \( \tau \) are obtained by the equation of the non-linear model of the process tank equation (22) and equation (23). As for the linear model, from the simulation results and comparing the results of non-linear model calculations by using equation (18) and equation (19), the water level at steady state \( h_{ss} \) is obtained, where the level at steady state is 0.2 m.

The flow rate of water at steady state \( q_{oss} \) using equation (20) and equation (21) is obtained, \( q_{oss} = 0.0001577 \) \( m^3/s \).

For validation of the linear tank model, where \( h_{ss} = 0.2 \) m and \( q_{in,ss} = 0.0001577 \) \( m^3/s \), by simulation using Simulink, the linear model tank response curve and non-linear model of water level in the tank are compared the flow rate of water input is in steady state, transfer function of linear model using value of \( K \) and \( \tau \) obtained by equation of non linear model as follows:

\[
G(s) = \frac{2536.46}{124.44s + 1}
\]

The Simulink programming for validation is shown in Fig. 12.

Then, result of the simulation is shown in Fig. 13 is known that the steady state of level response curve for linear tank model is equal to the non-linear tank model, or the steady state of level response curve for linear tank model is coinciding with the non-linear tank model.
Thus, the linearity of the tank model with $K = 2536.46$ and time constant $\tau = 124$ seconds is validated. The block diagram of Fig. 4 can be simplified as shown in Fig. 14.

**B. Level Transmitter**

In the block diagram of Fig. 8, the $G_1$ is obtained as follows: by using the equation (24), where $\rho \approx 1000$ kg/m$^3$ and $g = 9.8$ m/s$^2$, $P(t) = 9810 \ h(t)$. Then, using equation (25), $G_1$ gain is obtained, $G_1 = 9810 \ Pa/m$

Furthermore $G_2$, based on the differential pressure transmitter specification, it is known, that is, $\Delta P_{\text{max}} = 100$ IN H$_2$O $\approx 25000$ Pa, $\Delta P_{\text{min}} = 0$

Then, using equation (26) is obtained $\Delta P$, $\Delta P = 25000$ Pa.

The maximum level measured by the differential pressure transmitter be calculated by using equation (27) and the result is, $\Delta h = 2.55$ meter.

Then, using equation (28) and Fig. 6, where LH = 0.8 m and LL = 0.2 m, the span measuring the water level in the tank $h_{\text{m}} = 0.6$ meter.

Thus, using equation (29), the maximum pressure measured by the differential pressure transmitter $P_{\text{m}}$ is obtained $P_{\text{m}} = 5864.55$ Pa and the pressure range measured by the differential pressure transmitter is 0 to 5864.55 Pa.

The pressure input is converted by the differential pressure transmitter into an output current signal of 4 mA to 20 mA. By using equation (30), $G_2$ gain is obtained, $G_2 = 0.00273 \ mA/Pa$

Then, using equation (31), the gain of $G_{LT}$ is obtained, $G_{LT} = 26.76 \ mA/m$.

So the block diagram of level measurement using differential pressure transmitter becomes as shown in Fig. 1:

Now the block diagram for $G_{LT}$ in the form of Laplace transform can be presented as shown in Fig. 16.

**C. Final Control Element (FCE)**

The final control element (FCE) in this system is a control valve with a linear characteristic.

The block diagram $G_3$ as shown in Fig.10, consists of current input signal $I$ between 4 mA to 20 mA to I/P. It produces a pressure output $P_c$ between 0.2 bar to 1 bar = 20 kPa to 100 kPa. By using the equation (32), $G_3$ gain is obtained, $G_3 = 5 \ kPa/\ mA$

Assume that the diaphragm area $A_d$ is 0.022 m$^2$ and the spring constant $K_s$ is 173228 N/m. By using equation (34), $G_4$ gain is obtained, $G_4 = 1.27 \times 10^{-7} \ m^3/N$

Or by using equation (35) $G_4$ gain is $1.2 \times 10^{-7} \ m/Pa$. 

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Fig.13. Water level response curve in linear model tank, and non-linear model, to steady-state flow rate of water input $q_{\text{ss}} = 0.0003154 \ m^3/s$

Fig.14. The tank model has been linear

Fig.15. The component values of the LT block diagram

Fig.16. Block diagram of LT with gain
In block diagram G5, the maximum flow rate of water flowing in the industrial plant process simulator is 19 liters/min = 0.0003154 m³/s. Because the input changes of the stem is in meter and it is known that the size of the diameter of the used valve is 0.5 Inch (as shown in table 1), the full range input change of the stem is 0 to 0.0127 m. G5 can be obtained for change valve stem of 1 mm or 0.001 m with 7.874% of the full range.

By using equation (38), G5 gain is obtained, 
\[ G_5 = 0.02 \text{ m}^3/\text{s/m} \]

By using equation (40), \( G_v \) gain is also obtained, 
\[ G_v = 1.577 \times 10^{-5} \text{ m}^3/\text{s/m} \text{A} \]

Then, diagram block diagram in Fig. 9 can be presented as follow:

\[ G_5 = 0.02 \text{ m}^3/\text{s/m} \]
\[ G_v = 1.577 \times 10^{-5} \text{ m}^3/\text{s/m} \text{A} \]

Fig. 17. The component values of the control valve block diagram and \( G_v \) diagram block in Laplace transform is shown in Fig.18.

Fig. 18. Block diagram of control valve with gain

**D. Stability**

By entering the values that have been obtained for \( G_v, G_{LT}, \text{Level} \), also the value of \( K \) for \( G_C = 1 \) into the block diagram of Fig. 3, then block diagram can be presented as shown in Fig.19.

To test the stability of the simulator, in this paper, the Root Locus of the closed-loop transfer function is used as shown in Fig. 20. The detailed analysis in MATLAB is shown in Fig. 21.

Fig. 19. Component values of block diagram water level control in the tank on the simulator

Fig. 20. Root locus of water level control on simulator

Fig. 21. The response curves to the step input of the water level control in the simulator
V. CONCLUSION

The water level in the tank of the simulator can reach a maximum of 80 cm it is obtained with the input of flow rate of water filling the tank as much as 19 liters/minute, and the valve opening area for water out of the tank of \(7.97 \times 10^{-5}\) m\(^2\).

So it is also obtained that the tank has a gain of 2536.46 and the time constant 124 seconds.

Furthermore, the stability of the simulator based on Root Locus and the response to the step input is stable.

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Note:
Since June 24, 2014 the name of Research Center for Calibration, Instrumentation and Metrology - Indonesian Institute of Sciences (Research Center for KIM-LIPI), changed to Research Center for Metrology - Indonesian Institute of Sciences (Research Center for Metrology-LIPI).

Source:
http://pui.ristekdikti.go.id/index.php/lembaga/profil/q2O7rm7

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