RESEARCH ARTICLE OPEN ACCESS

MODELLING AND SIMULATION OF SUBSEA UMBILICAL DYNAMICS: A NUMERICAL APPROACH

O. O. Ajayi[1] , O. O. Aribike[2]
Department of Mechanical Engineering
University of Lagos
Lagos, Nigeria.

--***--

ABSTRACT

In this paper, we describe the mathematical method we used to model and simulate the dynamics of a subsea umbilical. The subsea umbilical dynamics are modelled as a tensioned beam with hydrodynamic forces and other forces acting on it. The forces and moments acting on the umbilical were considered in 3-D (three-dimension) numerical form using the Newtonian method so as to obtain a model that will predict the motion of the umbilical cable in subsea environment. The derived model consists of three (3) uncoupled, non-linear, partial differential equations (i.e., a set of two 4th-order and one 2nd-order non-linear partial differential equations). The equations were non-dimensionalized and simulated using Mathematica. The results of the simulations are presented in this paper. Based on our simulation of the effect of tension on the subsea umbilical, we conclude that an increase in tension results in an increase in the frequency of oscillations of the umbilical cable in the normal and transverse directions keeping the effects of current/water forces constant.

Keywords :- **Subsea umbilical, boundary conditions, numerical model, hydrodynamic forces, tension and bending moments**

--***--

NOMECLATURE

FPSO = Floating Production Storage Offloading system

FSO = Floating Storage Offloading system

FPU = Floating Production Unit

PDE = Partial Differential Equation

p = cable displacement in the tangential (longitudinal) direction;

q = cable displacement in the normal direction;

z = cable displacement in the transverse direction;

s = cable length measured from bottom end of cable upwards;

t = time;

Vn = sea current velocity component in the cable normal direction;

\

Vt = component of sea current velocity in the cable tangential direction;

Vz = component of sea current velocity in the transverse direction;

an = sea current acceleration component in the cable normal direction;

az = sea current acceleration component in the transverse direction;

m = uniform mass per unit length of umbilical cable;

d =outer diameter of cable;

di = inner diameter of cable;

To = tension at bottom end of cable;

T = tension at any length s of the cable;

ρc = density of umbilical cable;

ρw = density of sea water;

ε = strain;

E = modulus of elasticity of umbilical cable;

I= second moment of area of cable about its axis;

g = acceleration due to gravity;

φ = angle sea current makes with umbilical axis

ϕ = angle umbilical makes with the horizontal at any length s;

$\theta 1$ = deflection angle of normal cable displacement with respect to cable axis;

$\theta 2$ = deflection angle of transverse cable displacement with respect to cable axis;

Cdn = coefficient of normal drag;

Cdt = coefficient of tangential drag;

Cdz = coefficient of transverse drag;

Ca = added mass coefficient;

c = coefficient of structural damping;

Mn = bending moment induced in the cable normal direction;

Mz = bending moment induced in the transverse direction;

Also,

$$\text{Cos } \phi = \frac{dx}{ds}$$

$$\text{Sin } \phi = \frac{dy}{ds}$$

Where, x and y are horizontal and vertical catenary functions of the umbilical length (s).

$$I = \frac{\pi(d^4 - d_i^{\,4})}{32}$$

1. INTRODUCTION

Subsea umbilical is widely used in the deepwater offshore operations for transmitting power (hydraulic or electrical), fluid/chemical injections and control to subsea well [1].Subsea umbilicals in-conjunction with risers connect subsea structures (subsea wells and trees, etc.) to floating systems (FPSO, FSO, FPU) or shore-basedfacilities. They are therefore indispensible components of offshore oil and gas production/developments. The stability and reliability of subsea umbilicals under different environmental conditions (load, forces, tension, pressure, etc.) is thus essential. In spite of this, limited articles have been published on the dynamics/behavior of subsea umbilicals [2 - 6].

Some papers have been published on static analysis [7 - 8] due to the fact that once marine cable such as subsea umbilicals are installed (simultaneously layed and trenched) they operate under static conditions (no waves and current induced motions, etc)… and therefore not subject to the highest loads of service life at that situations (risk at this period is somewhat minimal).

A few papers have also been published on dynamic behaviours of umbilical [9] and its non-linear response [10]

Other research on umbilical focuses on stress and fatigue analysis [11 - 13] to determine how numerical model will perform using test bed (test work) as a means of verification and calibration of the model results.

Our focus is on umbilical dynamics because it is important to understand the performance of umbilicals during loading conditions (as a result of environmental loading and host vessel motions during installation operations and/or an unexpected delay(s) (such as installation equipment failure, adverse weather conditions, etc.) which will go a long way in minimizing risks. In addition, it is difficult to design and manufacture fully dynamic umbilicals that meet all environmental conditions/applications. Thus, we focus on umbilical analysis due to motions of the lay vessel and this involves hydrostatic & hydrodynamic considerations (analysis) on the umbilical (as a result of induced motion). This will enable good design and manufacturing of umbilicals by understand its performance during deployment and confirm its suitability for specific applications (and enhance field subsea layout).

This paper focuses on modeling and simulation of subsea umbilical dynamics using a numerical approach.The write-up is structured into five sections covering introduction, modeling, simulation, results & discussions, and finally conclusions. The introduction section presents the rationale and context for the work while the second section provides the general overview on umbilicals and their linkage to this study. The next section discusses modeling of umbilical dynamics by considering the forces acting on it in three directions (dimensions) and this was used to form the equations of the umbilical dynamics. This section was followed by the simulation parts, while the results of the simulations are presented and discussed in the results & discussions section. This paper concludes with the conclusion section based on the findings from this work with suggestions on further work.

1.1 Background

Flowlines, umbilical and risers systems are major parts of deepwater structure that links the subsea wells to surface structures (offshore platforms, etc.). The length of an umbilical that lies on the seabed is referred to as the static section. However, the part of the umbilical running from the host facility through the water column to the seabed is known as the dynamic section if it is free-hanging. This is the case when the host facility is a floating system, while for the umbilical deployed from a fixed platform at shore; the dynamic section is simply the length of the umbilical that is free-hanging in water. The dynamic section is subject to substantial forces that do not impinge significantly on the static section, such as the water current, and, in the case of a floater host, the motions of the platform or vessel itself.

If the dynamic section hangs freely between the platform and the seabed, the configuration is known as a free-hanging catenary. Different installation configurations that provide support for the umbilical in the water column may also be used, such as lazy wave, pliant wave, reverse pliant wave and steep wave. In general, the complexity and severity of the requirements for a dynamic umbilical increases with increasing water depth.

Tension is applied at the top of the umbilical which allows it to resist lateral loads. Its effects on the dynamic response of the umbilical have been studied in this project.

The umbilical is subjected to a time-varying distributed load due to the ocean current. This results in undesirable transverse and longitudinal flow and vortex induced vibrations. These vibrations cause stresses in the cable which may result in fatigue problems. Examples of fatigue problems include cyclic loads (which may lead to ultimate failure), propagation of cracks (which will require inspections and costly repairs), and as a worst case, environmental pollution due to chemical leakage from damaged areas.

For the above reasons and also to determine the position and the values of the extreme responses of the cable and to ensure that the performance of the system is not compromised by this dynamic response, it becomes imperative to analyze the dynamics of the subsea umbilical cable.

Two types of umbilical analyses can be employed. One is the analysis in the frequency domain using spectral analysis and the other is the analysis in the time domain using numerical simulation method. The first method yields valid solutions for linear systems. But the analysis of subsea umbilical dynamics requires that the system be modelled in a non-linear manner. The non-linearity is introduced by Morison's model of the hydrodynamic force. Thus, the time domain modeling and simulation of a subsea umbilical is used in this study. Since a fourth-order non-linear PDE is being used, analytical means becomes increasingly difficult to apply. Therefore, a numerical method is most applicable to generate a solution.

2. MODELLING

Consider a flexible umbilical cable submerged in a sea and subject to a variety of forces (Figure 1). The forces acting on it can be classified into hydrostatic, hydrodynamic, tension and bending forces (Figures 2 and 3). All these forces must be considered in order to effectively describe the motion of the umbilical cable as a function of time and the cable length. Thus, the problem analyzed is a non-linear dynamics problem and its equations have been made uncoupled to avoid highly unnecessary complexities in the mathematics of the problem in view.

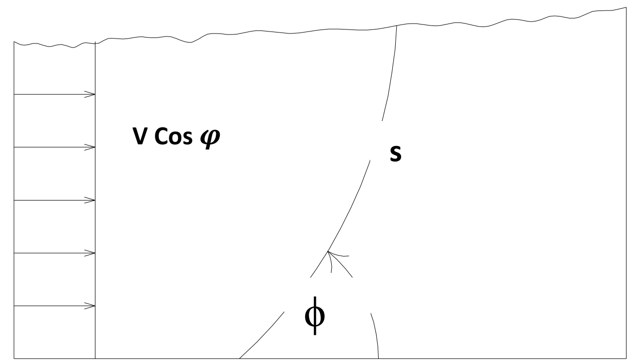
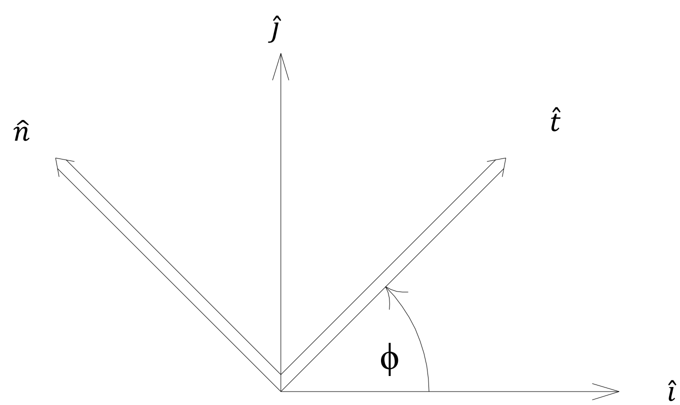


Figure 1: Schematic of an umbilical cable under-sea

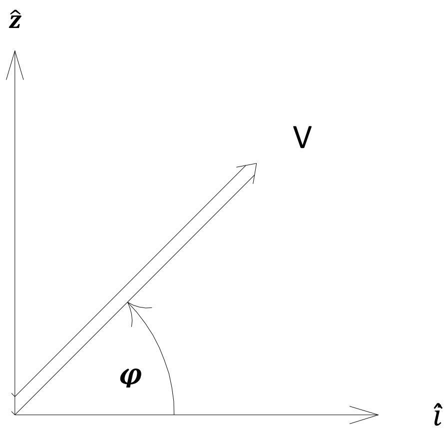
PARAMETERS

Major parameters that will be considered in this paper include

- parametric catenary equations of the umbilical
- mass per unit length of the umbilical
- density of the umbilical
- geometry of the umbilical
- tension in the umbilical
- elasticity of the umbilical
- velocity profile of subsea
- density of subsea
- drag and buoyancy
- acceleration due to gravity



Cable normal and tangential directions



Position vector of sea velocity

Figure 2:Schematic showing the orientation of the $\hat{n}$ ***,*** $\hat{t}$ ***and*** $\hat{z}$ ***axes***

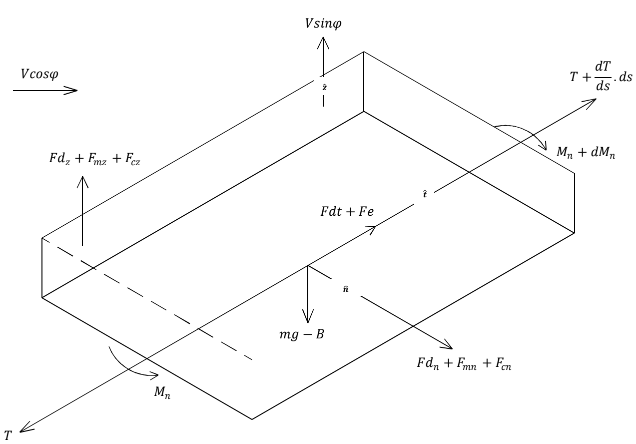


Figure 3: Forces and moments acting on a cable element

Applying Newton's second law of motion, the inertia force of the cable in each direction is equal to the net sum of the forces acting on the cable in that direction.

2.1 Forces

2.1.1 Normal Direction

Hydrostatic Force

The hydrostatic force is the buoyancy acting on the cable. The total buoyancy force acting on a cable segment Δs is given by:

$\frac{-mg.\rho_w}{\rho_c}$.Δs acting in the vertical direction. This force as all other forces that will be considered must be resolved into the normal and tangential directions of the cable which are the directions in which the dynamic analysis has been resolved. Thus, the buoyancy, B is also given by

$$\frac{-mg.\rho_w}{\rho_c}(\cos\phi\hat{n} + \sin\phi\hat{t}).\Delta s$$

Weight: The weight per unit length is given by mg.ds. And resolving also, gives

$$mg(-\cos\phi\hat{n} - \sin\phi\hat{t}).\Delta s$$

So, the net hydro static force on the cable element is

$$\frac{mg\,(\rho_c-\rho_w)}{\rho_c}(-\cos\phi\hat{n} - \sin\phi\hat{t}).\Delta s$$

The net hydrostatic force in the normal direction is thus given as:

$$\frac{mg\,(\rho_w-\rho_c)}{\rho_c}\text{Cos}\,\phi.\,\Delta s \qquad \hat{n}$$

Drag Force

The drag force is also has components in the cable normal and tangential directions.

The normal drag forces according to Morison [14] are directly proportional to the square of the corresponding relative normal velocity.

Thus, the normal drag on a cable segment Δs is given by:

$$F_{dn} = -\,0.5C_{dn}.\,d.\,\rho_w.\,V_{rn}|V_{rn}|\,.\,\Delta s \qquad \hat{n}$$

where V_{rn} is the relative velocity of the flow with respect to the cable velocity in the $\hat{n}$ direction given by :

$$V_{rn} = V_n + \frac{\partial q}{\partial t}$$

And $V_n = (V.\,Cos\,\varphi).\,\text{Sin}\,\phi = V.\,Cos\,\varphi.\frac{dy}{ds}$

Hydrodynamic Inertia Forces

The hydrodynamic inertia force exists only in the normal directions and is given by Morisonas

$$-\,0.25\,C_a.\,\pi\,.\,d.\,\rho_w.\,a_{rn}\,.\,\Delta s \qquad \hat{n}$$

Where, $a_{r\,n}$ is the relative acceleration of the flow with respect to the cable acceleration in the $\hat{n}$ direction which is

$$\mathbf{a_{r\,n} = a_n} + \frac{\partial^2 q}{\partial t^2}$$

And $\quad a_n = a.\ \mathrm{Sin}\ \phi = a.\ \frac{dy}{ds}$

Structural Damping Force

The structural damping force is modelled as a linear function of the local velocity of the cable.

$$\mathbf{F_{sn} = -\ c.\frac{\partial q}{\partial t}.\ \Delta s} \qquad \hat{n}$$

Tension

In specifying tension, the spatially varying effects of buoyancy and weight should be considered. This results in a spatially varying tension T(s) whose vertical component is:

$$\mathbf{T(s) = T_o + m.g.s}$$

Where, s is the length along the cable with its origin at the cable bottom.

The normal direction $\hat{q}$ is a radial coordinate that is always perpendicular to the static cable axis at any length s.

Therefore, the force in this direction due to tension at a length s of the cable is given from trigonometry as T. sin θ_1 where θ_1 is the angle of deflection of the cable in the $\hat{q}$ direction with respect to the cable axis and T is the tension along the cable. But as $\Delta s \rightarrow 0$, sin $\theta \rightarrow$ tan θ_1; and tan $\theta_1 = \frac{\partial q}{\partial s}$. Therefore, at a length s, this normal force is $T.\frac{\partial q}{\partial s}$. The total normal force on a cable element, Δs due to tension is thus given by the difference in this force at length s and at length s+Δs which is given by:

$$[T\frac{\partial q}{\partial s}|s+\Delta s - T\frac{\partial q}{\partial s}|s] = \Delta(\mathbf{T.\frac{\partial q}{\partial s}})$$

Therefore, normal force due to tension on an element Δs of the cable becomes $\Delta(\mathbf{T.\frac{\partial q}{\partial s}})$.

Bending Moment Force

For displacements in a direction normal to a cable, there is a bending moment which can be gotten from the Euler-Bernoulli's relation for straight beams. Though the cable is curved in the static mode, the Euler-Bernoulli's relation will still apply since its cross-sectional dimensions are quite small compared to the static curvature of the cable. The deflection angle is inversely proportional to the curvature (κ_n) introduced to a flexible cable due to the normal displacements, the constant of proportionality being the cable's flexural rigidity EI. The curvature introduced can also be approximated as a derivative of the deflection angle with respect to the cable length while this deflection angle is the angle of the normal displacement with respect to the cable axis which as stated previously is the partial derivative of normal displacement with respect to the cable length. So, the bending moment (M_n) in the normal direction imposed on the cable at length s by a deflection Δq is given as:

$$M_n = -EI\frac{\partial \theta_1}{\partial s} \text{But } \theta_1 = \frac{\partial q}{\partial s},$$

$$M_n = -EI\ \frac{\partial^2 q}{\partial s^2}$$

The force due to bending moment at length s is given by $F_{bn} = \frac{\partial M_n}{\partial s}$.

Now, the net force $\Delta(F_{bn})$ on a cable element Δs, due to this moment is given by:

$$[EI\frac{\partial M_n}{\partial s}|s+\Delta s - EI\frac{\partial M_n}{\partial s}|s] = \Delta(\mathbf{EI.\frac{\partial M_n}{\partial s}})$$

2.1.2 Tangential Direction

Hydrostatic Force

Since the total net hydrostatic force on the cable is

$$\frac{mg\ (\rho_c - \rho_w)}{\rho_c}\mathbf{.\ (-cos\ \phi\hat{n} - sin\ \phi\hat{t})\ .}\ \Delta s$$

The component of this force resolved in the direction tangential to the cable is seen to be:

$$\frac{mg\ (\rho_w - \rho_c)}{\rho_c}\ \mathbf{Sin\ \phi\ .}\ \Delta s \qquad \hat{t}$$

Drag Force

The tangential drag is proportional to the square of the relative tangential velocity. The drag force in the tangential direction is thus given as:

$$\mathbf{F_{dt} = 0.5C_{dt}.\pi\ .\ d.\ \rho_w.\ V_{rt}|V_{rt}|.}\ \Delta s \qquad \hat{t}$$

where $V_{r\,t}$ is the relative velocity of the flow with respect to the cable velocity in the $\hat{t}$ direction which is:

$$\mathbf{V_{r\,t} = V_t - \frac{\partial p}{\partial t}}$$

And $\quad V_t = (V.\ Cos\ \varphi).\ \mathrm{Cos}\ \phi = V.\ Cos\ \varphi.\ \frac{dx}{ds}$

Elastic Force

This force applies only in the tangential direction. In this direction, the displacement of the cable elements in this direction results in an elastic force since the umbilical cable is elastic. This force is directly proportional to the negative displacement according to Hooke's law.

So, $\Delta F_e = \Delta(EA\varepsilon)$

Now strain $(\varepsilon) = \frac{\Delta p}{\Delta s}$

Thus, for an element Δs as Δs→0;

$[EA\frac{\partial p}{\partial s}|s+\Delta s - EA\frac{\partial p}{\partial s}|s] = \Delta(\mathbf{EA}.\frac{\partial p}{\partial s})$

$\Delta (\mathbf{F_e}) = \Delta(\mathbf{EA}.\frac{\partial p}{\partial s})$

2.1.3 Transverse Direction

Drag Force

The transverse drag force according to Morrison's model is also proportional to the square of the relative velocity of the flow with respect to the local velocity of the cable in the transverse direction. On a cable segment Δs, it is given by:

$$\mathbf{0.5}C_{dz}.\,\mathbf{d}.\,\rho_w.\,V_{rz}|V_{rz}|.\,\Delta s \qquad \hat{z}$$

Where, V_{rz} is the relative velocity of the flow with respect to the cable velocity in the $\hat{z}$ direction which is:

$$\mathbf{V_{rz}} = \mathbf{V_z} - \frac{\partial z}{\partial t}$$

And $V_z = V.\,Sin\,\varphi$

Hydrodynamic Inertia Forces

In the transverse direction ($\hat{z}$), the hydrodynamic inertia force is directly related to the relative acceleration of sea current flow with respect to cable local velocity in this direction and it is given as:

$$\mathbf{0.25}C_a.\,\pi.\,\mathbf{d}.\,\rho_w.\,a_{rz}.\,\Delta s \qquad \hat{z}$$

where a_{rz} is the relative acceleration of the flow with respect to the cable acceleration in the $\hat{z}$ direction which is:

$$\mathbf{a_{rz}} = \mathbf{a_z} + \frac{\partial^2 z}{\partial t^2}$$

And $a_z = a.\,\text{Sin}\,\varphi$

Structural Damping Force

Just as in the normal direction, the structural damping force in this direction is directly proportional to the local transverse velocity of the cable and is given by:

$$\mathbf{F_{sz}} = -\mathbf{c}.\frac{\partial z}{\partial t}.\,\Delta s \qquad \hat{z}$$

Tension

The tension as previously described is $T(s) = T_o + m.g.s$

where s is the length along the cable with its origin at the cable bottom.

Now, the transverse direction $\hat{z}$ is perpendicular to the static cable axis. So, the transverse force at a length s of the cable is given from trigonometry as T. sin θ_2 where θ_2 is the angle of deflection of the cable in the $\hat{z}$ direction with respect to the cable axis and T is the tension along the cable. The transverse force on a cable element, Δs due to tension is thus given by the difference in this force at length s and at length s+Δs which is equal to Δ(T. sin θ_2). But as $\Delta s \rightarrow 0$, sin $\theta_2 \rightarrow$ tan θ_2; and tan $\theta_2 = \frac{\partial z}{\partial s}$.

$[T\frac{\partial z}{\partial s}|s+\Delta s - T\frac{\partial z}{\partial s}|s] = \Delta(\mathbf{T}.\frac{\partial z}{\partial s})$

Therefore, force due to tension on an element Δs of the cable in the $\hat{z}$ direction becomes $\Delta(\mathbf{T}.\frac{\partial z}{\partial s})$.

Bending Moment Force

Just as in the normal direction, to a cable, there is a bending moment which according to Euler and Bernoulli is inversely proportional to the curvature (κ_z) introduced to a flexible cable due to a deflection in the transverse direction $\hat{z}$, the constant of proportionality being the cable's flexural rigidity EI. The curvature itself can be approximated as a derivative of the deflection angle with respect to the cable length while this deflection angle is the angle of the second cable normal displacement with respect to the cable axis when tension was being considered is the partial derivative of the transverse displacement with respect to the cable length. So, the bending moment (M_z) imposed on the cable at length s by a deflection Δz in the $\hat{z}$ direction, is given as:

$M_z = -EI\frac{\partial \theta_2}{\partial s}$ and $\theta_2 = \frac{\partial z}{\partial s}$,

So, $M_z = -EI\frac{\partial^2 z}{\partial s^2}$. The force in the $\hat{z}$ direction due to this bending moment at length s is given by $F_{bz} = \frac{\partial M_z}{\partial s}$.

Now, the net force $\Delta(F_{bz})$ on a cable element Δs, due to this moment is given by:

$[\mathrm{EI}\frac{\partial M_z}{\partial s}|s+\Delta s - \mathrm{EI}\frac{\partial M_z}{\partial s}|s] = \Delta(\mathbf{EI}.\frac{\partial M_z}{\partial s})$

2.2 Equations of Umbilical Dynamics

Summing up all forces according in the normal direction,

$$\mathbf{m}\frac{\partial^2 q}{\partial t^2}.\Delta s = -\mathbf{c}\frac{\partial q}{\partial t}.\Delta s - \Delta(\mathbf{EI}\ \frac{\partial^3 q}{\partial s^3}) + \Delta(\mathbf{T}\frac{\partial q}{\partial s}) + (f_{\mathrm{dn}} + f_{\mathrm{mn}} + f_{\mathrm{gn}}).\ \Delta s$$

where $f_{dn} = 0.5C_{dn}.\ \mathrm{d}.\ \rho_w.\ V_{rn}|V_{rn}|$

$f_{mn} = 0.25C_a.\ \pi\ .\ \mathrm{d}.\ \rho_w.\ a_{rn}$

$f_{gn} = \frac{mg\ (\rho_w - \rho_c)}{\rho_c}.\ \cos\phi$

Dividing all through by Δs and as Δs→0; the equation above becomes

$$\mathbf{m}\frac{\partial^2 q}{\partial t^2} = -\mathbf{c}\frac{\partial q}{\partial t} - \mathbf{EI}\ \frac{\partial^4 q}{\partial s^4} + \frac{\partial}{\partial s}\left(\mathbf{T}\ \frac{\partial q}{\partial s}\right) + (f_{\mathrm{dn}} + f_{\mathrm{mn}} + f_{\mathrm{gn}})$$

Thus, the equation governing the umbilical cable dynamics in the normal direction becomes:

$$\mathbf{m}\frac{\partial^2 q}{\partial t^2} + \mathbf{c}\frac{\partial q}{\partial t} + \mathbf{EI}\ \frac{\partial^4 q}{\partial s^4} = \frac{\partial}{\partial s}\left(\mathbf{T}\ \frac{\partial q}{\partial s}\right) + f_{\mathrm{dn}} + f_{\mathrm{mn}} + f_{\mathrm{gn}}\hat{n}$$

In deriving the governing equations above, the tension induced strain has been neglected as this strain value is very small (usually smaller than 10^{-6})

In the direction tangential to the cable, the hydrodynamic and hydrostatic forces have already been stated.

Thus, the equation governing the umbilical cable dynamics in the tangential direction is given by:

$$m\frac{\partial^2 p}{\partial t^2}.\Delta s = \Delta(\mathbf{EA}\ .\frac{\partial p}{\partial s}) + (f_{\mathrm{dt}} + f_{\mathrm{gt}}).\ \Delta s$$

where $f_{\mathrm{dt}} = -\ 0.5C_{dt}.\ \pi.\ \mathrm{d}.\ \rho_w.\ V_{rt}|V_{rt}|$

and $f_{\mathrm{gt}} = \frac{mg\ (\rho_w - \rho_c)}{\rho_c}.\ \sin\phi$

Dividing through by Δs and as $\Delta s \rightarrow 0$;

The equation governing the dynamics of the umbilical cable in its tangential direction is:

$$m\frac{\partial^2 p}{\partial t^2} = \mathbf{EA}\ \frac{\partial^2 p}{\partial s^2} + f_{\mathrm{dt}} + f_{\mathrm{gt}} \qquad \hat{t}$$

In the transverse direction, weight and buoyancy do not have a component this direction since it is a horizontal direction. But all other type of forces acting on the cable in the normal direction except are also at play in this direction as has been shown previously. Therefore the equation of the umbilical dynamics in this direction is given by:

$$\mathbf{m}\frac{\partial^2 z}{\partial t^2}.\Delta s = -\mathbf{c}\frac{\partial z}{\partial t}.\Delta s - \Delta(\mathbf{EI}\ \frac{\partial^3 z}{\partial s^3}) + \Delta(\mathbf{T}\frac{\partial z}{\partial s}) + (f_{\mathrm{dz}} + f_{\mathrm{mz}}).\ \Delta s$$

where $f_{\mathrm{dz}} = 0.5.\ C_{dz}.\ \mathrm{d}.\ \rho_w.\ V_{rz}|V_{rz}|$

$f_{\mathrm{mz}} = 0.25C_a.\ \pi.\ \mathrm{d}.\ \rho_w.\ a_{rz}$

Dividing all through by Δs and as Δs→0 ;

$$\mathbf{m}\frac{\partial^2 z}{\partial t^2} = -\mathbf{c}\frac{\partial z}{\partial t} - \mathbf{EI}\ \frac{\partial^4 z}{\partial s^4} + \frac{\partial}{\partial s}\left(\mathbf{T}\ \frac{\partial z}{\partial s}\right) + f_{\mathrm{dz}} + f_{\mathrm{mz}}$$

This results in the equation below for the transverse direction:

$$\mathbf{m}\frac{\partial^2 z}{\partial t^2} + \mathbf{c}\frac{\partial z}{\partial t} + \mathbf{EI}\ \frac{\partial^4 z}{\partial s^4} = \frac{\partial}{\partial s}\left(\mathbf{T}\ \frac{\partial z}{\partial s}\right) + f_{\mathrm{dz}} + f_{\mathrm{mz}} \qquad \hat{z}$$

Thus, the complete model of the subsea umbilical dynamics goes thus:

2.2.1 Cable Normal Direction

$$\mathbf{m}\frac{\partial^2 q}{\partial t^2} + \mathbf{c}\frac{\partial q}{\partial t} + \mathbf{EI}\ \frac{\partial^4 q}{\partial s^4} = \frac{\partial}{\partial s}\left(\mathbf{T}\ \frac{\partial q}{\partial s}\right) + f_{\mathrm{dn}} + f_{\mathrm{mn}} + f_{\mathrm{gn}}\ \hat{n}$$

with the following initial and boundary conditions

Boundary conditions

q (0, t) = 0

$\mathrm{EI}\,\frac{\partial^2 q}{\partial s^2}(0, t) = 0$

q (L, t) = 0

$\mathrm{EI}\,\frac{\partial^2 q}{\partial s^2}(L, t) = 0$

Initial conditions

q (s, 0) = 0

$\frac{\partial q}{\partial t}(s, 0) = 0$

2.2.2 Cable Tangential Direction

$$m\frac{\partial^2 p}{\partial t^2} = \mathbf{EA}\ \frac{\partial^2 p}{\partial s^2} + f_{\mathrm{dt}} + f_{\mathrm{gt}} \qquad \hat{t}$$

with the following initial and boundary conditions

Boundary conditions

p(0, t) = 0

p(L, t) = 0

Initial conditions

p (s, 0) = 0

$\frac{\partial p}{\partial t}$ (s, 0) = 0

2.2.3 Cable Transverse Direction

$$\mathbf{m}\frac{\partial^2 z}{\partial t^2} + \mathbf{c}\frac{\partial z}{\partial t} + \mathbf{EI}\frac{\partial^4 z}{\partial s^4} = \frac{\partial}{\partial s}\left(\mathbf{T}\frac{\partial z}{\partial s}\right) + f_{dz} + f_{mz} \qquad \hat{z}$$

with the following initial and boundary conditions:

Boundary conditions

z(0, t) = 0

EI $\frac{\partial^2 z}{\partial s^2}$ (0, t) = 0

z(L, t) = 0

EI $\frac{\partial^2 z}{\partial s^2}$ (L, t) = 0

Initial conditions

z (s, 0) = 0

$\frac{\partial z}{\partial t}$ (s, 0) = 0

3. SIMULATION

Before the simulation, the non-dimensionalizing of the partial differential equations was done. This is to ensure that errors due to units do not arise in the simulation. More importantly, a computer simulation takes a longer time with increasing values of parameters involved in the simulation. The non-dimensionalization of the equations effectively puts away the delay that would have arisen as a result of these large values during a computer simulation.

In the equations governing the normal directions,

s= L;

the scale for time was gotten by equating the overall inertia term to the tension force term.

The scale for sea velocity was gotten by equating the overall inertia term to the drag force.

The scale for sea acceleration was gotten by equating the overall inertia term to the hydrodynamic inertia force.

In the tangential direction, the scale for time was gotten by equating the overall inertia term to the elasticity modulus force term while the scale for sea velocity and acceleration followed the same procedure as in the dynamics equation in the normal direction.

The non-dimensionalized form of the umbilical dynamics model is as follows:

$$\frac{\partial^2 q'}{\partial t'^2} + \mathbf{c}'\frac{\partial q'}{\partial t'} + \frac{\mathbf{EI}}{\mathbf{L^2.T_o}}\frac{\partial^4 q'}{\partial s'^4} = \frac{\partial}{\partial s'}\left(\mathbf{T}'\frac{\partial q'}{\partial s'}\right) + f'_{dn} + f'_{mn} + f'_{gn}\,\hat{n}$$

$$\frac{\partial^2 z'}{\partial t'^2} + \mathbf{c}'\frac{\partial z'}{\partial t'} + \frac{\mathbf{EI}}{\mathbf{L^2.T_o}}\frac{\partial^4 z'}{\partial s'^4} = \frac{\partial}{\partial s'}\left(\mathbf{T}'\frac{\partial z'}{\partial s'}\right) + f'_{dz} + f'_{mz}\,\hat{z}$$

$$\frac{\partial^2 p'}{\partial t''^2} = \frac{\partial^2 p'}{\partial s'^2} + f'_{dt} + f'_{gt} \qquad \hat{t}$$

where

$q' = \frac{2m}{\rho_w . d} . q$

$p' = \frac{2m}{\rho_w . d} . p$

$z' = \frac{2m}{\rho_w . d} . z$

$s' = \frac{s}{L}$

$t' = \frac{1}{L} . \sqrt{\frac{T_o}{m}} . t$

$t'' = \frac{1}{L} . \sqrt{\frac{EA}{m}} . t$

$T' = \frac{T}{T_o}$

$c' = c . L . \sqrt{\frac{T_o}{m}}$

$V'_{rn} = \frac{\sqrt{T_o . m}}{0.5\rho_w . d . L} . V_n + \frac{\partial q'}{\partial t'}$

$V'_{rz} = \frac{\sqrt{T_o . m}}{0.5\rho_w . d . L} . V_z - \frac{\partial z'}{\partial t'}$

$$V'_{rz} = \frac{\sqrt{E.A.m}}{0.5\rho_w.d.L} . V_t \quad - \quad \frac{\partial p'}{\partial t''}$$

$$a'_{n} = \frac{\rho_w{}^2.d^3.\pi.L^2}{T_o.m} . a_n$$

$$a'_{z} = \frac{\rho_w{}^2.d^3.\pi.L^2}{T_o.m} . a_z$$

$$f'_{dn} = - C_{dn} \; V'_{rn} |V'_{rn}|$$

$$f'_{dz} = C_{dz} \; V'_{rz} |V'_{rz}|$$

$$f'_{dt} = C_{dt} . \pi . V'_{rt} |V'_{rt}|$$

$$f'_{mn} = - C_a .[a'_n + \frac{\rho_w.\pi.d^3}{8m} . \frac{\partial^2 q'}{\partial t'^2}]$$

$$f'_{mz} = C_a .[a'_z - \frac{\rho_w.\pi.d^3}{8m} . \frac{\partial^2 z'}{\partial t'^2}]$$

$$f'_{gn} = f_{gn} . \frac{\rho_w.d.m.L^2}{2T_o}$$

$$f'_{gt} = f_{gt} . \frac{\rho_w.d.m.L^2}{2EA}$$

The Mathematica software from Wolfram Research Inc. was used for the simulation. The software works on the PDEs by reducing them to several ODEs and then solving them.

Details of a particular steel umbilical and environmental conditions in West Africa Deepwater Operations are used in simulating the dynamic response in 300s was simulated is provided in Table 1.

Table 1: Typical data for a Steel Umbilical Cable

PARAMETER	VALUE
Mass per unit length	$1500kgm^{-1}$
Length	10 000m
Bottom tension	10KN
Outer diameter	0.8m
Inner diameter	0.6m
Elastic modulus	$2.1 \times 10^{11} Nm^{-2}$
Structural damping coefficient	0.003
Gravity	$9.81ms^{-2}$
Velocity of Sea	$1000ms^{-1}$
Water depth	1200m
C_{dt}	0.03
C_{dn}	0.7
C_a	1
φ	50°

4. RESULTS & DISCUSSIONS

In the study, we varied the water depth to see the effect on tension from 350m (for shallow water), 1000m - 1500m (for deepwater depth in West Africa (such as in Cameroun, Malabo, Bonga, Akpo) which are typically within that water depth range, and for ultra-deepwater (with water depth in the vicinity of 2600m (in Gulf of Mexico) so as to verify our numerical simulation results To do this, the

The umbilical is tuned to meet any water depth or environmental condition by increasing the umbilical mass (i.e., which in real-life can be achieved by adding weight elements or stiffness elements into the cross section of the umbilicals).

The resulting graph of the normal displacement plotted against umbilical length and time is shown in Figure 4.

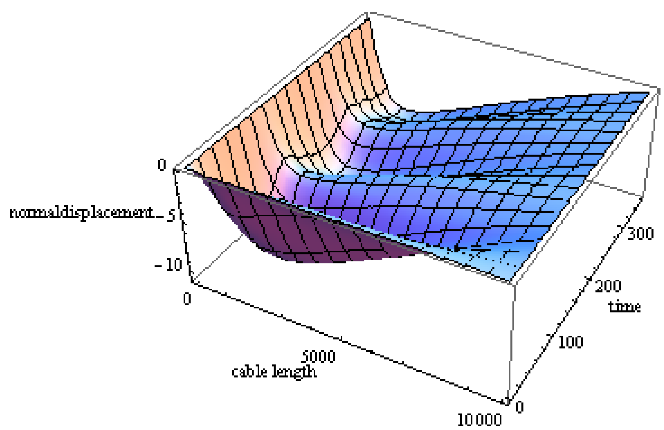


Figure4: Graph of normal displacement against time and length

From Figure 4, it can be seen that the highest normal response occurs at 1500m. The period of oscillation is about 200seconds.

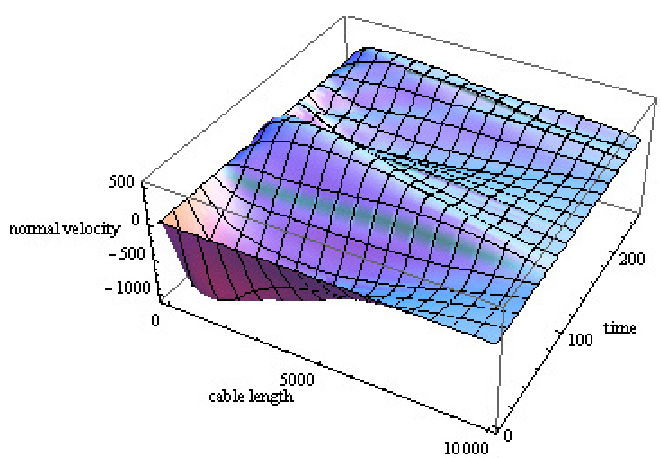


Figure5: Graph of normal velocity against time and length

The amplitude of the normal velocity is highest at 1000m (Figure 5).

The graph of the normal response (in Figure 5), show conformity with the expected oscillatory motion of a vibrating tensioned beam in the transverse direction.

Figure 6illustrates a graph of the tangential displacement against cable length and time.

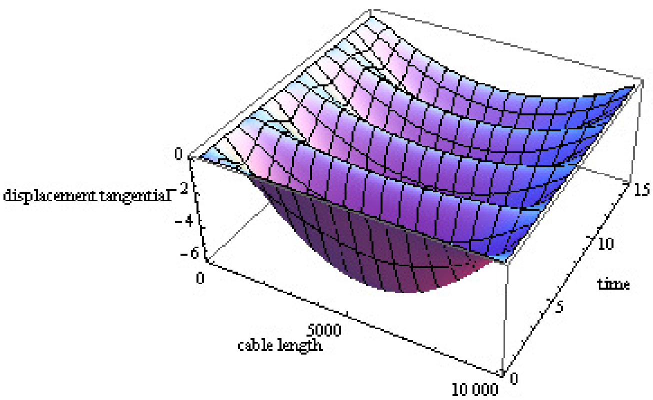


Figure 6: Graph of tangential displacement against time and length

In the tangential direction, the mid-point of the umbilical cable is the most responsive.

Also, the graph of the tangential velocity of the cable is shown in Figure 7.

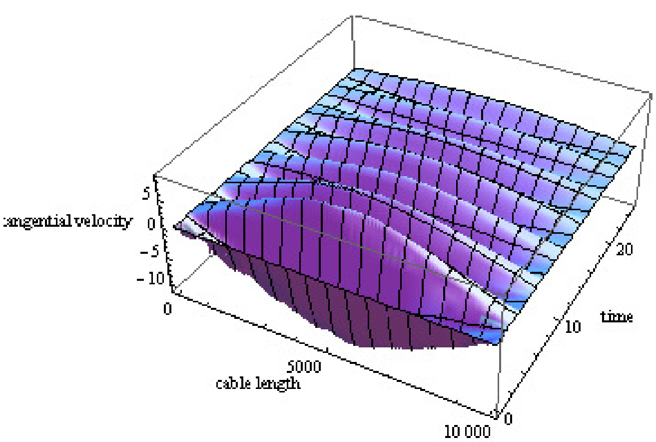


Figure 7: Graph of tangential velocity against time and length

The tangential velocity just like the tangential displacement has the highest amplitude at the mid-length of the umbilical cable.

Figure 8 depicts a graph of the transverse displacement against time and length.

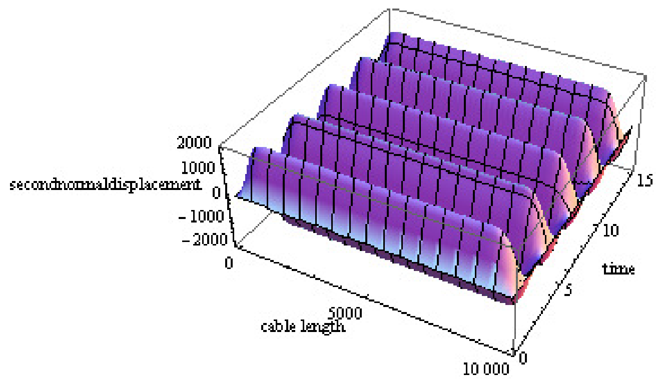


Figure 8: Graph of transversedisplacement against time and length

The response of the cable in the transverse displacement ($\hat{\mathbf{z}}$) is uniform along its length from about 500m to 9500m. This response is quite fast as can be seen in the graph above. The period of oscillation is about 5 seconds.

The effects of tension on the frequency of oscillations of the normal and transverse directions for a water depth of 1200m are presented in Figures 9 and 10 respectively.

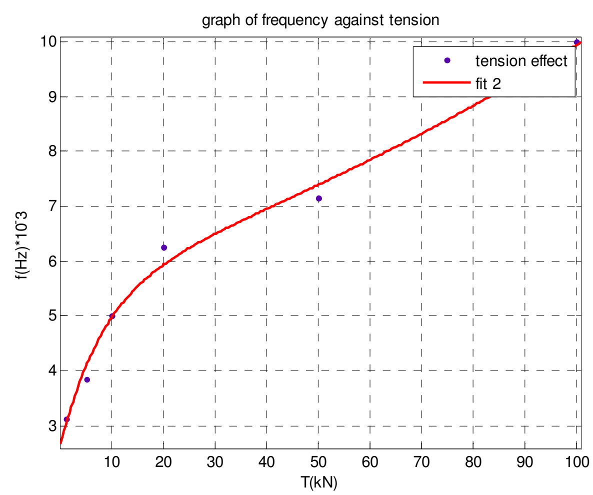


Figure 9: Graph of frequency against tension in normal direction

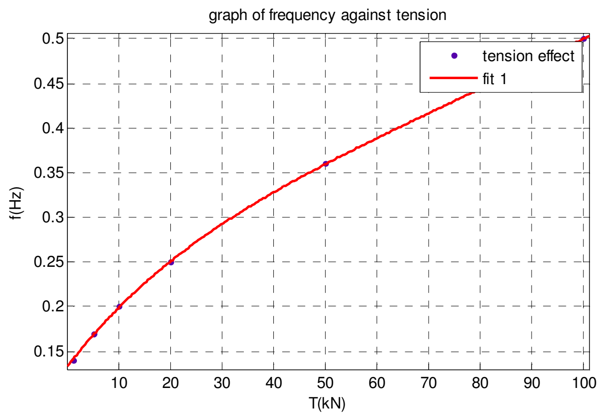


Figure 10: Graph of frequency against tension in transverse direction

When we tuned the umbilicals (by increasing the mass, i.e. 30kg/m and 60kg/m) we observed the same pattern in Figures 11 and 12 as in Figure 10. The polynomial equation fit was of higher orders (i.e., increasing the mass results in an increase in frequency of oscillation in the umbilical cable in the normal and transverse directions). Thus, the plots show that umbilicals filled with heavier materials will adjust its tension to compensate for the non-linearity of the umbilical dynamics.

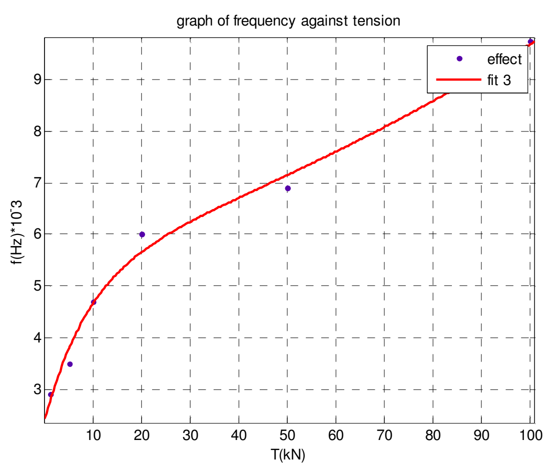


Figure 11: Graph of frequency against tension in transverse direction based on 30kg/m

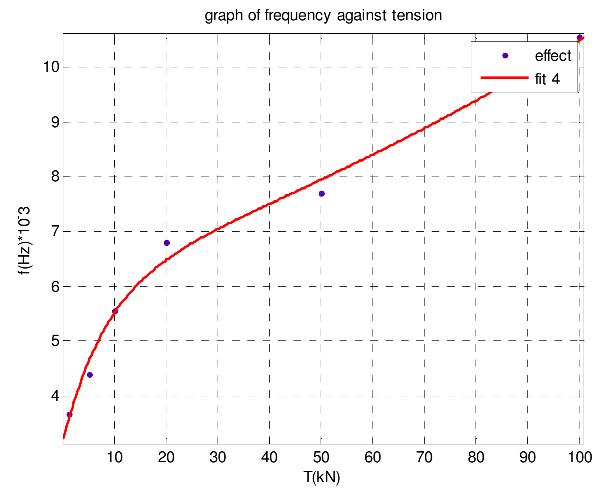


Figure 12: Graph of frequency against tension in transverse direction based on 60kg/m

5. CONCLUSIONS

The reliability of the subsea umbilical is quite critical to oil production in terms of control, quality and flow assurance. The subsea umbilical's dynamics is modelled as a set of two 4th order and one 2nd order non-linear PDEs. It is seen from Figure 8 that the response of the umbilical cable in the transverse direction is almost uniform along its length and indeed it is a fast response. The response of the umbilical cable in the normal direction shown in Figure 4 is not as fast as that of the transverse direction. It takes a period of about 200 seconds as against 5 seconds of the transversedisplacement. The tangential displacement also has a fast response as seen from Figure 6.

In the normal direction, the displacement amplitude is highest around s = 1500m as seen from Figure 4, while the amplitude of the velocity (Figure 5) is highest at about s =1000m. In the tangential direction, the highest dynamic response occurs at mid-length. It is interesting to note that the amplitude of the tangential velocity of the cable reduces with time (Figure 7). Based on the results, we can conclude that an increase in tension leads to an increase in the frequency of oscillations of the umbilical cable in the normal and transverse directions keeping the effects of current/water forces constant. Similar effect is observed when the umbilical is tuned to meet any water depth or environmental condition by increasing the umbilical mass.

Lastly, vortex induced vibration (VIV) and control of subsea umbilicals is an area where a lot still needs to be done in future.

6. REFERENCES

[1] Swanson, R. C., Rae, V. S., Langner, C. G., and Venkataraman, G. (1995); Metal Tube Umbilicals - Deepwater and Dynamic Considerations. OTC 7713, Houston.

[2] Martinelli,L., Lamberti, A., Ruol, P., Ricci, P., Kirrane, P., Fenton, C., and Johanning, L., 2010, Power Umbilical for Ocean Renewable Energy Systems-Feasibility and Dynamic Response Approach, 3rd International Conference on Ocean Energy, Bilbao, pp 4 -10

[3] Bleik A. and Triantafyllou M.S., 1983, The Dynamics of Taut and Slack Marine Cables 15th Offshore Technology Conference.

[4] How, B.V.E.,Ge, S.S.,and Choo,Y.S. (2009); Active Control of Flexible Marine Risers, Journal of Sound and Vibration, vol. 320, pp. 758-776

[5] Brouwers, J. J. H., 1982, Analytical Method for predicting the response of Marine Risers.

[6] Howell C. T., 1992, Numerical Application of 2-D Nonlinear Equations with Applications to Low-Tension Problems, International Journal of Ocean and Offshore Engineering, Vol. 2, No.2.

[7] Huang, S.and. Vassalos, D. 1993, A Semi-Analytic Treatment Of Three Dimensional Statics Of Marine Cables, in Ocean Engng, Vol. 20, No 4, pp. 409-420

[8] Prat, J., del Rio, J. and Arbos, A., 2011, Preliminary study of moored power cables, in Oceans 2011 IEEE/OES : Oceans of energy for sustainable future. "Actas". IEEE, 2011-Spain, 6-9 June 2011, pp 1-6.

[9] Park, H.I., Kwon, D.Y. and D.H. Jung, 2005, A Numerical Analysis for the Dynamic Behavior of ROV Launcher and Umbilical Cable under Combined Excitations, in Proc. Of the 15th Int'l offshore and Polar Eng. Conf, South Korea June 19-24, pp 313-318

[10] Custodio, A.B., and Vaz, M.A. , 2002, A nonlinear formulation for the axisymmetric response of umbilical cables and flexible pipes, in Applied Ocean Research 24, pp 21–29

[11] Sakamto, K., Fujimoto Y. and Osawa H., 2008, Development of Fatigue-Less Umbilical Cable for Full Ocean Depth 12000m in Proceedings of the 57th IWCS, International Wire & Cable Symposium, pp 488-492

[12] Brown, E., 2011, Steel Tube Umbilical Design Life Verification by Full-Scale Fatigue Test, in 2011 Offshore Technology Conference, OTC 21819

[13] Saevik, S. and Ekeberg, K. I., 2002, Non Linear Stress Analysis of Complex Umbilical Cross-Section in 21St Int'l Conf on Offshore Mechanics and Artic Engineering, June 23-28, Oslo, Norway, pp 211-217

[14] Morison, J. R., O'brien, M. P., Johnson, J. W., and Schaaf, S. A. (1950); The Force Exerted by Surface Waves on Piles, Petroleum Transactions, AIME, Vol. 189, pp. 149 -154