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Tuning of a 2DOF PI-PID Controller for use with Second-order-like Processes

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Abstract:

This paper presents a tuning approach for a 2DOF PI-PID controller to be used with second-order-like processes. The MATLAB computer program is used through its optimization toolbox to tune the controller subjected to four functional constraints. The functional constraints are selected to produce good performance of the control system regarding the transient and steady state characteristics. The tuning technique used is compared with another technique and the effectiveness of the controller was investigated through comparison with using other two controllers.

Keywords — **2DOF PI-PID controller, controller tuning, control system performance**.

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I. INTRODUCTION

This is one of the controllers investigated by the author to improve the performance of linear control systems and get red of the kick characteristic encountered with the use of the conventional PID controller.

Araki and Taguchi (2003) in their paper about 2DOF PID controllers presented a structure of the 2DOF PID controller with a PD sub-controller in a feedforward loop and a PID sub-controller in the forward path of the control system. They tuned the controller for seven types of processes [1]. Viteckova and Vitecek (2008) studied the tuning process of 2DOF controller used with integral plus time delay plants. They used a second order filter receiving the reference input signal and a PID sub-controller in the feed forward path receiving the error signal [2]. Alfaro, Vilanova and Arrieta (2009) presented some considerations on set-point weight choice for 2DOF PID controllers. The 2DOF controller structure they used consisted of a PI sub-controller receiving the reference input signal and a PID sub-controller with derivative filter in the feedback path. Their analysis considered the robustness of the control system [3]. Viteckova and Vitecek (2010) investigated the tuning of PI and PID controllers with 2DOF using the multiple dominant pole method. The 2DOF controller structure they used had a first order filter receiving the reference signal and a PI sub-controller receiving the error signal in the forward path [4].

Ruiz (2012) in his Ph. D. Thesis about trade-off issues in 2DOF PID control design tuned 1DOF and 2DOF PI and PID controllers. He considered a filter receiving the reference input signal and a PI or PID sub-controller receiving the error signal of the control system considering control system robustness [5]. Alfaro and Vilanova (2013) presented a simple robust tuning of 2DOF PID controller from performance/robustness trade-off analysis. They considered a 2DOF controller structure having a PI sub-controller receiving the reference input and a PID sub-controller receiving the error signal. They used the desired robustness level to tune the 2DOF controller [6], Kumar and Patel (2015) in their tuning technique for 2DOF PID controller used with second order processes used a controller structure with PI sub-controller receiving the reference input signal of the control system and a PID sub-controller set in the feedback path of the control system and receiving the process output signal as its input [7].

Abdelaty, Ahmed and Ouda (2018) studied the analysis and design of a set point weighting 2DOF PID controller. They used a 2/2 sub-controller receiving the reference input signal with set point weight on the proportional and derivative terms and a PID sub-controller in the forward path receiving the error signal of the control system with filter on its derivative part [8]. Hassaan (2018) studied the tuning process of a 2DOF PID controller for use with second-order-like processes. He used a 2DOF controller consisting of PD sub-controller receiving a reference input signal and set in a feedforward loop and a PID sub-controller set in the forward path of the control system receiving the error signal of the system. He tuned the controller for use with second order-like processes of damping ratio from 0.05 to 2 and a natural frequency up to 10 rad/s [9].

II. PROCESS

The controlled process is second-order-like process having the transfer function, $G_p(s)$:

$$G_p(s) = (\omega_n^2 / (s^2 + 2\zeta\omega_n s + \omega_n^2) \qquad (1)$$

Where:

ω_n = process natural frequency

ζ = process damping ratio

The damping ratio of the process classifies the process as an underdamped, critically damped or overdamped one.

III. THE PI-PID CONTROLLER

The structure of a 2DOF PI-PID controller for the control of a process is shown in Fig.1 [1]. The PI-PID controller shown in Fig.1 has two sub-controllers: one in the feed forward path with R(s) as its input with $G_{c1}(s)$ transfer function and another sub-controller in the main forward path of the closed-loop control system with the error signal E(s) as its input with $G_{c2}(s)$ transfer function. The output signal of the controller is U(s) superimposed by a disturbance signal D(s) all fed to the process to be controlled.

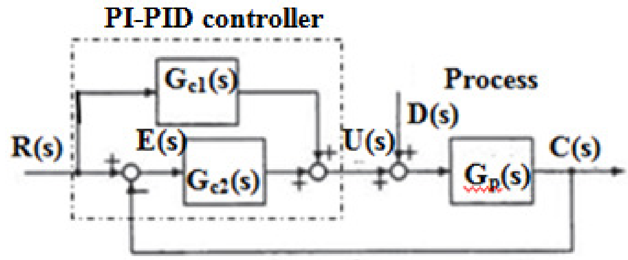

Fig.1 PI-PID controller structure [1].

In our present work, the first sub-controller is selected as a PI controller while the second sub-controller is selected as a PID controller. The transfer functions of both sub-controllers are:

$$G_{c1}(s) = K_{pc1} + K_{i1} /s \qquad (2)$$

And $$G_{c2}(s) = K_{pc2} + (K_{i2}/s) + K_{d2}\, s \qquad (3)$$

Where:

It has five parameters:

- Proportional gains, K_{pc1} and K_{pc2}
- Derivative gains, K_{d2}
- Integral gain, K_{i1} and K_{i2}

IV. CONTROL SYTEM TRANSFER FUNCTION

Using the block diagram of the closed-loop control system shown Fig.1 with R(s) as the reference input, and C(s) is the process output and the process transfer function of Eq.1 and the sub-controllers transfer functions of Eqs.2 and 3, the transfer function of the closed-loop control system for set-point tracking is M(s) = C(s)/R(s) given by:

$$M(s) = (b_0 s^2 + b_1 s + b_2) / (s^3 + a_1 s^2 + a_2 s + a_3) \qquad (4)$$

Where:

$b_0 = \omega_n^2 K_{d2}$

$b_1 = \omega_n^2 (K_{pc1} + K_{pc2})$

$b_2 = \omega_n^2 (K_{i1} + K_{i2})$

$a_1 = 2\zeta\omega_n + \omega_n^2 K_{d2}$

$a_2 = \omega_n^2 (1 + K_{pc2})$

$a_3 = \omega_n^2 K_{i2}$

V. CONTROLLER TUNING AND SYTEM TIME RESPONSE

The controller has five parameters: K_{pc1}, K_{i1} , K_{pc2}, K_{i2} and K_{d2} The controller parameters are tuned as follows:

- The control and optimization toolboxes of MATLAB are used to assign the five parameters of the controller [10].
- The integral of time multiplied by absolute error (ITAE) is chosen as an objective function for the optimization process.
- Four functional constraints are set for the closed-loop control system: maximum percentage overshoot, settling time , a stability constraint derived from the Routh-Hurwitz criterion for control system stability and a steady-state error constraint.
- The step response of the closed-loop control system is plotted using the command '*step*' of MATLAB [11].
- The controller is tuned using the above approach for second order-like processes having equivalent damping ratio in the range: $0.05 \leq \zeta \leq 2$ and equivalent natural frequency in the range: $0.5 \leq \omega_n \leq 10$ rad/s.
- The time-based specifications of the closed-loop control system are extracted using the MATLAB command '*stepinfo*' [12].
- The tuning parameters of the 2DOF PI-PID controller and some of the performance measures of the closed-loop control system incorporating the controller and the second order-like process are given in Tables 1 through10.

TABLE 1
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 0.05$

ω_n (rad/s)	**0.5**	**1**	**2.5**	**5**	**7.5**	**10**
K_{pc1}	0.9605	0.9660	0.9447	0.9593	0.9657	0.9579
K_{i1}	0.0102	0.01	0.01	0.0199	0.01	0.0191
K_{pc2}	29.5567	68.5448	2.0847	2.0077	499.726	27.1385
K_{i2}	1.0191	1	1.0088	1.0088	1.0005	30.5541
K_{d2}	498.831	456.740	5.0061	5.0061	64.3593	10.0985
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0367	0.01	0.0541	0.0541	0.0012	0.0038
e_{ss}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.0006

TABLE 2
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 0.1$

ω_n (rad/s)	**0.5**	**1**	**2.5**	**5**	**7.5**	**10**
K_{pc1}	0.6575	0.9627	0.9510	0.9724	0.9650	0.9313
K_{i1}	0.0793	0.01	0.01	0.01	0.01	0.0111
K_{pc2}	126.6043	125.078	4.3925	500	500	499.9372
K_{i2}	7.9320	1.0032	0.9281	1.00	1	1.1098
K_{d2}	481.406	452.206	6.362	500	27.9990	67.0961
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0371	0.01	0.103	0.0004	0.0025	0.0006
e_{ss}	-0.01	-0.01	-0.011	-0.01	-0.01	-0.01

TABLE 3
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 0.25$

ω_n (rad/s)	**0.5**	**1**	**2.5**	**5**	**7.5**	**10**
K_{pc1}	0.9604	0.9156	0.9387	0.9674	0.9700	0.9700
K_{i1}	0.01	0.0216	0.01	0.01	0.01	0.01
K_{pc2}	130.872	234.154	499.986	500	499.932	499.991
K_{i2}	1.0011	2.1592	1	1	1	1
K_{d2}	499.701	256.803	310.278	165.017	295.115	299.446
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0366	0.0173	0.0024	0.0011	0.0003	0.0002
e_{ss}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.0006

TABLE 4
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 0.5$

ω_n (rad/s)	**0.5**	**1**	**2.5**	**5**	**7.5**	**10**
K_{pc1}	0.6609	0.9669	0.9760	0.9659	0.9633	0.9701
K_{i1}	0.0796	0.01	0.01	0.01	0.01	0.01
K_{pc2}	482.683	452.481	500	499.989	77.1300	499.980
K_{i2}	7.9638	1	1	1	1	1
K_{d2}	318.508	422.019	184.093	91.2897	7.7331	294.432
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0483	0.0108	0.004	0.0020	0.0096	0.0002
e_{ss}	-0.01	-0.01	-0.011	-0.01	-0.01	-0.01

TABLE 5
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 0.75$

ω_n (rad/s)	0.5	1	2.5	5	7.5	10
K_{pc1}	0.6945	0.9659	0.9705	0.9652	0.9702	0.9699
K_{i1}	0.07	0.01	0.01	0.01	0.01	0.01
K_{pc2}	488.811	485.488	490.200	499.959	499.991	491.419
K_{i2}	7.0014	1	1.0006	1	1	0.9993
K_{d2}	301.654	310.406	299.851	63.5319	285.176	289.409
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0517	0.0147	0.0025	0.0029	0.0003	0.0002
e_{ss}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

TABLE 6
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 1$

ω_n (rad/s)	0.5	1	2.5	5	7.5	10
K_{pc1}	0.7620	0.9616	0.9641	0.9647	0.9647	0.9700
K_{i1}	0.0522	0.01	0.01	0.01	0.01	0.01
K_{pc2}	486.252	499.968	499.894	499.989	500	499.997
K_{i2}	5.2239	1	1	1	1	0.9996
K_{d2}	295.017	244.397	81.1038	48.579	32.3683	285.338
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0546	0.0187	0.0087	0.0038	0.0025	0.00016
e_{ss}	-0.01	-0.01	-0.011	-0.01	-0.01	-0.01

TABLE 7
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 1.25$

ω_n (rad/s)	0.5	1	2.5	5	7.5	10
K_{pc1}	0.9583	0.9600	0.9622	0.9644	0.9644	0.9701
K_{i1}	0.01	0.0118	0.0106	0.01	0.01	0.01
K_{pc2}	445.586	499.976	499.995	499.971	500	498.905
K_{i2}	1.0007	1.1759	1.0635	1.0004	1	0.9766
K_{d2}	352.767	267.384	49.847	34.919	26.1704	284.977
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0518	0.0181	0.0125	0.0051	0.0031	0.0002
e_{ss}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

TABLE 8
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 1.5$

ω_n (rad/s)	0.5	1	2.5	5	7.5	10
K_{pc1}	0.9489	0.9615	0.9694	0.9640	0.9644	0.9642
K_{i1}	0.0116	0.01	0.0104	0.01	0.01	0.01
K_{pc2}	450.046	499.369	498.518	499.998	499.963	500
K_{i2}	1.1571	1	1.0354	1	1.001	1.001
K_{d2}	296.944	164.697	284.510	32.918	21.695	10.8917
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0615	0.0277	0.0028	0.0055	0.0037	0.0035
e_{ss}	-0.01	-0.01	-0.011	-0.01	-0.01	-0.01

TABLE 9
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 1.75$

ω_n (rad/s)	0.5	1	2.5	5	7.5	10
K_{pc1}	0.9467	0.9601	0.9623	0.96370.	0.9640	0.9644
K_{i1}	0.0117	0.0101	0.01	0.01	0.01	0.01
K_{pc2}	489.539	496.599	495.372	499.933	499.976	498.871
K_{i2}	1.165	1.0112	1	1	1	1
K_{d2}	277.733	140.478	39.607	27.640	18.897	29.3017
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0658	0.0325	0.0153	0.0065	0.0043	0.0019
e_{ss}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

TABLE 10
2DOF PI-PID CONTROLLER TUNING FOR $\zeta = 2$

ω_n (rad/s)	0.5	1	2.5	5	7.5	10
K_{pc1}	0.9431	0.9576	0.9620	0.9632	0.9638	0.9632
K_{i1}	0.0119	0.01	0.01	0.01	0.01	0.01
K_{pc2}	490.080	493.7164	450.1456	499.997	499.995	273.126
K_{i2}	1.1943	1	1	1.0023	1	1
K_{d2}	243.690	92.404	44.021	19.078	16.564	5.5115
OS_{max} (%)	0	0	0	0	0	0
T_s(s)	0.0749	0.0414	0.163	0.0081	0.0049	0.0068
e_{ss}	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01

- The step time response of the control system for set-point tracking for a second order-like process using two sets of process parameters and using a tuned 2DOF PI-PID controller using Tables 1 and 10 is shown in Fig.2. The process parameters are:

Set 1: $\zeta = 0.1$, $\omega_n = 2.5$ rad/s
Set 2: $\zeta = 2.0$, $\omega_n = 10$ rad/s

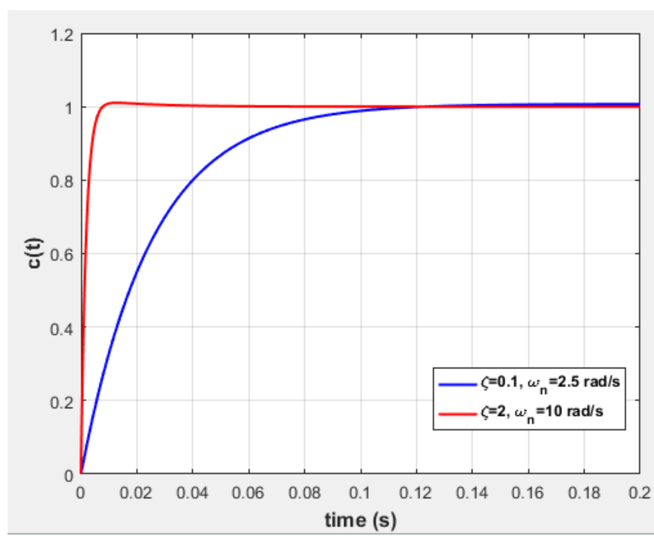

Fig.2 2DOF PI-PID controlled second order process

VI. COMPARIZON WITH MINIMUM ITAE STANDARD FORMS TUNING

- The parameters of the 2DOF PID controller is tuned using the minimum ITAE standard forms of Graham and Lathrop [13]. The resulting controller parameters are:

$K_{pc2} = 0.9579$

$K_{i2} = 30.5541$

$K_{d2} = 0.4210$

- The parameters K_{pc1}, K_{i1} and K_{i2} are taken as tuned in Table 1.
- The step response of the closed loop control system incorporating the 2DOF PI-PID controller and the process with $\zeta = 0.05$ and $\omega_n = 10$ rad/s is shown in Fig.3 for both tuning techniques.
- The performance characteristics of the control system for both tuning techniques are as follows:

- The maximum percentage overshoot is zero using the presented tuning technique compared with 23.819 % using the minimum ITAE standard forms of Graham and Lathrop.
- The settling time (within ± 2 % band) is 0.0038 s compared with 0.236 s using the minimum ITAE standard forms of Graham and Lathrop.

VII. COMPARIZON WITH OTHER CONTROLLERS

- To investigate the effectiveness of the studied controller, it was compared with using a PI-PD controller [14] and a 2DOF-PID controller [9] for a second order process having 0.05 damping ratio and a 10 rad/s natural frequency.
- The PI-PD controller was tuned by the auther and had the tuned parameters [14]:

$K_c = 18.9517$

$K_f = 1.0037$

$K_i = 44.9216$

$K_d = 0.7119$

- The 2DOF PID controller was tuned by the author and had the tuned controller parameters [9]:

$K_{pc} = 0.9990$

$K_{d1} = 0.0100$

$K_{pc2} = 0.6140$

$K_{i2} = 36.4770$

- The unit step response of the closed loop control system incorporating the three controllers used with the highly oscillating process is shown in Fig.4.

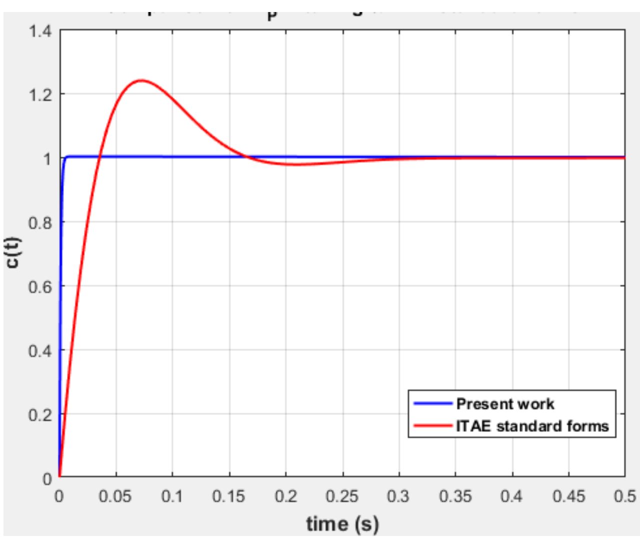

Fig.3 Comparison between two tuning techniques.

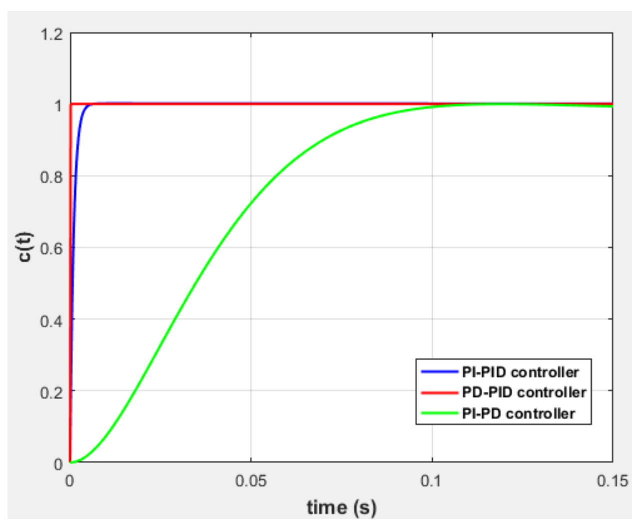

Fig.5 Comparison between three controllers.

- The time based characteristics of the control system using the three controllers are compared in Table 11.

TABLE 11
CHARACTERISTICS COMPARISON USING THREE CONTROLLERS

Controller	PI-PID	PD-PID	PI-PD
OS_{max} (%)	0	0	0
T_s (s)	0.0038	0.00001	0.0920
e_{ss}	-0.01	0	0

VIII. Conclusions

- The tuning process of a 2DOF PI-PID controller for use with second-order-like processes was investigated.
- A PI and PID sub-controllers were used as elements of the 2DOF PI PID controller.
- The 2DOF PI-PID controller was proposed in this paper to control second order-like processes of damping ratio between 0.05 and 2 and natural frequency between 0.5 and 10 rad/s.
- The 2DOF PI-PID controller had 5 gain parameters to be tuned for optimal performance of the closed loop control system incorporating the controller and the process.
- The 2DOF PI-PID controller was tuned using the optimization and control toolboxes of MATLAB.
- The controller succeeded to suppress completely the oscillations of the underdamped process to step input change and compensate their slow step time response for overdamped second order-like processes.
- The drawback of using the PI-PID controller was that it generates a steady state error for the closed loop control system incorporating a second-order-like process. However, this could be controlled through a functional constraint on the steady-state error such that it has to be less than a specific value.
- All the tuning parameters of the 2DOF PI - PID controller generated step time response with zero overshoot and settling time less than 0.13 s.
- The tuning process of the 2DOF PI-PID controller was based on using the optimization and control toolboxes of the MATLAB as a constrained optimization problem.
- An ITAE objective function and four functional constraints were used to tune the controller and maintain a stable control system with good performance measures.
- The used tuning technique was compared with a tuning technique based on the minimum ITAE standard forms.
- The effectiveness of the proposed 2DOF PI-PID controller in controlling a highly oscillating second order process was compared with using a PD-PID and PI-PD controllers. The 2DOF PI-PID controller could compete well with the PI-PD controller.

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BIOGRAPHY

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