RESEARCH ARTICLE OPEN ACCESS

Exponential Stabilization of the Coupled Dynamical Neural Networks with Nodes of Different Dimensions and Time Delays

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Abstract:

A class of coupled neural networks with different internal time-delays and coupling delays is investigated, which consists of nodes of different dimensions. By constructing suitable Lyapunov functions and using the linear matrix inequality, the criteria of exponential stabilization for the coupled dynamical system are established, and formulated in terms of linear matrix inequality. Finally, numerical examples are presented to verify the feasibility and effectiveness of the proposed theoretical results.

Keywords----**nodes of different dimensions; coupled dynamical system with time delays; exponential stabilization; linear matrix inequality; Lyapunov function**

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I. INTRODUCTION

With the rapid development of science and technology, neural networks are gradually and widely applied to the optimal control, combinatorial optimization, pattern recognition, and other fields. Due to the finite speeds of transmission and traffic congestion, time delays are common in actual systems, and the delays may be constant, time-varying with parameter or mixed (see [1]).

In the past two decades, stability or stabilization problems have been the hot topics for neural networks with time delays, including isolated neural networks and coupled neural networks (see [2]-[17] and references therein). For instance, the stabilization problem of delayed recurrent neural networks is investigated by a state estimation based approach in [2]. The exponential stability problem for a class of impulsive cellular neural networks with time delay is investigated by Lyapunovfunctions and the method of variation of parameters in[3]. Some studies on exponential stability or stabilization problem are about neural networks with time-varying delays (see [5]-[8]). Some sufficient conditions to ensure the existence, uniqueness and global exponential stability of the equilibrium point of cellular neural networks with variable delays are derived in[5]. For the problem of exponential stabilization for a class of non-autonomous neural networks with mixed discrete and distributed time-varying delays, some new delay-dependent conditions for designing a memoryless state feedback controller which stabilizes the system with an exponential convergence of the resulting closed-loop system are established in [6]. A new class of stochastic Cohen-Grossberg neural networks with reaction-diffusion and mixed delays is studied in [7]. Since the form of coupling is various, there exist different coupled neural networks, e.g. coupled term with or without time delays, or hybrid both. Some researches on stability and synchronization of different coupled neural networks are investigated (see [9]-[11]). Some global stability criteria for arrays of linearly coupled delayed neural networks with nonsymmetric coupling are established on the basis of linear matrix inequality method, in which the coupling configuration matrix may be arbitrary matrix with appropriate dimensions in [9]. Since there may be some chaotic behaviours as the dynamics performance of time-delay neural network in certain situations, chaotic neural networks are formed (see [12]-[14] and references therein). Some researches on stability and synchronization of chaotic neural network are investigated in [12]-[14]. Furthermore, sufficient condition for exponential stability of the equilibrium solution of uncontrolled stochastic interval system is also presented in [15]. The decentralized continuous adaptive controller can be designed to make the solutions of the closed-loop system exponentially convergent to a ball, which can be rendered arbitrary small by adjusting design parameters in [16]. Mostof researches on synchronization problem of neural network are also based on linear matrix inequality method and Lyapunovfunctions.

Although there are lots of researches for the exponential stability or stabilization problems of neural networks with time delay, most of them concern with the same dimension of the states. If a network is constructed by nodes with different state dimension, the network will exhibit different dynamical behaviours (see [17]-[19] and references therein). Dimensions of nodes are actually different in many practical situations, so such coupled complex networks need more in-depth study.

Motivated by the above discussion, in this paper we consider the exponential stabilization problem of the coupled dynamical system. The dimensions of states in each isolated network are different, and the internal time-delays are different from the coupling delays. In Sec.3, the criteria of exponential stabilization for the coupled dynamical system are given and proved on basis of the linear matrix inequality and Lyapunovfunction. In Sec.4, two numerical simulation examples are given to demonstrate the effectiveness of the proposed theoretical results. In Sec.5, some concluding remarks are given.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider a coupled dynamical system with nodes of different dimensions and time delays:

$$\begin{aligned}\frac{dx_i(t)}{dt} &= -D_i x_i(t) + A_i f_i(x_i(t)) + B_i f_i(x_i(t-\tau_{i1})) \\ &\quad + \alpha_i \sum_{j=1}^{N} g_{ij} C_{ij} x_j(t) + \beta_i \sum_{j=1}^{N} g_{ij} \Gamma_{ij} x_j(t-\tau_{i2}) + u_i(t),\end{aligned} \quad (1)$$

in which each isolated node network with different dimensions and time delays is considered as:

$$\frac{dx_i(t)}{dt} = -D_i x_i(t) + A_i f_i(x_i(t)) + B_i f_i(x_i(t-\tau_{i1})), \quad (2)$$

where $i = 1, 2, \cdots, N$; $x_i(t) = (x_{i1}(t), x_{i2}(t), \cdots, x_{in_i}(t))^T \in \Re^{n_i}$ is the state vector of the i th node; $A_i, B_i \in \Re^{n_i \times n_i}$ are constant matrices that representing the feedback matrix without and with time delays respectively; $D_i = diag(d_{i1}, d_{i2}, \cdots, d_{in_i}) > 0$ is a constant and diagonal matrix; $f_i(*)$ is the activation function; $G = (g_{ij})_{N\times N}$ is an outer coupling matrix representing the coupling strength and the topological structure of the neural networks; $C_{ij}, \Gamma_{ij} \in \Re^{n_i \times n_j}$ are inner coupling matrices respectively representing the inner-linking strengths between the cells without and with time delays,when $n_i \neq n_j$, $C_{ij}, \Gamma_{ij} \in \Re^{n_i \times n_j}$ are arbitrary matrices; α_i, β_i are the strengths of the constant coupling and delayed coupling, respectively; τ_{i1}, τ_{i2} are the constant and positive delays, $\tau_{i1} > 0, \tau_{i2} > 0$ and $\tau_{i1} < \tau, \tau_{i2} < \tau$; $u_i(t) = (u_{i1}(t), u_{i2}(t), \cdots, u_{in_i}(t))^T \in \Re^{n_i}$ is an external input.

The initial condition associated with (1) is given as follows:

$$x_{ij}(s) = \phi_{ij}(s) \in C([-\tau, 0], \Re), \quad (3)$$

where $\tau_i = \max\{\tau_{i1}, \tau_{i2}\}$, $\tau = \max\{\tau_1, \tau_2, \cdots, \tau_N\}$, $i = 1, 2, \cdots, N$, $j = 1, 2, \cdots, n_i$.

For convenience, the notations are givens as follows:

I denotes $M \times M$ identity matrix; I_{n_i} denotes $n_i \times n_i$ identity matrix; $\|y\| = \sqrt{y^T y}$ denotes the norm of y ; $\lambda_{\max}(*)$ and $\lambda_{\min}(*)$ respectively denote the maximum and minimum eigenvalue of $*$; $M = n_1 + n_2 + \cdots + n_N$;

$X(t) = (x_1^T(t), x_2^T(t), \cdots, x_N^T(t))^T$,

$U(t) = (u_1^T(t), u_2^T(t), \cdots, u_N^T(t))^T$,

$F(X(t)) = (f_1^T(x_1(t)), f_2^T(x_2(t)), \cdots, f_N^T(x_N(t)))^T$,

$F(X(t-\tau_1)) = (f_1^T(x_1(t-\tau_{11})), f_2^T(x_2(t-\tau_{21})), \cdots, f_N^T(x_N(t-\tau_{N1})))^T$,

$X(t-\tau_2) = (x_1^T(t-\tau_{12}), x_2^T(t-\tau_{22}), \cdots, x_N^T(t-\tau_{N2}))^T$,

$diag(\cdots)$ denotes a block-diagonal matrix;

$D = diag(D_1, D_2, \cdots, D_N)$; $A = diag(A_1, A_2, \cdots, A_N)$;

$B = diag(B_1, B_2, \cdots, B_N)$; $W_i = diag(w_{i1}, w_{i2}, \cdots, w_{in_i})$;

$W = diag(W_1, W_2, \cdots, W_N)$;

$\begin{bmatrix} X & Y \\ * & Z \end{bmatrix}$ is defined as a matrix in form of $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$.

In order to study the exponential stabilization of the coupled dynamical system (1), the linear controller isdesigned as

$U(t) = -KX(t)$,i.e., $u_i(t) = -K_i x_i(t)$, (4)

where $K_i = diag(k_{i1}, k_{i2}, \cdots, k_{in_i})$, $K = diag(K_1, K_2, \cdots, K_N)$.

Hence, equation (1) can be rewritten as

$$\begin{aligned}\frac{dX(t)}{dt} &= -(D+K)X(t) + AF(X(t)) + BF(X(t-\tau_1)) \\ &+ H_1 X(t) + H_2 X(t-\tau_2),\end{aligned} \quad (5)$$

where

$$H_1 = \begin{bmatrix} \alpha_1 g_{11} C_{11} & \alpha_1 g_{12} C_{12} & \cdots & \alpha_1 g_{1N} C_{1N} \\ \alpha_2 g_{21} C_{21} & \alpha_2 g_{22} C_{22} & \cdots & \alpha_2 g_{2N} C_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_N g_{N1} C_{N1} & \alpha_N g_{N2} C_{N2} & \cdots & \alpha_N g_{NN} C_{NN} \end{bmatrix},$$

$$H_2 = \begin{bmatrix} \beta_1 g_{11} \Gamma_{11} & \beta_1 g_{12} \Gamma_{12} & \cdots & \beta_1 g_{1N} \Gamma_{1N} \\ \beta_2 g_{21} \Gamma_{21} & \beta_2 g_{22} \Gamma_{22} & \cdots & \beta_2 g_{2N} \Gamma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \beta_N g_{N1} \Gamma_{N1} & \beta_N g_{N2} \Gamma_{N2} & \cdots & \beta_N g_{NN} \Gamma_{NN} \end{bmatrix}.$$

Assume that $X^* = (x_1^{*T}, x_2^{*T}, \cdots, x_N^{*T})^T$ is the equilibrium point of the coupled dynamical system (1), then itsatisfies

$$\frac{dX^*}{dt} = -(D+K)X^* + Af(X^*) + Bf(X^*) + H_1 X^* + H_2 X^*, \quad (6)$$

i.e., $-(D+K)X^* + Af(X^*) + Bf(X^*) + H_1 X^* + H_2 X^* = 0$,

where $x_i^* = (x_{i1}^*, x_{i2}^*, \cdots, x_{in_i}^*)^T$.

Define the linear coordinate transformation $E(t) = X(t) - X^*$, andthe new dynamical systems can be described as follows:

$$\begin{aligned}\frac{de_i(t)}{dt} &= -D_i e_i(t) + A_i \phi_i(e_i(t)) + B_i \phi_i(e_i(t-\tau_{i1})) \\ &+ \alpha_i \sum_{j=1}^{N} g_{ij} C_{ij} e_j(t) + \beta_i \sum_{j=1}^{N} g_{ij} \Gamma_{ij} e_j(t-\tau_{i2}) + U_i(t)\end{aligned} \quad (7)$$

That is,

$$\begin{aligned}\frac{dE(t)}{dt} &= -DE(t) + A\Phi(E(t)) + B\Phi(E(t-\tau_1)) \\ &+ H_1 E(t) + H_2 E(t-\tau_2) + U(t)\end{aligned}, \quad (8)$$

where

$$\begin{aligned}e_i(t) &= (e_{i1}(t), e_{i2}(t), \cdots, e_{in_i}(t))^T \\ &= (x_{i1}(t) - x_{i1}^*, x_{i2}(t) - x_{i2}^*, \cdots, x_{in_i}(t) - x_{in_i}^*)^T,\end{aligned}$$

$\phi_i(e_i(t)) = f_i(x_i(t)) - f_i(x_i^*)$, $U_i(t) = -K_i e_i(t)$,

$\phi_i(e_i(t-\tau_{i1})) = f_i(x_i(t-\tau_{i1})) - f_i(x_i^*)$, $U(t) = -KE(t)$,

$E(t) = (e_1^T(t), e_2^T(t), \cdots, e_N^T(t))^T$, $E(t) = X(t) - X^*$,

$$\begin{aligned}\Phi(E(t)) &= (\phi_1^T(e_1(t)), \phi_2^T(e_2(t)), \cdots, \phi_N^T(e_N(t)))^T \\ &= F(X(t)) - F(X^*),\end{aligned}$$

$\Phi(E(t-\tau_1)) = F(X(t-\tau_1)) - F(X^*)$,

$E(t-\tau_2) = (e_1^T(t-\tau_{12}), e_2^T(t-\tau_{22}), \cdots, e_N^T(t-\tau_{N2}))^T$.

Assumption 1. The activation function

$$f_i(x_i(t)) = (f_{i1}(x_{i1}(t)), f_{i2}(x_{i2}(t)), \cdots, f_{in_i}(x_{in_i}(t)))^T$$

isLipschitz continuous, i.e., there exist constants $w_{il} > 0$, such that

$$|f_{il}(\xi_1) - f_{il}(\xi_2)| \le w_{il} |\xi_1 - \xi_2|$$

holdsfor any different $\xi_1, \xi_2 \in \Re$, and $\xi_1 \ne \xi_2$, where $i = 1, 2, \cdots, N$, $l = 1, 2, \cdots, n_i$.

Definition 1[13]For given $k > 0$, the dynamical system (1) is said to be exponential stabilization, if there exist constant $Z > 0$ such that the following inequality $\|E(t)\|^2 \le Z\|\varphi\|^2 e^{-kt}$ holds for all initial conditions $e_{ij}(s)(i = 1, \cdots, N; j = 1, \cdots, n_i)$ of system (7) and any $t \ge T$ (sufficiently large $T > 0$), where

$$\|\varphi\| = \sup_{-\tau \le s \le 0} \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{n_i} |e_{ij}(s)|^2}.$$

Lemma 1 [4] For any $x, y \in \Re^n$ and positive definite matrix $Q \in \Re^{n \times n}$, the following matrix inequality holds:

$$2x^T y \le x^T Q x + y^T Q^{-1} y.$$

Lemma 2[6]The linear matrix inequality

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

where $Q^T(x) = Q(x)$, $R^T(x) = R(x)$, is equivalent to $R(x) > 0$, and $Q(x) - S(x)R^{-1}(x)S^T(x) > 0$.

Lemma 3 [5]If Q, R are real symmetric matrices, and $Q > 0$, $R \ge 0$, then a positive constant σ exist, such that the following inequality holds:

$$-Q + \sigma R < 0.$$

III. EXPONENTIAL STABILIZATION ANALYSIS

Theorem 1. Under the assumption 1, the coupled dynamical system (1) is exponential stabilization,if there exist $M \times M$ positive definite diagonal matrices P , Q , R and $\bar{K}$,such that the following linear matrix inequalities hold: $\Xi > 0$,where

$$\Xi = \begin{bmatrix} \psi & PA & PB & PH_1 & PH_2 \\ * & Q & 0 & 0 & 0 \\ * & * & R & 0 & 0 \\ * & * & * & Q & 0 \\ * & * & * & * & Q \end{bmatrix} > 0 \quad (9)$$

$\psi = PD + D^T P + \bar{K} + \bar{K}^T - WQW - WRW - 2Q$,

$P_i = diag(p_{i1}, p_{i2}, \cdots, p_{in_i}), P = diag(P_1, P_2, \cdots, P_N)$,

$Q_i = diag(q_{i1}, q_{i2}, \cdots, q_{in_i}), Q = diag(Q_1, Q_2, \cdots, Q_N)$,

$R_i = diag(r_{i1}, r_{i2}, \cdots, r_{in_i}), R = diag(R_1, R_2, \cdots, R_N)$,

$W_i = diag(w_{i1}, w_{i2}, \cdots, w_{in_i}), W = diag(W_1, W_2, \cdots, W_N)$.

Moreover, the gain matrix of a desired controller of the form (4) is given by $K = P^{-1}\bar{K}$.

Proof. From Lemmas 2 and 3, we get the following conclusion: there exist a positive constant λ, such that

$$\Xi_1 = \begin{bmatrix} \psi^* & PA & PB & PH_1 & PH_2 \\ * & -Q & 0 & 0 & 0 \\ * & * & -R & 0 & 0 \\ * & * & * & -Q & 0 \\ * & * & * & * & -Q \end{bmatrix} < 0,$$

where $\psi^* = -\psi + \lambda P + (e^{\lambda\tau} - 1)(Q + WRW)$.

Consider the Lyapunovfunction as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (10)$$

$$V_1(t) = e^{\lambda t}\sum_{i=1}^{N} e_i^T(t) P_i e_i(t)$$

$$V_2(t) = \sum_{i=1}^{N}\int_{t-\tau_{i2}}^{t} e^{\lambda(s+\tau)} e_i^T(s) Q_i e_i(s) ds$$

$$V_3(t) = \sum_{i=1}^{N}\int_{t-\tau_{i1}}^{t} e^{\lambda(s+\tau)} \phi_i^T(e_i(s)) R_i \phi_i(e_i(s)) ds.$$

Calculating the time derivatives of $V_1(t)$, $V_2(t)$ and $V_3(t)$ along the trajectories of system (7), we have

$$\begin{aligned} \dot{V}_1(t) &= \lambda e^{\lambda t}\sum_{i=1}^{N} e_i^T(t) P_i e_i(t) + 2e^{\lambda t}\sum_{i=1}^{N} e_i^T(t) P_i \dot{e}_i(t) \\ &= \lambda e^{\lambda t} E^T(t) P E(t) + 2e^{\lambda t} E^T(t) P \dot{E}(t) \end{aligned} \quad (11)$$

$$\begin{aligned} \dot{V}_2(t) &= \sum_{i=1}^{N} e^{\lambda(t+\tau)} e_i^T(t) Q_i e_i(t) \\ &\quad - \sum_{i=1}^{N} e^{\lambda(t-\tau_{i2}+\tau)} e_i^T(t-\tau_{i2}) Q_i e_i(t-\tau_{i2}) \\ &\le e^{\lambda(t+\tau)} E^T(t) Q E(t) - e^{\lambda t} E^T(t-\tau_2) Q E(t-\tau_2) \end{aligned} \quad (12)$$

$$\begin{aligned} \dot{V}_3(t) &= \sum_{i=1}^{N} e^{\lambda(t+\tau)} \phi_i^T(e_i(t)) R_i \phi_i(e_i(t)) \\ &\quad - \sum_{i=1}^{N} e^{\lambda(t-\tau_{i1}+\tau)} \phi_i^T(e_i(t-\tau_{i1})) R_i \phi_i(e_i(t-\tau_{i1})) \\ &\le e^{\lambda(t+\tau)} \Phi^T(E(t)) R \Phi(E(t)) \\ &\quad - e^{\lambda t} \Phi^T(E(t-\tau_1)) R \Phi(E(t-\tau_1)) \end{aligned} \quad (13)$$

From Lemma 1 and Assumption 1, we obtain

$$\begin{aligned} & 2E^T(t) PA \Phi(E(t)) \\ &\le E^T(t) PAQ^{-1}A^T P E(t) + \Phi^T(E(t)) Q \Phi(E(t)) \\ &\le E^T(t) PAQ^{-1}A^T P E(t) + E^T(t) WQW E(t), \end{aligned} \quad (14)$$

$$\begin{aligned} & 2E^T(t) PB \Phi(E(t-\tau_1)) \\ &\le E^T(t) PBR^{-1}B^T P E(t) + \Phi^T(E(t-\tau_1)) R \Phi(E(t-\tau_1)), \end{aligned} \quad (15)$$

$$\begin{aligned} & 2E^T(t) PH_2 E(t-\tau_2) \\ &\le E^T(t) PH_2 Q^{-1}(H_2)^T P E(t) + E^T(t-\tau_2) Q E(t-\tau_2) \end{aligned} \quad (16)$$

$$\begin{aligned} & 2E^T(t) PH_1 E(t) \\ &\le E^T(t) PH_1 Q^{-1}(H_1)^T P E(t) + E^T(t) Q E(t) \end{aligned} \quad (17)$$

Substituting (11)~(17) into $\dot{V}(t)$, it yields

$$\begin{aligned} \dot{V}(t) \le e^{\lambda t}\{ & E^T(t)[\lambda P - PD - D^T P + WQW + e^{\lambda\tau} Q \\ & + e^{\lambda\tau} WRW + Q + PAQ^{-1}A^T P + PBR^{-1}B^T P \\ & + PH_1 Q^{-1}(H_1)^T P + PH_2 Q^{-1}(H_2)^T P - 2PK] E(t)\} \end{aligned} \quad (18)$$

From Lemma 2, $\Xi_1 < 0$ is equivalent to

$$\begin{aligned} \Delta = & \lambda P - P(D+K) - (D+K)^T P + WQW + e^{\lambda\tau} Q \\ & + e^{\lambda\tau} WRW + Q + PAQ^{-1}A^T P + PBR^{-1}B^T P \\ & + PH_1 Q^{-1}(H_1)^T P + PH_2 Q^{-1}(H_2)^T P \\ & < 0 \end{aligned} \quad (19)$$

It is derived that $\dot{V}(t) \le 0$. Therefore, we know $V(t)$ is decreasing from t=0, and $V(t) \le V(0)$.

From the initial conditions (3) of the coupled dynamical system (1),we obtain the initial conditions of thenew dynamical system (8) are $e_{ij}(s) = \varphi_{ij}(s) - x_{ij}^*(s) \in C([-\tau, 0], \Re)$.

It follows from (10) that

$$\begin{aligned} V(0) &= e^0 \sum_{i=1}^{N} e_i^T(0) P_i e_i(0) + \sum_{i=1}^{N}\int_{-\tau_{i2}}^{0} e^{\lambda(s+\tau)} e_i^T(s) Q_i e_i(s) ds \\ &\quad + \sum_{i=1}^{N}\int_{-\tau_{i1}}^{0} e^{\lambda(s+\tau)} \phi_i^T(e_i(s)) R_i \phi_i(e_i(s)) ds \\ &= E^T(0) P E(0) + \sum_{i=1}^{N}\int_{-\tau_{i2}}^{0} e^{\lambda(s+\tau)} e_i^T(s) Q_i e_i(s) ds \\ &\quad + \sum_{i=1}^{N}\int_{-\tau_{i1}}^{0} e^{\lambda(s+\tau)} \phi_i^T(e_i(s)) R_i \phi_i(e_i(s)) ds \end{aligned} \quad (20)$$

$$\sum_{i=1}^{N}\int_{-\tau_{i2}}^{0} e^{\lambda(s+\tau)}e_i^T(s)Q_ie_i(s)ds$$
$$\le \sum_{i=1}^{N}\int_{-\tau}^{0} e^{\lambda(s+\tau)}e_i^T(s)Q_ie_i(s)ds = e^{\lambda\tau}\int_{-\tau}^{0} e^{\lambda s}E^T(s)QE(s)ds \quad (21)$$
$$\le e^{\lambda\tau}\lambda_{\max}(Q)\int_{-\tau}^{0} e^{\lambda s}ds\cdot\|\varphi\|^2 = \frac{e^{\lambda\tau}-1}{\lambda}\cdot\lambda_{\max}(Q)\|\varphi\|^2$$

$$\begin{aligned}&\sum_{i=1}^{N}\int_{-\tau_{i2}}^{0} e^{\lambda(s+\tau)}\phi_i^T(e_i(s))R_i\phi_i(e_i(s))ds\\ &\le e^{\lambda\tau}\int_{-\tau}^{0} e^{\lambda s}E^T(s)WRWE(s)ds\\ &\le e^{\lambda\tau}\lambda_{\max}(WRW)\int_{-\tau}^{0} e^{\lambda s}ds\cdot\|\varphi\|^2\\ &= \frac{e^{\lambda\tau}-1}{\lambda}\cdot\lambda_{\max}(WRW)\|\varphi\|^2\end{aligned} \quad (22)$$

Substituting (21)~(22) into $V(0)$, we get

$$\begin{aligned}V(0) &\le \lambda_{\max}(P)\|\varphi\|^2 + \frac{e^{\lambda\tau}-1}{\lambda}\cdot[\lambda_{\max}(Q)+\lambda_{\max}(WRW)]\|\varphi\|^2\\ &= \{\lambda_{\max}(P) + \frac{e^{\lambda\tau}-1}{\lambda}\cdot[\lambda_{\max}(Q)+\lambda_{\max}(WRW)]\}\|\varphi\|^2\end{aligned} \quad (23)$$

From $e^{\lambda t}\cdot\lambda_{\min}(P)\|E(t)\|^2 \le V_1(t) \le V(t)$ and $V(t)\le V(0)$,we have

$$e^{\lambda t}\cdot\lambda_{\min}(P)\|E(t)\|^2 \le V(0). \quad (24)$$

Combining with (23) and (24), we obtain

$$\begin{aligned}\|E(t)\|^2 &\le \frac{1}{\lambda_{\min}(P)}\cdot\{\lambda_{\max}(P)\\ &+\frac{e^{\lambda\tau}-1}{\lambda}\cdot[\lambda_{\max}(Q)+\lambda_{\max}(WRW)]\}\cdot\|\varphi\|^2 e^{-\lambda t}\\ &= Z\|\varphi\|^2 e^{-\lambda t}\end{aligned} \quad (25)$$

where $Z = \frac{1}{\lambda_{\min}(P)}\cdot\{\lambda_{\max}(P)+\frac{e^{\lambda\tau}-1}{\lambda}\cdot[\lambda_{\max}(Q)+\lambda_{\max}(WRW)]\}$.

That is, the coupled dynamical system (1) with nodes of different dimensions and time delays is exponential stabilization.This completes the proof.

IV. NUMERICAL EXAMPLES

Example 1. Consider a coupled dynamical system (1) is composed of two isolated node networks, in which the parameters are given as follows:

$N=2$, $n_1=2$, $n_2=3$, $f(x_{il})=\sin(x_{il})$, $i=1,2$, $l=1,\dots,n_i$,

$$D_1=\begin{bmatrix}8 & 0\\ 0 & 9\end{bmatrix}, A_1=\begin{bmatrix}0.2 & -0.1\\ -0.5 & 0.9\end{bmatrix}, B_1=\begin{bmatrix}-0.9 & 0.8\\ 0.7 & 0.6\end{bmatrix},$$

$$D_2=\begin{bmatrix}3 & 0 & 0\\ 0 & 7 & 0\\ 0 & 0 & 6\end{bmatrix}, A_2=\begin{bmatrix}0.1 & -0.1 & -0.2\\ 0.1 & 0.2 & -0.3\\ -0.5 & 0.4 & 0.9\end{bmatrix},$$

$$B_2=\begin{bmatrix}-0.8 & 0.3 & 0.7\\ -0.5 & -0.7 & 0.3\\ -0.3 & 0.4 & -0.1\end{bmatrix}, C_{21}=\begin{bmatrix}-0.6 & 0.6\\ -0.2 & -0.8\\ 0.7 & -0.2\end{bmatrix},$$

$$C_{11}=\begin{bmatrix}0.1 & -0.8\\ -0.1 & 0.3\end{bmatrix}, C_{12}=\begin{bmatrix}0.4 & 0.3 & 0.9\\ -0.7 & 0.1 & 0.3\end{bmatrix},$$

$$\Gamma_{11}=\begin{bmatrix}-0.2 & -0.4\\ 0.9 & 0.4\end{bmatrix}, \Gamma_{12}=\begin{bmatrix}0.9 & -0.9 & 0.7\\ -0.6 & 0.1 & 0.3\end{bmatrix},$$

$$C_{22}=\begin{bmatrix}-0.8 & -0.1 & 0.8\\ -0.2 & -0.3 & -0.9\\ -0.6 & 0.9 & 0.5\end{bmatrix}, \Gamma_{21}=\begin{bmatrix}0.3 & 0.1\\ -0.2 & -0.1\\ -0.6 & -0.7\end{bmatrix},$$

$$\Gamma_{22}=\begin{bmatrix}0.9 & 0.1 & 0.7\\ 0.4 & -0.8 & 0.6\\ -0.3 & 0.8 & -0.4\end{bmatrix}, G=\begin{bmatrix}0.2 & 0.6\\ 0.7 & 0.1\end{bmatrix},$$

$\alpha_1=0.1$, $\alpha_2=0.2$, $\beta_1=0.9$, $\beta_2=0.1$,

$\tau_{11}=0.5$, $\tau_{12}=0.7$, $\tau_{21}=0.6$, $\tau_{22}=0.4$,

By calculating, we know that equilibrium points of the two isolated node networks (2) are

$$\tilde{x}_1 = 10^{-7}*(0.0111,-0.0034)^T$$
$$\tilde{x}_2 = 10^{-7}*(0.0756,-0.0117,0.1720)^T,$$

respectively. Therefore, it is easy to derive that $w_{il}=1$, and $W=I_5$, for $i=1,2,\cdots,N$, $l=1,2,\cdots,n_i$.

By using the MATLAB LMI Control Toolbox, solving the linear matrix inequalities in Theorem 1, it yields the following feasible solutions:

$$P = diag(3.7960, 3.4276, 3.9682, 4.5893, 5.0527)$$
$$Q = diag(20.2148, 20.3078, 20.4814, 20.5548, 20.2048)$$
$$R = diag(20.2106, 20.2726, 20.3884, 20.4373, 20.2040)$$
$$\bar{K} = diag(20.1689, 19.9210, 19.4580, 19.2622, 20.1957)$$

Then, we have

$$K = diag(5.3132, 5.8119, 4.9035, 4.1972, 3.9970).$$

According to Theorem 1, given the initial condition:

$$X(0)=(0.5,\ -0.2,\ -0.3,\ 0.2,\ 0.5)^T,$$

the coupled dynamical system (1) can achieve the exponential stabilization, and the equilibrium point is

$$X^* = 10^{-10}*(0.0913,-0.0236,0.1704,-0.0514,0.3277)^T,$$

the simulation results are plotted in Fig. 1. All state trajectories converge to the equilibrium point, and the equilibrium point of the coupled dynamical system (1) is different from that of each isolated node network (2).

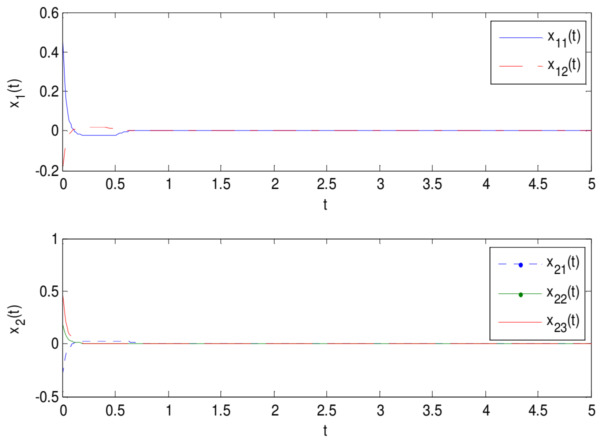

Fig. 1.The state trajectories of coupled dynamical system in Example 1.

Example 2. Consider a coupled dynamical system (1) is composed of two isolated node networks, in which the parameters are given as follows:

$$N = 2,\ n_1 = 2,\ n_2 = 3,\ f(x_{il}) = \sin(x_{il}) + 1,$$

$$i = 1,2,\ l = 1,\ldots,n_i,$$

$$D_1 = \begin{bmatrix} 29 & 0 \\ 0 & 36 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & -1 \\ 5 & -4 \end{bmatrix},\ B_1 = \begin{bmatrix} -3 & 1 \\ 4 & 5 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 33 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 34 \end{bmatrix},\ A_2 = \begin{bmatrix} -3 & 1 & 4 \\ 1 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} -5 & -5 & 4 \\ 1 & 2 & 4 \\ 2 & -4 & 3 \end{bmatrix},$$

$$C_{11} = \begin{bmatrix} -5 & -1 \\ 1 & -2 \end{bmatrix},\ C_{12} = \begin{bmatrix} 4 & -2 & -4 \\ -2 & -4 & 2 \end{bmatrix},$$

$$\Gamma_{11} = \begin{bmatrix} -1 & -2 \\ -1 & 2 \end{bmatrix},\ \Gamma_{12} = \begin{bmatrix} 3 & -5 & -5 \\ -1 & -3 & 1 \end{bmatrix},$$

$$C_{21} = \begin{bmatrix} 1 & 4 \\ -5 & -2 \\ 2 & 4 \end{bmatrix},\ C_{22} = \begin{bmatrix} -4 & 2 & 1 \\ 3 & -5 & 4 \\ -4 & 4 & 4 \end{bmatrix},$$

$$\Gamma_{21} = \begin{bmatrix} 3 & -3 \\ 2 & 1 \\ -3 & 5 \end{bmatrix},\ \Gamma_{22} = \begin{bmatrix} -4 & 3 & -3 \\ -5 & 2 & 3 \\ -2 & 4 & 3 \end{bmatrix},\ G = \begin{bmatrix} 0.9 & 0.7 \\ 0.1 & 0.4 \end{bmatrix},$$

$$\alpha_1 = 0.1,\ \alpha_2 = 0.4,\ \beta_1 = 0.9,\ \beta_2 = 0.6,$$

$$\tau_{11} = 0.5,\ \tau_{12} = 1,\ \tau_{21} = 0.8,\ \tau_{22} = 0.4,$$

By calculating, we know that equilibrium points of the two isolated node networks (2) are

$$\tilde{x}_1 = (-0.0645, 0.2690)^T$$

$$\tilde{x}_2 = (-0.3305, 0.3848, -0.0039)^T,$$

respectively. Therefore, it is easy to derive that $w_{il} = 2$, and $W = 2I_5$,for $i = 1,2,\cdots,N$, $l = 1,2,\cdots,n_i$.

By using the MATLAB LMI Control Toolbox, solving the linear matrix inequalities $\Xi > 0$ in Theorem 1, it yields the following feasible solutions:

$$P = diag(3.5079, 2.2782, 2.0888, 4.6528, 3.3211)$$

$$Q = diag(24.8830, 23.2186, 22.5922, 24.6529, 26.5236)$$

$$R = diag(24.6942, 22.3220, 21.7158, 26.2661, 29.3649)$$

$$\bar{K} = diag(44.4094, 53.7464, 62.1294, 47.1680, 39.8634)$$

Then, we have

$$K = diag(12.6598, 23.5912, 29.7447, 10.1376, 12.0031).$$

Given the initial condition $X(0) = (1,-1,-2,2,1)^T$,according to Theorem 1,the coupled dynamical system (1) can achieve the exponential stabilization and the equilibrium point is

$$X^* = (-0.0376, 0.1092, -0.1181, 0.1190, 0.0145)^T,$$

the simulation results are plotted in Fig. 2. All state trajectories converge to the equilibrium point, and the equilibrium point of the coupled dynamical system (1) is different from that of each isolated node network (2).

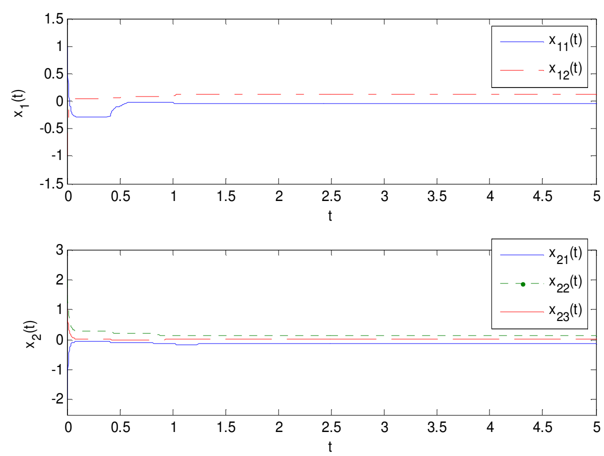

Fig. 2.The state trajectories of coupled dynamical system in Example 2.

V. CONCLUSION

In this paper, the exponential stabilization problem has been investigated for the coupled neural networks. Thedimensions of states in each isolated network can be different, and dimensions of nodes are actually differentin many practical situations, so the derived results will have wider applicability. The criteria of exponentialstabilization for the coupled dynamical system are established on basis of the linear matrix inequality andLyapunovfunction. The effectiveness of the proposed theoretical results has been demonstrated by two numerical simulation examples. In addition, finite-time control could be an alternative method for the exponentialstabilization of such kind of coupled dynamical system, which deserves our further investigation.

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