interval valued fuzzy ideals of \( \Gamma \)-near-rings

V. Chinnadurai and K. Arulmozhi

1. Introduction

The notion of fuzzy sets was introduced by Zadeh \[12\] in 1965, and he \[13\] also generalized it to interval valued fuzzy subsets (shortly i.v fuzzy subsets), whose of membership values are closed subinterval of \([0, 1]\). Near-ring was introduced by Pilz \[8\] and \( \Gamma \)-near-ring was introduced by Satyanarayana \[9\] in 1984. The idea of fuzzy ideals of near-rings was presented by Kim \textit{et al.} \[6\]. Fuzzy ideals in Gamma-near-rings was proposed by Jun \textit{et al.} \[5\] in 1998. Moreover, Thillaigovindan \textit{et al.} \[10\] studied the interval valued fuzzy quasi-ideals of semigroups. Chinnadurai \textit{et al.} \[3\] characterized of fuzzy weak bi-ideals of \( \Gamma \)-near-rings. Thillaigovindan \textit{et al.} \[11\] worked on interval valued fuzzy ideals of near-rings. Rao \[7\] carried out a study on anti-fuzzy \( k \)-ideals and anti-homomorphism of \( \Gamma \)-near-rings. In this paper, we define a new notion of interval valued fuzzy ideals of \( \Gamma \)-near-rings, which is a generalized concept of an interval valued fuzzy ideals of near-rings. We also investigate some of its properties and illustrate with examples.

2. Preliminaries

In this section, we list some basic definitions.

**Definition 2.1.** (\[13\]) Let \( X \) be any set. A mapping \( \eta : X \rightarrow D[0, 1] \) is called an interval valued fuzzy subset (briefly, an i.v fuzzy subset) of \( X \), where \( D[0, 1] \) denotes the family of closed subintervals of \([0, 1]\) and \( \tilde{\eta}(x) = [\eta^-(x), \eta^+(x)] \) for all

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\( x \in X \), where \( \eta^-(x) \) and \( \eta^+(x) \) are fuzzy subsets of \( X \) such that \( \eta^-(x) \leq \eta^+(x) \) for all \( x \in X \).

**Definition 2.2.** ([12]) An interval number \( \tilde{a} \), we mean an interval \([a^-, a^+]\)
such that \( 0 \leq a^- \leq a^+ \leq 1 \) and where \( a^- \) and \( a^+ \) are the lower and upper limits of \( \tilde{a} \) respectively. The set of all closed subintervals of \([0, 1]\) is denoted by \( D[0, 1] \). We also identify the interval \([a, a]\) by the number \( a \in [0, 1] \). For any interval numbers \( \tilde{a}_j = [a_j^-, a_j^+] \), \( \tilde{b}_j = [b_j^-, b_j^+] \in D[0, 1], j \in \Omega \) (where \( \Omega \) is index set), we define

\[
\max^j \{\tilde{a}_j, \tilde{b}_j\} = [\max \{a_j^-, b_j^-, a_j^+, b_j^+\}],
\min^j \{\tilde{a}_j, \tilde{b}_j\} = [\min \{a_j^-, b_j^-, a_j^+, b_j^+\}],
\inf^j \tilde{a}_j = [\bigcap_{j \in \Omega} a_j^-, \bigcap_{j \in \Omega} a_j^+],
\sup^j \tilde{a}_j = [\bigcup_{j \in \Omega} a_j^-, \bigcup_{j \in \Omega} a_j^+].
\]

and let

(i) \( \tilde{a} \leq \tilde{b} \Leftrightarrow a^- \leq b^- \) and \( a^+ \leq b^+ \),
(ii) \( \tilde{a} = \tilde{b} \Leftrightarrow a^- = b^- \) and \( a^+ = b^+ \),
(iii) \( \tilde{a} \nless \tilde{b} \Leftrightarrow \tilde{a} \leq \tilde{b} \) and \( \tilde{a} \neq \tilde{b} \),
(iv) \( k\tilde{a} = [ka^-, ka^+] \), whenever \( 0 \leq k \leq 1 \).

**Definition 2.3.** ([10]) Let \( \tilde{\eta} \) be an i.v fuzzy subset of \( X \) and \([t_1, t_2] \in D[0, 1] \). Then the set \( \tilde{U}(\tilde{\eta} : [t_1, t_2]) = \{ x \in X | \tilde{\eta}(x) \geq [t_1, t_2] \} \) is called the upper level subset of \( \tilde{\eta} \).

**Definition 2.4.** ([8]) A near-ring is an algebraic system \((R, +, \cdot)\) consisting of a non empty set \( R \) together with two binary operations called + and \( \cdot \) such that \((R, +)\) is a group not necessarily abelian and \((R, \cdot)\) is a semigroup connected by the following distributive law: \((x + z) \cdot y = x \cdot y + z \cdot y \) valid for all \( x, y, z \in R \). We use the word ‘near-ring’ to mean ‘right near-ring’. We denote \( xy \) instead of \( x \cdot y \).

**Definition 2.5.** ([9]) A \( \Gamma \)-near-ring is a triple \((M, +, \Gamma)\) where

(i) \((M, +)\) is a group,
(ii) \( \Gamma \) is a nonempty set of binary operations on \( M \) such that for each \( \alpha \in \Gamma \),
\((M, +, \alpha)\) is a near-ring,
(iii) \( x\alpha(y\beta z) = (x\alpha y)\beta z \) for all \( x, y, z \in M \) and \( \alpha, \beta \in \Gamma \).

In what follows, let \( M \) denote a \( \Gamma \)-near-ring unless otherwise specified.

**Definition 2.6.** ([9]) A subset \( A \) of a \( \Gamma \)-near-ring \( M \) is called a left (resp. right) ideal of \( M \) if

(i) \((A, +)\) is a normal divisor of \((M, +)\), (i.e) \( x - y \in A \) for all \( x, y \in A \) and \( y - x \in A \) for all \( x, y \in A \) and \( y \in M \)
(ii) \( u\alpha(x + v) - u\alpha v \in A \) (resp. \( x\alpha u \in A \)) for all \( x \in A, \alpha \in \Gamma \) and \( u, v \in M \).

**Definition 2.7.** ([9]) Let \( M \) be a \( \Gamma \)-near-ring. Given two subsets \( A \) and \( B \) of \( M \), we define \( A\Gamma B = \{ a\alpha b | a \in A, b \in B \text{ and } \alpha \in \Gamma \} \) and also define another operation \( \ast \) on the class of subset of \( M \) as
\[
A\Gamma \ast B = \{ a\gamma(a' + b) - a\gamma a' | a, a' \in A, \gamma \in \Gamma , b \in B \}.
\]
**Definition 2.8.** ([10]) Let \( I \) be a subset of a near-ring \( M \). Define a function \( f_I : M \to D[0, 1] \) by

\[
f_I(x) = \begin{cases} 
1 & \text{if } x \in I \\
0 & \text{otherwise.}
\end{cases}
\]

**Definition 2.9.** ([4]) If \( \tilde{\eta} \) and \( \tilde{\lambda}, \tilde{\mu}(i \in \Omega) \) are i.v fuzzy subsets of \( X \). The following are defined by

(i) \( \tilde{\eta} \leq \tilde{\lambda} \Leftrightarrow \tilde{\eta}(x) \leq \tilde{\lambda}(x) \).

(ii) \( \tilde{\eta} = \tilde{\lambda} \Leftrightarrow \tilde{\eta}(x) = \tilde{\lambda}(x) \).

(iii) \( \tilde{\eta} \cap \tilde{\lambda}(x) = \min\{\tilde{\eta}(x), \tilde{\lambda}(x)\} \).

(iv) \( \tilde{\eta} \cup \tilde{\lambda}(x) = \max\{\tilde{\eta}(x), \tilde{\lambda}(x)\} \).

(v) \( \bigcup_{i \in \Omega} \tilde{\eta}(x) = \sup\{\tilde{\eta}(x)\mid i \in \Omega\} \).

(vi) \( \bigcap_{i \in \Omega} \tilde{\eta}(x) = \inf\{\tilde{\eta}(x)\mid i \in \Omega\} \) for all \( x \in X \).

where \( \inf\{\tilde{\eta}\mid i \in \Omega\} = [\inf_{i \in \Omega} \eta_i^{-}(x)], \inf_{i \in \Omega} \{\eta_i^{+}(x)\}] \) is the i.v infimum norm and \( \sup\{\tilde{\eta}\mid i \in \Omega\} = [\sup_{i \in \Omega} \eta_i^{-}(x)], \sup_{i \in \Omega} \{\eta_i^{+}(x)\}] \) is the i.v supremum norm.

### 3. Interval valued fuzzy ideals of \( \Gamma \)-near-rings

In this section, we introduce the notion of i.v fuzzy left(right)ideal of \( M \) and discuss some of its properties.

**Definition 3.1.** An i.v fuzzy subset \( \tilde{\eta} \) in a \( \Gamma \)-near-ring \( M \) is called an i.v fuzzy left (resp. right) ideal of \( M \) if

(i) \( \tilde{\eta} \) is an i.v fuzzy normal divisor with respect to addition,

(ii) \( \tilde{\eta}(\alpha(p + d) - cod) \geq \tilde{\eta}(p) \), (resp. \( \tilde{\eta}(pcod) \geq \tilde{\eta}(p) \)) for all \( p, c, d \in M \) and \( \alpha \in \Gamma \).

The condition (i) of definition 3.1 means that \( \tilde{\eta} \) satisfies:

(i) \( \tilde{\eta}(p - q) \geq \min\{\tilde{\eta}(p), \tilde{\eta}(q)\} \).

(ii) \( \tilde{\eta}(q + p - q) \geq \tilde{\eta}(p) \), for all \( p, q \in M \)

Note that \( \tilde{\eta} \) is an i.v fuzzy left (resp. right) ideal of \( \Gamma \)-near-ring \( M \), then \( \tilde{\eta}(0) \geq \tilde{\eta}(p) \) for all \( p \in M \), where \( 0 \) is the zero element of \( M \).

**Example 3.1.** Let \( M = \{0, a, b, c\} \) be a non-empty set with binary operation \( + \) and \( \Gamma = \{\alpha, \beta\} \) be the non-empty set of binary operations as shown in the following tables:

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**Table 1.**

Let \( \tilde{\eta} : M \to D[0, 1] \) be an i.v fuzzy subset defined by \( \tilde{\eta}(0) = [0.8, 0.9] \), and \( \tilde{\eta}(a) = [0.6, 0.7], \tilde{\eta}(b) = \tilde{\eta}(c) = [0.2, 0.3] \). Then \( \tilde{\eta} \) is an i.v fuzzy ideal of \( M \).
Theorem 3.1. Let \( \tilde{\eta} = [\eta^-, \eta^+] \) be an i.v fuzzy subset of a \( \Gamma \)-near-ring \( M \), then \( \tilde{\eta} \) is an i.v fuzzy left(right) ideal of \( M \) if and only if \( \eta^-, \eta^+ \) are fuzzy left (right) ideal of \( M \).

Proof. Let \( \tilde{\eta} \) be an i.v fuzzy left ideal of \( M \). For any \( p, q, r \in M \).

\[
\tilde{\eta}(p - q) = \min \{ \tilde{\eta}(p), \tilde{\eta}(q) \} \\
= \min \{ [\eta^-(p), \eta^+(p)], [\eta^-(q), \eta^+(q)] \} \\
= \min \{ [\eta^-(p), \eta^-(q)], \min \{ [\eta^+(p), \eta^+(q)] \} \}
\]

It follows that \( \eta^-(p) \geq \min \{ \eta^-(p), \eta^-(q) \} \) and

\[
[\eta^-(pq), \eta^+(pq)] = \tilde{\eta}(pq) \\
\geq \min \{ \tilde{\eta}(p), \tilde{\eta}(q) \} \\
= \min \{ [\eta^-(p), \eta^+(p)], [\eta^-(q), \eta^+(q)] \} \\
= \min \{ [\eta^-(p), \eta^-(q)], \min \{ [\eta^+(p), \eta^+(q)] \} \}
\]

Then
gamma (pq) \geq \min \{ \gamma (p), \gamma (q) \}

and

gamma (pq) \geq \min \{ \gamma (p), \gamma (q) \}.

Now

\[
[\eta^-(q - p - q), \eta^+(q - p - q)] = \tilde{\eta}(q - p - q) = \tilde{\eta}(p) = [\eta^-(p), \eta^+(q)].
\]

It follows that \( \eta^-(q + p - q) = \eta^-(p) \) and \( \eta^+(q + p - q) = \eta^+(p) \). Also

\[
[\eta^-(pq), \eta^+(pq)] \geq \tilde{\eta}(pq) \\
\geq \tilde{\eta}(p) \\
= [\eta^-(q), \eta^+(q)]
\]

So

\( \eta^-(pq) \geq \eta^-(q) \) and \( \eta^+(pq) \geq \eta^+(q) \).

Now

\[
[\eta^-(p + r), \eta^+(p + r)] = \tilde{\eta}(p + r) = \tilde{\eta}(r) = [\eta^-(r), \eta^+(r)].
\]

It follows that \( \eta^-(p + r) \geq \eta^-(r) \) and \( \eta^+(p + r) \geq \eta^+(r) \).
Conversely, assume that $\eta^-, \eta^+$ are fuzzy left(right) ideals of $M$. Let $p, q, r \in M$ Now

\[ \tilde{\eta}(p - q) = [\eta^-(p - q), \eta^+(p - q)] \]
\[ \geq \min^r\{[\eta^-(p)\eta^-(q)]\}, \min^l\{[\eta^+(p)\eta^+(q)]\} \]
\[ = \max^r\{[\eta^-(p)\eta^-(q)]\}, \min^l\{[\eta^+(p)\eta^+(q)]\} \]
\[ = \min^l\{[\tilde{\eta}(p), \tilde{\eta}(q)]\} \]
\[ \tilde{\eta}(p + q) = [\eta^-(p + q), \eta^+(p + q)] \]
\[ = [\eta^-(p), \eta^+(p)] \]
\[ = \tilde{\eta}(p) \]
\[ \tilde{\eta}(pq) = [\eta^-(pq), \eta^+(pq)] \]
\[ \\geq \max^r\{[\eta^-(p)\eta^-(q)]\}, \min^l\{[\eta^+(p)\eta^+(q)]\} \]
\[ = \max^r\{[\tilde{\eta}(p), \tilde{\eta}(q)]\} \]
\[ \tilde{\eta}((p + r)\alpha q - p\beta q) = [\eta^-((p + r)\alpha q - p\beta q), \eta^+((p + r)\alpha q - p\beta q)] \]
\[ \geq [\eta^-((p + r)\alpha q - p\beta q), \eta^+((p + r)\alpha q - p\beta q)] \]
\[ = \tilde{\eta}(r) \]

Hence $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of $M$.

**Theorem 3.2.** Let $I$ be a left (right) ideal of $\Gamma$-near-ring $M$. Then for any $\tilde{c} \in D[0, 1]$, there exists an i.v fuzzy left (right) ideal $\tilde{\eta}$ of $M$ such that $\tilde{U}(\tilde{\eta} : \tilde{c}) = I$.

**Proof.** Let $I$ be a left (right)ideal of $M$. Let $\tilde{\eta}$ be an i.v fuzzy subset of $M$ defined by

\[ \tilde{\eta}(p) = \begin{cases} \tilde{c} & \text{if } p \in I \\ \tilde{0} & \text{otherwise.} \end{cases} \]

Then $\tilde{U}(\tilde{\eta}; \tilde{c}) = I$. If $p, q \in I$, then $p, q \in I$ and

\[ \tilde{\eta}(p - q) = \tilde{c} = \min^r\{\tilde{c}, \tilde{c}\} = \min^r\{\tilde{\eta}(p), \tilde{\eta}(q)\}. \]

If $p, q \notin I$, then $\tilde{\eta}(p) = \tilde{0} = \tilde{q}$ and thus

\[ \tilde{\eta}(p - q) = \tilde{0} = \min^l\{\tilde{0}, \tilde{0}\} = \min^l\{\tilde{\eta}(p), \tilde{\eta}(q)\}. \]

Suppose that $p, q \in I$. Then

\[ \tilde{\eta}(p - q) = \tilde{0} = \min^l\{\tilde{c}, \tilde{0}\} = \min^l\{\tilde{\eta}(p), \tilde{\eta}(q)\}. \]
If $p \in I$ and $q \in M$, then $q + p - q \in I$ and so $\tilde{\eta}(q + p - q) = \tilde{c} = \tilde{\eta}(p)$. If $p \notin I$ and $q \in M$, then $\tilde{\eta}(p) = 0$ and thus $\tilde{\eta}(q + p - q) \geq 0 = \tilde{\eta}(p)$. If $q \notin I$ and $p \in M$, then $\eta(pq) \in I$ and so $\tilde{\eta}(pq) = \tilde{c} = \tilde{\eta}(q)$. If $q \notin I$ and $p \in M$, then $\tilde{\eta}(q) = 0$ and thus $\tilde{\eta}(pq) \geq 0 = \tilde{\eta}(q)$.

If $r \in I$ and $p, q \in M$, then $((p + r)aq - p\beta q) \in I$ and so $\tilde{\eta}((p + r)aq - p\beta q) = \tilde{c} = \tilde{\eta}(q)$. If $z \notin I$ and $p, q \in M$, then $\tilde{\eta}(r) = 0$ and $\tilde{\eta}((p + r)aq - p\beta q) \geq 0 = \tilde{\eta}(r)$. Hence $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of the $\Gamma$-near-ring $M$.

**Theorem 3.3.** Let $M$ be a $\Gamma$-near-ring and $\tilde{\eta}$ is an i.v fuzzy left(right) ideal of $M$, then the set $M_{\tilde{\eta}} = \{p \in M| \tilde{\eta}(p) = \tilde{\eta}(0)\}$ is left(right) ideal of $M$.

**Proof.** Let $\tilde{\eta}$ be an i.v fuzzy left ideal of $M$. Let $p, q \in M_{\tilde{\eta}}$. Then

$$\tilde{\eta}(p) = \tilde{\eta}(0), \tilde{\eta}(q) = \tilde{\eta}(0)$$

and

$$\tilde{\eta}(p - q) \geq \min\{\tilde{\eta}(p), \tilde{\eta}(q)\} = \min\{\tilde{\eta}(0), \tilde{\eta}(0)\} = \tilde{\eta}(0).$$

So $\tilde{\eta}(p - q) = \tilde{\eta}(0)$. Thus $p - q \in M_{\tilde{\eta}}$. For every $q \in M$ and $p \in M_{\tilde{\eta}}$ and $\alpha, \beta \in \Gamma$ we have $\tilde{\eta}(q + p - q) \geq \tilde{\eta}(p) = \tilde{\eta}(0)$. Hence $q + p - q \in M_{\tilde{\eta}}$ which shows that $M_{\tilde{\eta}}$ is a normal divisor of $M$ with respect to the addition. Let $p \in M_{\tilde{\eta}}, \alpha \in \Gamma$ and $c, d \in M$. Then $\tilde{\eta}(c\alpha(p + d) - cod) \geq \tilde{\eta}(p) = \tilde{\eta}(0)$ and hence $\tilde{\eta}(c\alpha(p + d) - cod) = \tilde{\eta}(0)$, i.e. $c\alpha(p + d) - cod \in M_{\tilde{\eta}}$. Therefore $M_{\tilde{\eta}}$ is a left ideal of $M$.

**Theorem 3.4.** Let $H$ be a non empty subset of a $\Gamma$-near-ring $M$ and $\tilde{\eta}_H$ be an i.v fuzzy set $M$ defined by

$$\tilde{\eta}_H(p) = \begin{cases} \tilde{s} & \text{if } p \in H \\ \tilde{t} & \text{otherwise} \end{cases}$$

for $p \in M$ and $\tilde{s}, \tilde{t} \in D[0, 1]$ and $\tilde{s} > \tilde{t}$. Then $\tilde{\eta}_H$ is an i.v fuzzy left(right) ideal of $M$ if and only if $H$ is a left ideal of $M$. Also $M_{\tilde{\eta}_H} = H$.

**Proof.** $\tilde{\eta}_H$ be an i.v fuzzy left(right) ideal of $M$ and let $p, q \in H$. Then $\tilde{\eta}_H(p) = \tilde{s} = \tilde{\eta}_H(q)$. Consider

$$\tilde{\eta}_H(p - q) \geq \min^I\{\tilde{\eta}_H(p), \tilde{\eta}_H(q)\}$$

$$= \min^I\{\tilde{s}, \tilde{s}\}$$

$$= \tilde{s}$$

and so $\tilde{\eta}_H(p - q) = \tilde{s}$ which implies that $p - q \in H$. For any $p \in H, \alpha \in \Gamma$ and $c, d \in M$. Then $\tilde{\eta}_H(c\alpha(p + d) - c\beta d) \geq \tilde{\eta}_H(p) = \tilde{s}$ (resp. $\tilde{\eta}_H(p\alpha c) \geq \tilde{\eta}_H(p) = \tilde{s}$) and hence $\tilde{\eta}_H(c\alpha(p + d) - c\beta d) = \tilde{s}$ (resp. $\tilde{\eta}_H(p\alpha c) = \tilde{s}$). This shows that $M$ is a left(right) ideal of $M$.

Conversely assume that $H$ is a left(right) ideal of $M$. Let $p, q \in M$, if at least one of $p$ and $q$ does not belong to $H$ then $\tilde{\eta}_H(p - q) \geq \tilde{t} = \min^I\{\tilde{\eta}_H(p), \tilde{\eta}_H(q)\}$.

If $p, q \in H$, then $p - q \in H$ and so $\tilde{\eta}_H(p - q) = \tilde{s} = \min^I\{\tilde{\eta}_H(p), \tilde{\eta}_H(q)\}$.

Clearly $\tilde{\eta}_H(p + q - p) = \tilde{t} = \tilde{\eta}_H(p)$ for all $p \notin M$ and $q \in M$. This shows that $\tilde{\eta}_H$ is an i.v fuzzy normal divisor w.r.to addition. Now let $p, c, d \in M$ and $\alpha \in \Gamma$. 

If \( p \in H \), then \( co(p + d) - cod \in A \) (resp. \( pac \in A \)) and thus \( \bar{\eta}_H(co(p + d) - c\beta d) = \bar{s} = \bar{\eta}_H(p) \) (resp. \( \bar{\eta}_H(pac) = \bar{s} = \bar{\eta}_H(p) \)).

If \( p \notin H \), then clearly

\[
\bar{\eta}_H(co(p + d) - c\beta d) = \bar{i} \geq \bar{\eta}_H(p)
\]

respective

\[
\bar{\eta}_H(pac) \geq \bar{i} = \bar{\eta}_H(p).
\]

Hence \( \bar{\eta}_H \) is an i.v fuzzy left(right) ideal of \( M \). Also

\[
M_{\bar{\eta}_H} = \{ p \in M | \bar{\eta}_H(p) = \bar{\eta}_H(0) \} = \{ p \in M | \bar{\eta}_H(p) = \bar{s} \} = \{ p \in M | p \in H \} = H
\]

\( \square \)

The following theorem given the relation between fuzzy subsets and crisp sub-sets of a \( \Gamma \)-near-ring.

**Theorem 3.5.** Let \( H \) be a subset of a \( \Gamma \)-near-ring \( M \). The characteristic function \( \bar{\eta}_H : M \to D[0, 1] \) is an i.v fuzzy left(right) ideal of \( M \) if and only if \( H \) is a left(right) ideal of \( M \).

**Proof.** Let \( H \) be a left ideal of \( M \), using the theorem 3.4. Conversely, assume that \( \bar{\eta}_H \) is an i.v fuzzy left(right) ideal of \( M \). Let \( p, q \in H \), then \( \bar{\eta}_H(p) = \bar{i} = \bar{\eta}_H(q) \) and so

\[
\bar{\eta}_H(p - q) \geq \min^i \{ \bar{\eta}_H(p), \bar{\eta}_H(q) \} = \min^i \{ \bar{i}, \bar{i} \} = \bar{i}
\]

so \( \bar{\eta}_H(p - q) = \bar{i} \). Therefore \( p - q \in H \). Hence \( H \) is an additive subgroup of \( M \).

Let \( p \in H \) and \( q \in M \), then \( \bar{\eta}_H(q + p - q) = \bar{\eta}_H(q) = \bar{i} \). Hence \( \bar{\eta}_H(q + p - q) = \bar{i} \) this implies that \( q + p - q \in H \). Thus \( H \) is a normal subgroup of \( M \). Let \( p \in M \) and \( q \in H \), then \( \bar{\eta}_H(paq) \geq \bar{\eta}_H(q) = \bar{i} \). Hence \( \bar{\eta}_H(paq) = \bar{i} \) this implies \( paq \in H \) and \( H \) is a left ideal of \( M \). Let \( p, q \in M \) and \( r \in H \). Then \( \bar{\eta}_H((p + r)aq - p\beta q) = \bar{\eta}_H(r) = \bar{i} \). Hence \( \bar{\eta}_H((p + r)aq - p\beta q) \in H \). Hence \( H \) is a left(right) ideal of \( M \).

\( \square \)

**Theorem 3.6.** Let \( \{ \bar{\eta}_i | i \in \Omega \} \) be family of i.v fuzzy ideals of a \( \Gamma \)-near-ring \( M \), then \( \bigcap_{i \in \Omega} \bar{\eta}_i \) is also an i.v fuzzy ideal of \( M \), where \( \Omega \) is any index set.

**Proof.** Let \( \{ \bar{\eta}_i | i \in \Omega \} \) be a family of i.v fuzzy ideals of \( M \). Let \( p, q, r \in M, \alpha, \beta \in \Gamma \) and \( \bar{\eta} = \bigcap_{i \in \Omega} \bar{\eta}_i \). Then,

\[
\bar{\eta}(p) = \bigcap_{i \in \Omega} \bar{\eta}_i(p) = \inf_{i \in \Omega} \bar{\eta}_i(p) = \inf_{i \in \Omega} \bar{\eta}_i(p).
\]
Now
\[ \tilde{h}(p - q) = \inf_{i \in \Omega} \tilde{h}_i(p - q) \]
\[ \geq \inf_{i \in \Omega} \min^i \{\tilde{h}_i(p), \tilde{h}_i(q)\} \]
\[ = \min^i \left\{ \inf_{i \in \Omega} \tilde{h}_i(p), \inf_{i \in \Omega} \tilde{h}_i(q) \right\} \]
\[ = \min^i \left\{ \bigcap_{i \in \Omega} \tilde{h}_i(p), \bigcap_{i \in \Omega} \tilde{h}_i(q) \right\} \]
\[ = \min^i \{\tilde{h}(p), \tilde{h}(q)\} \cdot \]
\[ \tilde{h}(poq) = \inf^i \{\tilde{h}_i(poq) : i \in \Omega\} \]
\[ \geq \inf^i \{\min^i \{\tilde{h}_i(p), \tilde{h}_i(q)\} : i \in \Omega\} \]
\[ = \min^i \left\{ \inf^i \{\tilde{h}_i(p) : i \in \Omega\}, \inf^i \{\tilde{h}_i(q) : i \in \Omega\} \right\} \]
\[ = \min^i \left\{ \bigcap_{i \in \Omega} \tilde{h}_i(p), \bigcap_{i \in \Omega} \tilde{h}_i(q) \right\} \]
\[ \bigcap_{i \in \Omega} \tilde{h}(q + p - q) = \inf^i \{\tilde{h}_i(q + p - q) : i \in \Omega\} \]
\[ = \inf^i \{\tilde{h}_i(p) : i \in \Omega\} \]
\[ = \left\{ \bigcap_{i \in \Omega} \tilde{h}_i(p) \right\} \cdot \]
\[ \bigcap_{i \in \Omega} \tilde{h}(poq) = \inf^i \{\tilde{h}_i(poq) : i \in \Omega\} \]
\[ \geq \inf^i \{\tilde{h}_i(q) : i \in \Omega\} \]
\[ = \left\{ \bigcap_{i \in \Omega} \tilde{h}_i(q) \right\} \]
\[ \bigcap_{i \in \Omega} \tilde{h}(p + r)aq - p\beta q = \inf^i \{\tilde{h}_i(p + r)aq - p\beta q) : i \in \Omega\} \]
\[ \geq \inf^i \{\tilde{h}_i(r) : i \in \Omega\} \]
\[ = \left\{ \bigcup_{i \in \Omega} \tilde{h}_i(r) \right\} \]

Therefore \( \bigcap_{i \in \Omega} \tilde{h}_i \) is an i.v fuzzy ideal of \( M \).

**Theorem 3.7.** Let \( \{\tilde{h}_i\}_{i \in \Omega} \) be family of i.v fuzzy ideals of a \( \Gamma \)-near-ring \( M \), then \( \bigcup_{i \in \Omega} \tilde{h}_i \) is also an i.v fuzzy ideal of \( M \), where \( \Omega \) is any index set.
Proof. Let \( \tilde{\eta}_i | i \in \Omega \) be a family of i.v fuzzy ideals of \( M \). Let \( p, q, r \in M, \alpha, \beta \in \Gamma \) and \( \tilde{\eta} = \bigcup_{i \in \Omega} \tilde{\eta}_i \). Then,

\[
\tilde{\eta}(p) = \bigcup_{i \in \Omega} \tilde{\eta}_i(p) = \left( \sup_{i \in \Omega} \tilde{\eta}_i \right)(p) = \sup_{i \in \Omega} \tilde{\eta}_i(p).
\]

Now

\[
\tilde{\eta}(p - q) = \sup_{i \in \Omega} \tilde{\eta}_i(p - q) \\
\geq \sup_{i \in \Omega} \max \{ \tilde{\eta}_i(p), \tilde{\eta}_i(q) \} \\
= \max \{ \sup_{i \in \Omega} \tilde{\eta}_i(p), \sup_{i \in \Omega} \tilde{\eta}_i(q) \} \\
= \max \left\{ \bigcup_{i \in \Omega} \tilde{\eta}_i(p), \bigcup_{i \in \Omega} \tilde{\eta}_i(q) \right\}
\]

\[
\tilde{\eta}(pq) = \sup \{ \tilde{\eta}_i(pq) : i \in \Omega \} \\
\geq \sup \{ \max \{ \tilde{\eta}_i(p), \tilde{\eta}_i(q) \} : i \in \Omega \} \\
= \max \{ \sup \{ \tilde{\eta}_i(p) : i \in \Omega \}, \sup \{ \tilde{\eta}_i(q) : i \in \Omega \} \}
\]

\[
\bigcup_{i \in \Omega} \tilde{\eta}(q + p - q) = \sup \{ \tilde{\eta}_i(q + p - q) : i \in \Omega \} \\
= \sup \{ \tilde{\eta}_i(p) : i \in \Omega \}
\]

\[
\bigcup_{i \in \Omega} \tilde{\eta}(p \alpha q) = \sup \{ \tilde{\eta}_i(p \alpha q) : i \in \Omega \} \\
\geq \sup \{ \tilde{\eta}_i(q) : i \in \Omega \} \\
= \left\{ \bigcup_{i \in \Omega} \tilde{\eta}_i(q) \right\}
\]

\[
\bigcup_{i \in \Omega} \tilde{\eta}(p + r) \alpha q = \sup \{ \tilde{\eta}_i(p + r \alpha q) : i \in \Omega \} \\
\geq \sup \{ \tilde{\eta}_i(r) : i \in \Omega \} \\
= \left\{ \bigcup_{i \in \Omega} \tilde{\eta}_i(r) \right\}
\]

Therefore \( \bigcup_{i \in \Omega} \tilde{\eta}_i \) is an i.v fuzzy ideal of \( M \). \( \square \)
Theorem 3.8. Let \( \tilde{\eta} \) be an i.v fuzzy subset of \( M \). \( \tilde{\eta} \) is an i.v fuzzy left (right) ideal of \( M \) if and only if \( \tilde{U}(\tilde{\eta} : [t_1, t_2]) \) is left (right) ideal of \( M \), for all \([t_1, t_2] \in D[0, 1]\).

Proof. Assume that \( \tilde{\eta} \) is an i.v fuzzy left(right) ideal of \( M \).

Let \([t_1, t_2] \in D[0, 1] \) such that \( p, q \in \tilde{U}(\tilde{\eta} : [t_1, t_2]) \).

Then \( \tilde{\eta}(p - q) \geq \min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\} \geq \min\{[t_1, t_2], [t_1, t_2] \} = [t_1, t_2] \). Thus \( p - q \in \tilde{U}(\tilde{\eta} : [t_1, t_2]) \). Let \( p \in \tilde{U}(\tilde{\eta} : [t_1, t_2]) \) and \( q \in M \) and \( \alpha, \beta \in \Gamma \). We have \( \tilde{\eta}(q + p - q) = \tilde{\eta}(p) \geq [t_1, t_2] \). Therefore \( q + p - q \in \tilde{U}(\tilde{\eta} : [t_1, t_2]) \). Hence \( \tilde{U}(\tilde{\eta} : [t_1, t_2]) \) is a normal subgroup of \( M \). Let \( q \in \tilde{U}(\tilde{\eta} : [t_1, t_2]) \) and \( p \in M \), thus \( \tilde{\eta}(p) \geq [t_1, t_2] \). Hence \( \tilde{U}(\tilde{\eta} : [t_1, t_2]) \) is a left (right) ideal of a \( \Gamma \)-near-ring of \( M \).

Conversely, assume \( \tilde{U}(\tilde{\eta} : [t_1, t_2]) \) is a left(right) ideal of \( M \), for all \([t_1, t_2] \in D[0, 1] \). Let \( p, q \in M \). Suppose \( \tilde{\eta}(p) \geq \min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\} \).

Choose \( \tilde{c} = [c_1, c_2] \in D[0, 1] \) such that \( \tilde{\eta}(p - q) < [c_1, c_2] < \min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\} \). This implies that \( \tilde{\eta}(p) \geq [c_1, c_2] \) and \( \tilde{\eta}(q) > [c_1, c_2] \), then we have \( p, q \in \tilde{U}(\tilde{\eta} : [t_1, t_2]) \), and since \( \tilde{U}(\tilde{\eta} : [c_1, c_2]) \) is a left(right) ideal of \( M \) then \( p - q \in \tilde{U}(\tilde{\eta} : [c_1, c_2]) \). Hence \( \tilde{\eta}(p - q) \geq [c_1, c_2] \) is a contradiction, this implies that \( \tilde{\eta}(p - q) \geq \min^i\{\tilde{\eta}(p), \tilde{\eta}(q)\} \).

Suppose \( \tilde{\eta}(q + p - q) < \tilde{p} \), choose an interval \( \tilde{c} = [c_1, c_2] \in D[0, 1] \) such that \( \tilde{\eta}(q + p - q) < [c_1, c_2] \tilde{\eta}(p) \). This implies that \( \tilde{\eta}(p) > [c_1, c_2] \) then we have \( p \in \tilde{U}(\tilde{\eta} : [c_1, c_2]) \), and since \( \tilde{U}(\tilde{\eta} : [c_1, c_2]) \) is a normal subgroup of \( (M, +) \) then \( q + p - q \in \tilde{U}(\tilde{\eta} : [c_1, c_2]) \). Hence \( q + p - q \geq [c_1, c_2] \) is a contradiction. Hence \( \tilde{\eta}(q + p - q) \geq \tilde{\eta}(p) \). Similarly \( \tilde{\eta}(q + p - q) \leq \tilde{\eta}(p) \). Hence \( \tilde{\eta}(p) \geq q + p - q = \tilde{\eta}(p) \).

Suppose \( \tilde{\eta}(p) < \tilde{\eta}(p) \), choose an interval \( \tilde{c} = [c_1, c_2] \in D[0, 1] \) such that \( \tilde{\eta}(p) < [c_1, c_2] \tilde{\eta}(q) \). This implies that \( \tilde{\eta}(q) > [c_1, c_2] \) then we have \( q \in \tilde{U}(\tilde{\eta} : [c_1, c_2]) \), and since \( \tilde{U}(\tilde{\eta} : [c_1, c_2]) \) is a left(right) ideal of \( M \) then \( \tilde{\eta}(p) \geq \tilde{\eta}(p) \).

Suppose \( \tilde{\eta}((p + r)aq - p\beta q) < \tilde{r} \), choose an interval \( \tilde{c} = [c_1, c_2] \in D[0, 1] \) such that \( \tilde{\eta}((p + r)aq - p\beta q) < [c_1, c_2] \tilde{\eta}(r) \). This implies that \( \tilde{\eta}(r) > [c_1, c_2] \) we have \( q \in \tilde{U}(\tilde{\eta} : [c_1, c_2]) \), and since \( \tilde{U}(\tilde{\eta} : [c_1, c_2]) \) is a left(right) ideal of \( M \) it follows that \( ((p + r)aq - p\beta q) \in \tilde{U}(\tilde{\eta} : [c_1, c_2]) \). Hence \( ((p + r)aq - p\beta q) \geq [c_1, c_2] \) which is a contradiction. Hence \( \tilde{\eta}((p + r)aq - p\beta q) \geq \tilde{\eta}(r) \). Thus \( \tilde{\eta} \) is an i.v fuzzy left(right) ideal of \( M \).

□

4. Homomorphism of interval valued fuzzy ideals of \( \Gamma \)-near-rings

In this section, we characterize i.v fuzzy ideals of \( \Gamma \)-near-rings using homomorphism.

Definition 4.1. \((\tilde{\eta}, \tilde{\delta})\) Let \( f \) be a mapping from a set \( M \) to a set \( S \). Let \( \tilde{\eta} \) and \( \tilde{\delta} \) be i.v fuzzy subsets of \( M \) and \( S \) respectively. Then \( f(\tilde{\eta}) \), the image of \( \tilde{\eta} \) under \( f \) is an i.v fuzzy subset of \( S \) defined by

\[
    f(\tilde{\eta})(y) = \begin{cases} 
    \sup_{x \in f^{-1}(y)} \tilde{\eta}(x) & \text{if } f^{-1}(y) \neq \emptyset, \\
    0 & \text{otherwise}
    \end{cases}
\]

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and the pre-image of \( \tilde{\eta} \) under \( f \) is an i.v fuzzy subset of \( M \) defined by \( f^{-1}(\tilde{\delta}(x)) = \tilde{\delta}(f(x)) \), for all \( x \in M \) and \( f^{-1}(y) = \{ x \in M | f(x) = y \} \).

**Definition 4.2.** ([5]) Let \( M \) and \( S \) be \( \Gamma \)-near-rings. A map \( \theta : M \rightarrow S \) is called a (\( \Gamma \)-near-ring) homomorphism if \( \theta(x + y) = \theta(x) + \theta(y) \) and \( \theta(x\alpha y) = \theta(x)\alpha\theta(y) \) for all \( x, y \in M \) and \( \alpha \in \Gamma \).

**Theorem 4.1.** Let \( f : M_1 \rightarrow M_2 \) be a homomorphism between \( \Gamma \)-near-rings \( M_1 \) and \( M_2 \). If \( \tilde{\delta} \) is an i.v fuzzy ideal of \( M_2 \), then \( f^{-1}(\tilde{\delta}) \) is an i.v fuzzy left (right) ideal of \( M_1 \).

**Proof.** Let \( \tilde{\delta} \) be an i.v fuzzy ideal of \( M_2 \). Let \( p, q, r \in M_1 \) and \( \alpha, \beta \in \Gamma \). Then

\[
\begin{align*}
\tilde{\delta}(f(p) - f(q)) &= \tilde{\delta}(p - q) \\
&= \min^i \{ \tilde{\delta}(f(p)), \tilde{\delta}(f(q)) \} \\
&= \min^i \{ f^{-1}(\tilde{\delta}(p)), f^{-1}(\tilde{\delta}(q)) \}.
\end{align*}
\]

\[
\begin{align*}
\tilde{\delta}(f(poq)) &= \tilde{\delta}(f(p) \cdot f(q)) \\
&= \min^i \{ \tilde{\delta}(f(p)), \tilde{\delta}(f(q)) \} \\
&= \min^i \{ f^{-1}(\tilde{\delta}(p)), f^{-1}(\tilde{\delta}(q)) \}.
\end{align*}
\]

\[
\begin{align*}
\tilde{\delta}(f(q + p - q)) &= \tilde{\delta}(f(q) + f(p) - f(q)) \\
&= \tilde{\delta}(f(p)) \\
&= f^{-1}(\tilde{\delta}(p)).
\end{align*}
\]

\[
\begin{align*}
\tilde{\delta}(f(poq)) &= \tilde{\delta}(f(p) \cdot f(q)) \\
&= \tilde{\delta}(f(p) \cdot f(q)) \\
&= f^{-1}(\tilde{\delta}(q)).
\end{align*}
\]

\[
\begin{align*}
\tilde{\delta}(f((p + r)aq - p\beta q)) &= \tilde{\delta}(f((p + r)aq - p\beta q)) \\
&= \tilde{\delta}(f((p + r) \cdot f(q) - f(p)\beta f(q)) \\
&= f^{-1}(\tilde{\delta}(r)).
\end{align*}
\]

Therefore \( f^{-1}(\tilde{\delta}) \) is an i.v fuzzy left(right) ideal of \( M_1 \). \( \square \)

**Theorem 4.2.** Let \( f : M_1 \rightarrow M_2 \) be an onto homomorphism of \( \Gamma \)-near-rings \( M_1 \) and \( M_2 \). Let \( \tilde{\delta} \) be an i.v fuzzy subset of \( M_2 \). If \( f^{-1}(\tilde{\delta}) \) is an i.v fuzzy left (right) ideal of \( M_1 \), then \( \tilde{\delta} \) is an i.v fuzzy left(right) ideal of \( M_2 \).
Proof. Let $p, q, r \in M_2$. Then $f(a) = p, f(b) = q$ and $f(c) = r$ for some $a, b, c \in M_1$ and $\alpha, \beta \in \Gamma$. It follows that

\[
\tilde{\delta}(p - q) = \tilde{\delta}(f(a) - f(b)) \\
= \tilde{\delta}(f(a - b)) \\
= f^{-1}(\tilde{\delta})(a - b) \\
\geq \min^t\{f^{-1}(\tilde{\delta})(a), f^{-1}(\tilde{\delta})(b)\} \\
= \min^t\{\tilde{\delta}(f(a)), \tilde{\delta}(f(b))\} \\
= \min^t\{\tilde{\delta}(p), \tilde{\delta}(q)\}.
\]

\[
\tilde{\delta}(paq) = \tilde{\delta}(f(a)f(b)) \\
= \tilde{\delta}(f(aab)) \\
= f^{-1}(\tilde{\delta})(aab) \\
\geq \min^t\{f^{-1}(\tilde{\delta})(a), f^{-1}(\tilde{\delta})(b)\} \\
= \min^t\{\tilde{\delta}(f(a)), \tilde{\delta}(f(b))\} \\
= \min^t\{\tilde{\delta}(p), \tilde{\delta}(q)\}.
\]

\[
\tilde{\delta}(q + p - q) = \tilde{\delta}(f(b) + f(a) - f(b)) \\
= \tilde{\delta}(f(b + a - b)) \\
= f^{-1}(\tilde{\delta})(b + a - b) \\
= f^{-1}\tilde{\delta}(a) \\
= \tilde{\delta}(f(a)) \\
= \tilde{\delta}(p).
\]

\[
\tilde{\delta}(paq) = \tilde{\delta}(f(a)f(b)) \\
= \tilde{\delta}(f(aab)) \\
= f^{-1}(\tilde{\delta})(aab) \\
\geq f^{-1}(\tilde{\delta})(b) \\
= \tilde{\delta}(f(b)) \\
= \tilde{\delta}(q).
\]

\[
\tilde{\delta}(p + r)aq - p\beta q = \tilde{\delta}(f(a) + f(c))\alpha f(b) - f(a)\beta f(b) \\
= \tilde{\delta}(f(a + c)ab - a\beta b) \\
= f^{-1}(\tilde{\delta})(a + c)ab - a\beta b) \\
\geq f^{-1}(\tilde{\delta})(c) \\
= \tilde{\delta}(f(c)) = \tilde{\delta}(f(r)).
\]
Hence $\tilde{\delta}$ is an i.v fuzzy left (right) ideal of $M_1$.

**Theorem 4.3.** Let $f: M_1 \rightarrow M_2$ be an onto $\Gamma$-near-ring homomorphism. If $\tilde{\eta}$ is an i.v fuzzy left (right) ideal of $M_1$, then $f(\tilde{\eta})$ is an i.v fuzzy left (right) ideal of $M_2$.

**Proof.** Let $\tilde{\eta}$ be an i.v fuzzy ideal of $M_1$. Since $f(\tilde{\eta})(p') = \sup f(p(\tilde{\eta}q))$, for $p' \in M_2$ and hence $f(\tilde{\eta})$ is nonempty. Let $p', q' \in M_2$ and $\alpha, \beta \in \Gamma$. Then we have \{p|x \in f^{-1}(p') and q \in f^{-1}(q')\} and \{p|p \in f^{-1}(p') \text{ and } q \in f^{-1}(q')\}.

\[
f(\tilde{\eta})(p' - q') = \sup f(p(\tilde{\eta}q)) \geq \sup f(p(\tilde{\eta}q)) = \sup f(p(\tilde{\eta}q)) = \sup f(p(\tilde{\eta}q)) \geq \min \{f(\tilde{\eta})(p'), f(\tilde{\eta})(q')\}.
\]

\[
f(\tilde{\eta})(q' + p' - q') = \sup f(p(\tilde{\eta}q)) \geq \sup f(p(\tilde{\eta}q)) = \sup f(p(\tilde{\eta}q)) = \sup f(p(\tilde{\eta}q)) = \sup f(p(\tilde{\eta}q)) = f(\tilde{\eta})(q').
\]

Therefore $f(\tilde{\eta})$ is an i.v fuzzy left (right) ideal of $M_2$.

**5. Anti-homomorphism of interval valued fuzzy ideals of $\Gamma$-near-rings**

In this section, we characterize i.v fuzzy ideals of $\Gamma$-near-rings using anti-homomorphism.
Definition 5.1. ([7]) Let $M$ and $S$ be $\Gamma$-near-rings. A map $\theta : M \rightarrow S$ is called a ($\Gamma$-near-ring) anti-homomorphism if $\theta(x + y) = \theta(y) + \theta(x)$ and $\theta(x\alpha y) = \theta(y)\alpha \theta(x)$ for all $x, y \in M$ and $\alpha \in \Gamma$.

Theorem 5.1. Let $f : M_1 \rightarrow M_2$ be an anti-homomorphism between $\Gamma$-near-rings $M_1$ and $M_2$. If $\tilde{\delta}$ is an i.v fuzzy ideal of $M_2$, then $f^{-1}(\tilde{\delta})$ is an i.v fuzzy left (right)ideal of $M_1$.

Theorem 5.2. Let $f : M_1 \rightarrow M_2$ be an onto anti-homomorphism of $\Gamma$-near-rings $M_1$ and $M_2$. Let $\delta$ be an i.v fuzzy subset of $M_2$. If $f^{-1}(\delta)$ is an i.v fuzzy left (right)ideal of $M_1$, then $\delta$ is an i.v fuzzy left (right) ideal of $M_2$.

Theorem 5.3. Let $f : M_1 \rightarrow M_2$ be an onto $\Gamma$-near-ring anti-homomorphism. If $\tilde{\eta}$ is an i.v fuzzy left (right)ideal of $M_1$, then $f(\tilde{\eta})$ is an i.v fuzzy left (right)ideal of $M_2$.

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