Performance Evaluation of Adaptive Filters for Noise Cancellation

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Abstract—VLSI implementation of adaptive noise canceller based on least mean square algorithm is implemented. First, the adaptive parameters are obtained by simulating noise canceller on MATLAB. Simulink model of adaptive noise canceller was developed and the noise is suppressed to an extreme extent in recovering the original signal. The objective of adaptive interference cancellation is to obtain an estimation of the interfering signal and to subtract it from the corrupted signal and hence obsess a noise free signal. For this purpose, the filter uses an adaptive algorithm to change the value of the filter coefficients, so that it acquires a better approximation of the signal after each iteration. The LMS, and its variant NLMS, RLS are the adaptive algorithms widely in urge. This paper presents a comparative analysis of the LMS(Least Mean Square), NLMS(Normalized LMS) and RLS(Recursive Least Square) Filters in case of noise cancellation and the effects of parameters- filter length and step size have been analyzed and the relation between the filters has been established. Finally, the performances of the algorithms in different cases have been compared.

Keywords—Noise cancellation, Adaptive filters, Adaptive Algorithms, LMS Filter, NLMS Filter, RLS Filter, Noise Cancellation

1 Introduction

Adaptive filters, as a segment of digital signal systems have been widely urged in communication industry as well as in applications such as adaptive noise cancellation, adaptive beam forming, and channel equalization. In general FIR structure has been used more magnificently than IIR structure in adaptive filters. The output FIR filters is the convolution of its input with its coefficients which have constant values. However, when the adaptive FIR filter was contrived, this required appropriate algorithm to update the filter’s coefficients. The algorithm urged to update the filter coefficient is the Least Mean Square (LMS) algorithm which is known for its simplification, low computational complexity, and better performance in different running environments. Recursive Least Squares algorithm is abstention in convergence than the LMS but its very complex to implement. Hence detaining system performance in terms of speed and FPGA area used.

1.1 Adaptive Filters

A filter is a device that maps its input signal to another output signal facilitating the extraction of the desired information contained in the input signal. Time-invariant filters have fixed internal parameters and structure.
An adaptive filter is time-varying since their parameters are continually changing in order to meet certain performance requirements. The general setup of an adaptive filtering environment is shown in Figure 1.1, where \( n \) is the iteration index, \( x(n) \) denotes the input signal, \( y(n) \) is the adaptive filter’s output signal, and \( d(n) \) defines the reference or desired signal. The error signal \( e(n) \) is the difference between the desired \( d(n) \) and filter output \( y(n) \). The error signal is used as a feedback to the adaptation algorithm in order to determine the appropriate updating of the filter’s coefficients.

### 1.2 Adaptive Noise Cancellation

Methods of adaptive noise cancellation were proposed by Widrow and Glover in 1975. The primary aim of an adaptive noise cancellation algorithm is to allow the noisy signal through a filter which suppresses the noise without disturbing the desired signal.

The basic block diagram is given in Fig. 1.2. An adaptive filter automatically adjusts its own impulse response through an LMS algorithm. Adaptive Noise Canceller (ANC) has two inputs – primary and reference. The primary input receives a signal from the signal source that is corrupted by the presence of noise, uncorrelated with the signal. The concept of adaptive noise cancelling, an alternative method of estimating a signal corrupted by additive noise or interference is to pass it through a filter. The mode uses a “primary” input which is epithetical an corrupted signal (source + noise) and a “reference” input containing noise correlated in some anonymous way with the primary noise.

### 1.3 Flow chart for adaptive noise canceller

The Flowchart for Adaptive noise canceller is shown in figure 1.3. Adaptive noise canceller works on principle of correlation cancellation. One is desired input \( D_{in} \) and another reference correlated noisy input \( X_{in} \). The desired input signal which is corrupted by noise can be obtained by appealing main input signal \( X(k) \) and noisy input signal \( N(k) \) to adder. This adder adds input signal and
noisy signal and endow corrupted signal. It further supplied to FIR filter as an input signal. The FIR filter has its endemic impulse response.

FIR filter convolutes input signal with its intrinsic impulse response. Then FIR filter consign a desired output signal. This is the first iteration which cannot solve the problem completely. And then forward to adaptive LMS algorithm as a input signal.

Figure 1.3. Flow chart for Adaptive noise canceller

Another reference noisy correlated input is also given to adaptive LMS algorithm. LMS algorithm continuously contemplate input signal with reference signal for adjusting the filter coefficients. After every iteration, the filter coefficients get upgraded. Its output is further applied to FIR filter again which will again perform filtering to give error output signal. After the certain number of iterations, we will get the required noise free output. The adaptive filter digress from a fixed filter in that it automatically amend its own impulse response. Reconcile is accomplished through an algorithm that responds to an error signal.
2 LMS Algorithm

The Least Mean Square or LMS algorithm is a stochastic gradient algorithm that iterates each tap weight in the filter in the direction of the gradient of squared amplitude of an error signal with respect to that tap weight.

The LMS is an approximation of the steepest descent algorithm, which uses an Instantaneous estimate of the gradient vector. The assess of the gradient is based on sample values of the tap input vector and an error signal. The algorithm iterates over each tap weight in the filter, moving it in the direction of the approximated gradient. The idea behind LMS filter is to use the method of steepest descent to find a coefficient vector which minimizes a cost function.

Least mean square (LMS) algorithm is stochastic gradient algorithm developed by Widrow and Hoff in 1959 and widely used in adaptive signal processing applications.

From Fig. 1.2, the output of the filter \( y(n) \) is given by

\[
Y(n) = w^T(n)x(n)
\]

(2.1)

Where \( w(n) \) is weight vector and the error signal is given by

\[
e(n) = d(n) - y(n)
\]

(2.2)

Substituting (2.1) in (2.2) yields

\[
e(n) = d(n) - w^T(n)x(n)
\]

(2.3)

According to the mean square error criterion Optimum filter parameters \( w_{opt} \) should make \( \xi = E\{e^2(n)\} \) as minimum.

The mean square error can be expressed as

\[
\xi = E\{d^2(n)\} - 2w^T r_{xd} + w^T R_{xx}w
\]

(2.4)

Where \( r_{xd} = E\{x(n)d(n)\} \), is cross-correlation vector and \( R_{xx} = E\{x(n)x^T(n)\} \), is autocorrelation matrix. It can be seen that the mean square error \( \xi \) is a quadratic function of \( W \), and the matrix \( R_{xx} \) is positive definite or positive semi definite, so it must have a minimum value. Due to this gradient of \( W \) is zero, the minimum when \( w_{opt} \) meet \( \nabla \xi = 0 \) and when \( R_{xx} \) get a unique solution \( w_{opt} = R_{xx}^{-1}r_{xd} \) is considered.

In LMS algorithm the gradient of the instantaneous squared error can be used instead of the gradient of the mean square error. To update the weights for each iteration of the adaptive filter a step size parameter \( \mu \) is introduced to control speed of convergence of the algorithm.

\[
w(n+1) = w(n) + 2\mu e(n)x(n)
\]

(2.5)

The step size parameter affects the stability, convergence speed and steady state error so to reduce steady state error small step size is used but it decreases the speed of the convergence of the algorithm. For better speed of convergence the step size value is increased but this affects the filter stability.

2.1 Simulink model for LMS filter
Environmental noise polluted sinusoidal signal is extracted. Noise signal is modeled as Gaussian noise. The two signals were added and subsequently fed into the simulation of LMS adaptive filter. The test block diagram of the noise canceller in Simulink is shown in Fig. 2.1. System inputs are Sinusoidal signal and Gaussian noise signal. The system outputs are the sinusoidal signal after filtering. By using manual switch LMS Adaptive filter Step size parameter is changed between high and low constant values. Gaussian noise generator is used for generating the noise signals. The step size value to the LMS filter was 0.001.

Select the Adapt port check box to create an Adapt port on the block. When the input to this port is nonzero, the block continuously updates the filter weights. When the input to this port is zero, the filter weights remain constant.

![Simulink model for LMS](image)

**Figure 2.1. Simulink model for LMS**

If the Reset port is enabled and a reset event occurs, the block resets the filter weights to their initial values. The LMS filter length used here is 32. If the step size parameter is at higher constant value the response is fast but showing less accurate and if step size factor is at lower constant value the response may be slow but it is showing more exact performance.

### 2.2 Experimental Results

Figure 2.2 shows the scope output of the LMS Filter. If the step size parameter is at higher constant value, the response is fast but showing less accurate results and if step size factor is at lower constant value the response may be slow but it is showing more exact performance.
In many adaptive filter algorithms Normalized least mean square algorithm (NLMS) is also derived from conventional LMS algorithm. The normalized LMS (NLMS), algorithm utilizes a variable convergence factor that minimizes the instantaneous error. Such a convergence factor usually reduces the convergence time but increases the misadjustment. In order to improve the convergence rate the updating equation of the conventional LMS algorithm can be employed variable convergence factor $\mu$. The value of $\mu \sigma^2 x$ directly affects the convergence rate and stability of the LMS adaptive filter. The NLMS algorithm is an effective approach to overcome this dependence, particularly when the variation of input signal power is large, by normalizing the update step-size with an estimate of the input signal variance, $\sigma^2 x(n)$. In practice, the correction term applied to the estimated tap-weight vector $w(n)$ at the $n$-th iteration is ‘normalized’ with respect to the squared Euclidean norm of the tap input $x(n)$ at the $(n-1)$-th iteration,

$$w(n + 1) = w(n) + \frac{\mu}{\|x(n)\|^2} e(n)x(n)$$  \hspace{1cm} (3.1)

Apparently, the convergence rate of the NLMS algorithm is directly proportional to the NLMS adaptation constant $\mu$, i.e. the NLMS algorithm is independent of the input signal power. Theoretically, by choosing $\mu$ so as to optimize the convergence rates of the algorithms, the NLMS algorithm converges more quickly than the LMS algorithm.

The variation of signal level at the filter input and selecting a normalized correction term, we get a stable as well as a potentially faster converging adaptation algorithm for both uncorrelated and correlated input signal. It has also been stated that the NLMS is convergent in the mean square if the adaptation constant $\mu$ (note that it is no longer called the step size) satisfies the following condition:

$$0 < \mu < 2$$

Despite this particular edge that NLMS exhibits, it has slight problem of its own. Consider the case when the input vector $x(n)$ is small. However, this can be easily overcome pending a positive constant to the denominator such that

$$w(n + 1) = w(n) + \frac{\mu}{\varepsilon + \|x(n)\|^2} e(n)x(n)$$  \hspace{1cm} (3.2)

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Figure 2.2 Scope output of LMS filter

3 NLMS Algorithm
Where the denominator is the normalization factor. With this, we obtain a more robust and reliable implementation of the NLMS algorithm.

4 Recursive Least Square (RLS) Algorithm

The Recursive Least Squares (RLS) filter is a better filter than the LMS filter, but it is not used as often as it could be because it requires more computational resources. The LMS filter requires $2N+1$ operation per filter update, whereas the RLS filter requires $2.5N^2 + 4N$. It has been successfully used in system identification problems and in time series analysis where its real-time performance is not an issue.

The Recursive least squares (RLS) adaptive filter is an algorithm which recursively finds the filter coefficients that minimize a weighted linear least squares cost function relating to the input signals. This is in contrast to other algorithms such as the least mean squares (LMS) that aim to reduce the mean square error. In the derivation of the RLS, the input signals are considered deterministic, while for the LMS and similar algorithm they are considered stochastic. Compared to most of its competitors, the RLS exhibits extremely fast convergence. However, this benefit comes at the cost of high computational complexity, and potentially poor tracking performance when the filter to be estimated changes.

In general, the RLS can be used to solve any problem that can be solved by adaptive filters. For example, suppose that a signal $d(n)$ is transmitted over an echoic, noisy channel that causes it to be received as

$$x(n) = \sum_{k=0}^{q} b(n-k)d(n-k) + v(n+1)$$  \hspace{1cm} (4.1)

Where $v(n)$ represents additive noise. To recover the desired signal $d(n)$ by use of a $p$-tap FIR filter,

$$d(n) = \sum_{k=0}^{p-1} W_n(k)x(n-k) = W_n^T x(n)$$  \hspace{1cm} (4.2)

Where

$$X_n = [x(n) \hspace{0.5cm} x(n-1) \hspace{0.5cm} ... \hspace{0.5cm} x(n-p+1)]^T$$  \hspace{1cm} (4.3)

is the vector containing the $p$ most recent samples of $x(n)$.

To estimate the parameters of the filter $W$, and at each time $n$ we refer to the new least squares estimate by $W_n$. As time evolves, to avoid completely redoing the least squares algorithm to find the new estimate from $w_{n+1}$, in terms of $W_n$.

5 Performance Evaluation of LMS, NLMS and RLS

Table 5.1 Comparison of LMS and RLS with mean variance

<table>
<thead>
<tr>
<th>Parameter</th>
<th>LMS</th>
<th>RLS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0057</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Algorithm</th>
<th>MSE</th>
<th>%noise reduction</th>
<th>Complexity</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS</td>
<td>$2.5 \times 10^{-2}$</td>
<td>98.62%</td>
<td>$2N+1$</td>
<td>Highly stable</td>
</tr>
<tr>
<td>NLMS</td>
<td>$2.1 \times 10^{-2}$</td>
<td>93.85%</td>
<td>$5N+1$</td>
<td>Stable</td>
</tr>
<tr>
<td>RLS</td>
<td>$1.7 \times 10^{-2}$</td>
<td>91.78%</td>
<td>$4N^2$</td>
<td>Less stable</td>
</tr>
</tbody>
</table>

Table 5.2. Comparison of LMS NLMS RLS Table

<table>
<thead>
<tr>
<th>Variance</th>
<th>0.86</th>
<th>0.55</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiplications</td>
<td>$2N+3$</td>
<td>$3(N+1)^2 + 3(N+1)$</td>
</tr>
<tr>
<td>Additions</td>
<td>$2N+2$</td>
<td>$3(N+1)^2 + 3(N+1)$</td>
</tr>
<tr>
<td>Time Elapsed</td>
<td>5.5ms</td>
<td>9.8ms</td>
</tr>
</tbody>
</table>
From the table 5.1 and 5.2, the LMS Filter have higher stability. The percentage of noise reduction is greater in LMS algorithm. The complexity is reduced in the LMS algorithm. NLMS and RLS have low complexity but the tracking performance is poor compared with LMS filter. Fixed filter length 10 is taken for evaluation. It can be observed that for both the LMS and the NLMS, a particular optimum step size produces best approximation of the original signal. The value of the optimum step size is larger for the LMS than the NLMS. However, as the filter length is increased, the gap closes down since the value decreases for the LMS and increases for the NLMS. Another notable difference is that the curve for the LMS ascends sharply before reaching optimum level, and descends slowly afterwards. For the NLMS, it is the reverse.

6 CONCLUSION

Efficient Adaptive Noise canceller has been designed and simulated using LMS Algorithm. The effects of the filter length and step size parameters have been analyzed to reveal the behavior of the two algorithms. On comparison of LMS with RLS, the adaptation rate is Equal. RLS has fast convergence rate and infinite memory and high complexity. The LMS algorithm has produced good results for noise cancellation problem. LMS Algorithm is easy to implement, simple and it has a fast convergence rates. The Least Mean Square (LMS) algorithm is known for its simplification, low computational complexity, and better performance in different running environment. When compared to other algorithms used for implementing adaptive filters, the LMS algorithm is seen to perform very well in terms of the number of iterations required for convergence.

REFERENCES:


