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LIMIT STATE FUNCTION FORMULATION AND TIME VARIANT RELIABILITY ANALYSIS

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Abstract- Most observable phenomena in the world contain certain amount of uncertainty. Probability analysis is used to estimate input and output variables of a system by considering uncertainty. Reliability is the probability that a product performs its intended function over a specified period of time and under specified service conditions. Time-variant stochastic loadings and random deterioration of material properties are inherent in engineering applications [1]. For this type of time dependent uncertainties time variant reliability analysis is used. In time variant analysis limit state function may vary over time. Deterioration of materials properties takes place due to corrosion. In this paper, a time variant limit state function developed to estimate reliability of a fluid flowing pipe over time. Monte Carlo simulation technique is used to compute reliability of pipe at different time with the help of "MATLAB R2008a" software. Due to corrosion reliability is decrease with time. So, the results of analysis can be used to determine time to maintain or change.

Keywords
Time variant reliability, Stress, Limit state function, Random variable, Probability distribution, Monte Carlo simulation

INTRODUCTION

Corrosion defined as the deterioration of material by reaction to its environment. The corrosion occurs because of the natural tendency for most metals to return to their natural state; e.g., iron in the presence of moist air will revert to its natural state, iron oxide. Extreme case of corrosion may lead to failure of the system [2]. Fluid flow is common phenomena for various chemical industries. Normally metallic pipes are used for flow of fluid. Some fluid is dangerous for environment and human being. So, proper and zero failure maintenance needed for hazard fluid flow system. Reliability analysis is used to estimate time to failure. The reliability of a product or system conveys the concept of dependability, successful operation or performance, and the absence of failure. The time-dependent reliability gives the reliability of the system over a specific time interval instead of the reliability at a certain time instant. And results of reliability analysis are used for timely maintenance.

RELIABILITY

Reliability is the ability that a system or component fulfills its intended function under given circumstances over a specified time period. One of the purposes of system reliability analysis is to identify the weakness in a system and to quantify the impact of component failures. It is usually measured by the probability of such ability. A system's reliability is modeled by what is known as its limit state function. The limit state function indicates the state of success or failure.

TIME VARIANT RELIABILITY

The time-dependent reliability gives the reliability of the system over a specific time interval instead of the reliability at a certain time instant. The strength and weakness of the component is not fixed. It changes with aging of system. The probability of failure/success is not just value, but also a function of time. For this, time variant reliability analysis needed for better estimation [3].

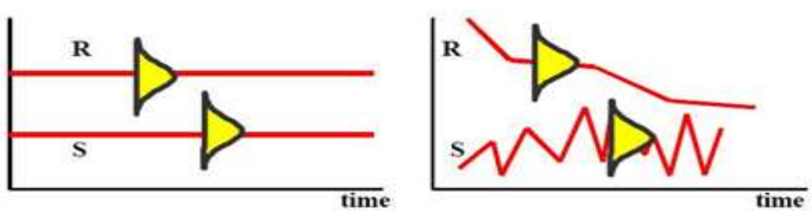

Figure 1: Time invariant and time variant system.

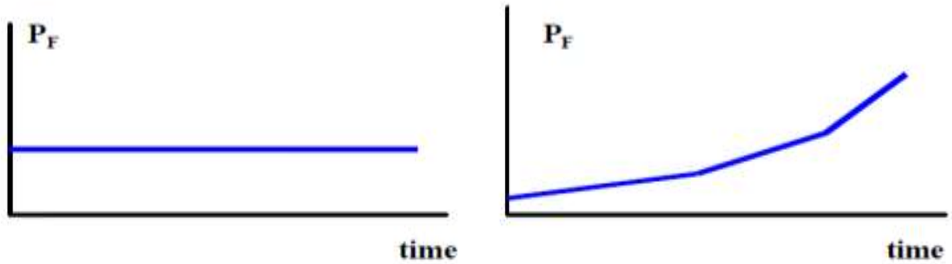

Figure 2: Time invariant and time variant probability of failure.

TIME VARIANT RELIABILITY ANALYSIS MAY BE TWO TYPES:

1. Load may be randomly varying in time: Stochastic processes are introduced in the analyses. It is allow accounting for environmental loading such as wind velocity, temperature or wave height, occupancy, traffic load etc.

2. Material properties may be decaying in time: The degradation mechanisms usually present as initiation phase and propagation phase. Both the initiation duration and propagation kinetics may be considered as random in the analyses. Example of these mechanisms are crack growth and propagation in fracture mechanics, corrosion in steel structures or in reinforce concrete rebar, concrete shrinkage and creep phenomena etc.

These two kinds of dependency may require different methods of analysis. In time-variant reliability analysis for a complicated engineering system may involve the use of many physics-based models and surrogate models, which bring in various types of uncertainty.

A general limit state function for reliability analysis is given by,

$$Z(R, S) = R - S$$

Where, Z = limit state function, R = resistance, S = load

$Z > 0$ FOR SAFE STATE OF SYSTEM.

$Z = 0$, FOR XON LIMIT STATE.

$Z < 0$, FOR FAILURE SET.

There are various types of time dependent problem. So, limit state function also various types for various cases.

1. Load variability
2. Time dependency of the resistance
3. Load and resistance time dependent, no interaction
4. Load and resistance time dependent, interaction

1. Load variability

In the case of only load variability with time, the limit state function can be formulated as:

$$Z(t) = R - S(t)$$

2. **Time dependency of the resistance**

In this case limit state function is:

$$Z(t) = R(t) - S$$

3. Load and resistance time dependent, no interaction

If both load S and resistance R are time dependent, and then limit state function formulated as:

$$Z(t) = R(t) - S(t)$$

4. Load and resistance time dependent, interaction

Generally, the resistance at time, t may affected by loading at any period of time. This for instance is what happens for mechanisms like fatigue and load of duration rupture.

$$Z(t) = R(S, t) - S(t)$$

Limit state function formulation

Let a thin wall cylindrical pipe which is given in below figure. And internal pressure of the cylinder is P.

Figure 3: Dimensions of cylindrical pipe.

WHERE, R_I = INNER RADIUS, R_0= OUTER RADIUS AND THICKNESS = T = R_0–R_I

WHEN THE PRESSURE INSIDE IS LARGER THAN THE PRESSURE OUTSIDE, THE CYLINDER WILL TEND TO SPILT ALONG A LENGTH AND A CIRCUMFERENCE. THE STRESS PRODUCED IN LONGITUDINAL DIRECTION IS CALLED LONGITUDINAL STRESS AND THE STRESS PRODUCE IN THE CIRCUMFERENTIAL DIRECTION IS CALLED CIRCUMFERENTIAL OR HOOP STRESS.

Longitudinal stress, σ_l

Consider the force trying to split the cylinder about a circumference. The areas acted on by the longitudinal stress, σ_l and the pressure P are calculated in the figure below.

Figure 4: Stress and pressure in longitudinal direction

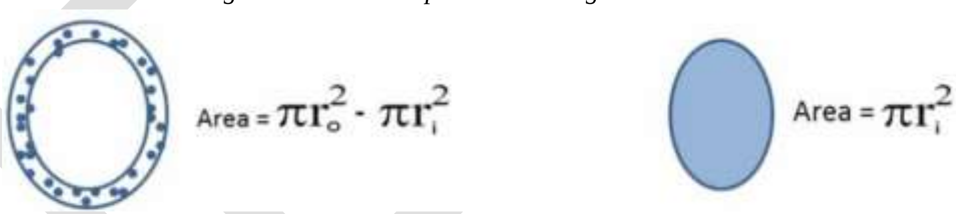

Figure 5: Area of thickness and area of fluid flow (Longitudinal direction)

From force equilibrium equation:

$$\text{Stress force} = \text{Pressure force}$$
$$(\pi r_o^2 - \pi r_i^2) \times \sigma_l = \pi r_i^2 \times P$$
$$\sigma_l = \frac{\pi r_i^2}{\pi r_o^2 - \pi r_i^2} \times P$$
$$= \frac{r_i^2}{(r_o + r_i)\big((r_o - r_i)\big)} \times P$$
$$= \frac{r_i^2}{2 r_m t} \times P$$

Circumferential or hoop stress, σ_c

Consider the force trying to split the cylinder about along a length. The areas acted on by the circumferential stress and the pressure are calculated in the figure below.

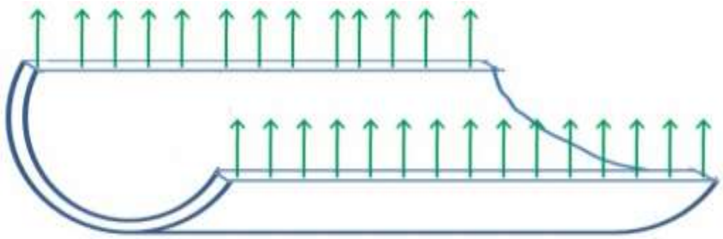

Figure 6: Stress and pressure in circumferential direction

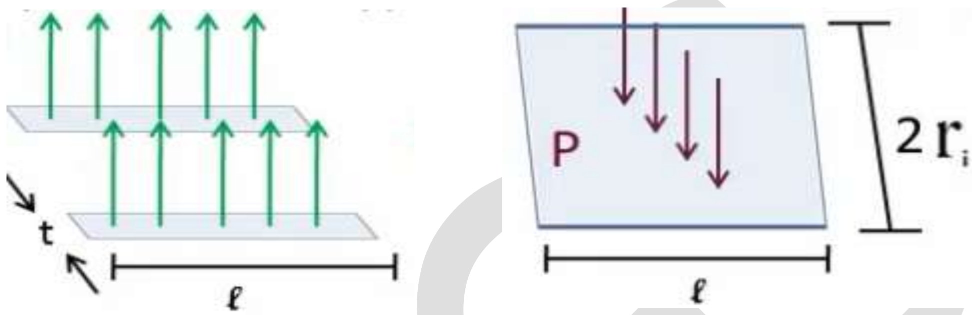

Figure 7: Area of thickness and area of fluid flow (circumferential direction)

From force equilibrium equation:

Stress force=Pressure force

$$2lt \times \sigma_c = 2r_i l \times P$$
$$\sigma_c = \frac{r_i}{t} \times P$$

It is clear that hoop stress is roughly twice the value of longitudinal stress. The wall thickness of the pipe, on almost all occasions is selected based on the thickness required for the internal pressure and allowance. Out of two pressure forces, hoop stress is used for pipe wall thickness calculation and longitudinal force mitigate by using sufficient support [4].

Limit state function

Failure of pipe due to hoop stress occurs when hoop stress force is higher than force [5],

$$F = \frac{4\pi^2 EI}{\left(\pi(r_o + r_i)\right)^2}$$

And $= \frac{Lt^3}{12}$,

$$= \frac{4E}{(r_o+r_i)^2} \times \frac{lt^3}{12}$$
$$= \frac{Elt^3}{3(2r_o - t)^2}$$

Where, $E\ Modulus\ of\ elasticity$

$I\ Moment\ of\ inertia$

$$So, hoop\ force = \sigma_c \times 2lt = \frac{(r_0 - t)}{t} \times P \times 2lt = 2l((r_0 - t)P$$

If corrosion rate of pipe materials $= R\ mm/y$. So, limit state function at time T is given by,

$$Z(P,T,R,E) = F - \sigma_c \times 2lt = \frac{El(t-RT)^3}{3(2r_o - t + RT)^2} - 2l((r_0 - t + RT)P$$

Where $P, R\ and\ E$ are random variable and $t, l\ and\ r_o$ are constant.

Monte Carlo simulation

Monte Carlo simulation is a type of simulation that relies on repeated random sampling and statistical analysis to compute the results. This method of simulation is very closely related to random experiments, experiments for which the specific result is not known in advance. In this context, Monte Carlo simulation can be considered as a methodical way of doing so-called what-if analysis. Monte Carlo simulation furnishes the decision maker with a range of possible outcomes and the probabilities they will occur for any choice of action [6].

In Monte Carlo simulation, we identify a statistical distribution which we can use as the source for each of the input parameters. Then, we draw random samples from each distribution, which then represent the values of the input variables. For each set of input parameters, we get a set of output parameters. The value of each output parameter is one particular outcome scenario in the simulation run. We collect such output values from a number of simulation runs.

Variables distribution and parameters

In this problem, for time dependent reliability analysis in limit state function there are seven (07) parameters including time. We considered thickness, outer diameter and length of pipe are constant. And modulus of elasticity, fluid pressure and corrosion rate are random variables. For Monte Carlo simulation, statistical distribution of input parameters is needed to identify from available historical data.

When sufficient data are available, a histogram or frequency diagram can be used to determine the underlying distribution. Also goodness of fit tests is used to assume distribution. Here, due to lack of available data we assume distribution of random variable.

Table 01: Random variables, distribution and Parameters

Variable	E (N/mm^2)	P(N/mm^2)	R(mm/y)
Statistical distribution	Normal	Normal	Normal
Mean	130000	0.5	.08
Standard deviation	8000	.01	.009

Reliability analysis

Reliability analysis can be divided into two broad categories: qualitative and quantitative. The former is intended to verify the various failure modes and causes that contribute to the unreliability of a product or system [7]. The latter uses real failure data (obtained, for example, from a test program or from field operations) in conjunction with suitable mathematical models to produce quantitative estimates of product or system reliability. Here, *"MATLAB R2008a"* software is used for reliability analysis by Monte Carlo simulation technique. Random number of each input variables repeatedly generated for each time (year) and value of limit state function is recorded. Total no. of positive value is divided by total repeated action to determine reliability of system in every year. Following table show data of reliability for thirty (30) years. Value of constant parameters:

$Inner\ redius\ of\ the\ pipe, r_i = 500mm$

$Thickness\ of\ the\ pipe, t = 25mm$

$Outer\ redius\ of\ the\ pipe, r_0 = 525mm$

$$Length\ of\ the\ pipe, l = 3m = 3000mm$$

Table 02: Reliability analysis result

Time (Year)	Reliability	Time (Year)	Reliability
1	1.0000	16	0.8933
2	1.0000	17	0.8783
3	1.0000	18	0.8400
4	0.9967	19	0.8067
5	0.9983	20	0.7667
6	0.9933	21	0.7167
7	1.0000	22	0.6433
8	0.9950	23	0.5717
9	0.9900	24	0.5183
10	0.9850	25	0.4683
11	0.9817	26	0.4133
12	0.9733	27	0.3500
13	0.9633	28	0.2867
14	0.9400	29	0.2700
15	0.9300	30	0.2017

Conclusion

Most of the engineering systems are time dependent. To determine reliability properly, time variant reliability analysis is essential. Probabilistic analysis is plays an important to estimate a system condition by considering uncertainty. Difficulty of reliability is depends of complexity of limit state function, number of random variable, interdependency of variables.

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