Noise Cancellation using Adaptive Filters Algorithms

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Abstract— Active Noise Control (ANC) involves an electroacoustic or electromechanical system that cancels the primary (unwanted) noise based on the principle of superposition. An anti-noise signal of equal amplitude and opposite phase is generated and combined with the primary noise, resulting in the cancellation of the noise. A fundamental problem to be considered in ANC systems is the requirement of highly precise control, temporal stability and reliability. To produce high degree of attenuation, the amplitude and phase of both the primary and the secondary noise must match with the close precision. The adaptive filters are used to control the noise and it has a linear input and output characteristic. If a transfer path of the noise has nonlinear characteristics it will be difficult for the filter to generate an optimal anti-noise. In this study, we propose a algorithm, delta rule algorithm which uses non linear output function. Delta rule is used for learning complex patterns in Artificial Neural Networks. We have implemented the adaptive filters using Least Mean Square (LMS) algorithm, Recursive Least Square (RLS) algorithm and compared the results.

Keywords— ANC, LMS, RLS, delta rule, error signal, neural networks, real-time

Introduction— Active noises are real time noise and they cannot be predictable (i.e., random). The traditional way to cancel the noise, which is called passive noise control, which technique based on the use of sound absorbing materials, is effective in higher frequency noise. However, significant power of the industrial noise often occurs in the frequency range between 50-250 Hz. Here the wavelength of sound is too long, so that passive techniques are no longer cost effective because they require material that is too heavy.

Active Noise Control System is working based on the principle of superposition. The system consists of a controller for which reference about the noise is given. The controller properly scales the reference noise and the phase reverses it. The phase-reversed signal is then added to the input signal that has some noise along the original message signal so that the noise gets cancelled out. There are many methods used for ANC system include both feedback and feed-forward control. ANC is based on either feed-forward control, where a coherent reference noise input is sensed before it propagates past the secondary source, or feedback control where the active noise controller attempts to cancel the noise without the benefit of an upstream reference input. The performance of the active control system is determined largely by the signal processing algorithm and the actual acoustical implementation. Effective algorithm design requires reasonable knowledge of algorithm behaviour for the desired operating conditions. Since the active noise is random, the proper prediction of the noise cannot be possible, the controller should contain a adaptive filter part whose filter coefficients will be changing based on the error signal which the difference between the output of the controller and the output from an unknown plant. To achieve reduction of noise in complicated multiple noise source, we must use active noise control by multiple reference channel. That is input signal to the each channel is con-elated and the output also con-elated.

LMS ALGORITHM— The Least Mean Square, or LMS, (Douglas and Pan, 1995) algorithm is a stochastic gradient algorithm that iterates each tap weight in the filter in the direction of the gradient of the squared amplitude of an error signal with respect to that tap weight as shown in Fig. 1. The LMS algorithm is an approximation of the steepest descent algorithm which uses an instantaneous estimate of the gradient vector.
The estimate of the gradient is based on sample values of the tap input vector and an error signal. The algorithm iterates over each tap weight in the filter, moving it in the direction of the approximated gradient (Woo, 2001). Widrow et al. and Mugdha, M. Dewasthale, 2014 devised the LMS algorithm in 1959. The objective is to change (adapt) the coefficients of an FIR filter, w(n), to match as closely as possible to the response of an unknown system, p(n). The unknown system and the adapting filter process the same input signal x(n) and have outputs d(n) (also referred to as the desired signal) and y(n), respectively.

The LMS algorithm basically has two processes. The first one is the filtering process and the next is adaptive process. In filtering process, the reference signal is filtered in the adaptive filter and it is combined with desired signal. The error signal is the difference of the desired signal and the output signal of the filter w(n) (Glentis et al., 1999). In adaptive process, the reference signal and the error signal are fed to the LMS algorithm and the weights of the filter are modified based on the LMS algorithm.

It is assumed that all the impulse responses in this study are modeled by those of finite impulse response (FIR) filters. d(n) has the primary noise to be controlled and x(n) is the reference about the noise.

\[ d(n) = p^T(n) * x_1(n) \]  
(1)

where p(n) is the impulse response of the unknown plant and \( x_1(n) = [x(n) \ x(n-1)x(n-M+1)]^T \)

and M is the length of p(n). The y(n) is the output signal from the filter.

\[ Y(n) = w^T(n) * x_2(n) \]  
(2)

where \( w(n) = [w(n) \ w(n-1) \ w(n-2) \ldots \ w(n-N+1)]^T \) is the weight vector of the ANC controller with a length N and \( x_2(n)=[x(n) \ x(n-1) \ x(n-N+1)]^T \).

The error signal e(n) is difference of the desired signal d(n) and the output of the filter y(n).

\[ e(n) = d(n) - y(n) \]  
(3)

The weight of the filter w(n) is updated using the following equation;

\[ w(n+1) = w(n) + x(n) \ e(n) \]  
(4)

The termed as step size. This step size has a profound effect on the convergence behavior of the LMS algorithm. If is too small, the algorithm will take an extraordinary amount of time to converge. When is increased, the algorithm converges more quickly however, if is increased too much, the algorithm will actually diverge. A good upper bound on the value of is 2/3 over the sum of the eigen values of the autocorrelation matrix of the input signal. The correction that is applied in updating the old estimate of the coefficient vector is based on the instantaneous sample value of the tap-input vector and the error signal.
The correction applied to the previous estimate consists of the product of three factors: the (scalar) step-size parameter, the error signal e(n-1) and the tap-input vector u(n-1). The LMS algorithm requires approximately 20L iterations to converge in mean square, where L is the number of tap coefficients contained in the tapped-delay-line filter. The LMS algorithm requires 2L+1 multiplications, increasing linearly with L.

**RLS ALGORITHM**— Recursive Least Square algorithm (RLS) (Kuo and Morgan, 1996) can be used with an adaptive transversal filter to provide faster convergence and smaller steady state error than the LMS algorithm (Haykin, 1996 and Upal Mahbub et al., 2012). The RLS algorithm uses the information contained in all the previous input data to estimate the inverse of the autocorrelation matrix of the input vector. It uses this estimate to properly adjust the tap weights of the filter.

In the RLS algorithm the computation of the correction utilizes all the past available information. The correction consists of the product of two factors: the true estimation error (n) and the gain vector k(n). The gain vector itself consists of \( P^{-1}(n) \), the inverse of the deterministic correlation matrix, multiplied by the tap-input vector u(n). The major difference between the LMS and RLS algorithms is therefore the presence of the correction term of the RLS algorithm that has the effect of decorrelating the successive tap inputs, thereby making the RLS algorithm self-orthogonalizing. Because of this property, we find that the RLS algorithm is essentially independent of the eigen-value spread of the correlation matrix of the filter input. The RLS algorithm converges in mean square within less than 2L iterations. The rate of convergence of the RLS algorithm is therefore, in general, faster than that of the LMS algorithm by an order of magnitude. There are no approximations made in the derivation of the RLS algorithm. Accordingly, as the number of iterations approaches infinity, the least-squares estimate of the coefficient vector approaches the optimum Wiener value and correspondingly, the mean-square error approaches the minimum value possible. In other words, the RLS algorithm, in theory, exhibits zero misadjustment. The superior performance of the RLS algorithm compared to the LMS algorithm is attained at the expense of a large increase in computational complexity. The complexity of an adaptive algorithm for real-time operation is determined by two principal factors:

- The number of multiplications (with divisions counted as multiplications) per iteration and
- The precision required performing arithmetic operations. The RLS algorithm requires a total of 3L \((3+L)/2\) multiplications, which in creases as the square of L the number of filter coefficients. But the order of RLS algorithm can be reduced.

**DELTA RULE ALGORITHM**— Delta rule algorithm (Yegnarayana, 2001) is widely used in Artificial Neural Networks in pattern recognition and will differ from the LMS algorithms in the weight update equation. The change in weight vector of the delta rule algorithm is;

\[ wi = *(d(n)-f(y(n)))(y(n))*x(n) \] (5)

Where \( f(y(n)) \) is the output function.

The above equation is valid only for the differential output function. In LMS, the output function is linear \( f(x) = x \). In this case, we are using non-linear ear output functions, which possess sigmoid nonlinearity. Since the derivative of the function is also used, the output function used should be differentiable. Two examples of sigmoid nonlinear ear function are the logistic function and the hyperbolic tangent function.

The logistic function output lies between 0 to 1. Similarly the second is hyperbolic tangent function and it is given by the output of the hyperbolic function lies between -1 to 1. is the scaling factor. Since the maximum value of the logistic function and hyperbolic tangent function is 1, the divergence of the weights is avoided. By properly choosing value of, the faster convergence can be achieved. The computational complexity of delta rule algorithm is sum of computational complexity of LMS and computations involved in the calculation of \( f(x) \) and \( f(x) \) and multiplication of the function in the weight update vector. The computational complexity is not increased greatly compared to the LMS algorithm.
**SIMULATION AND RESULTS**— The weight update equation of LMS RLS and delta rule are (Woo, 2001), respectively. A white Gaussian noise with zero mean and unit variance is used as the reference signal. The primary path $p(n)$ is simulated by a filter of length 128.

![Learning Curve](image1)

Fig. 2 Output estimation error of LMS algorithm

![Error curve](image2)

Fig. 3 Error value of RLS algorithm

![System output](image3)

Fig. 4 True and estimated output of RLS algorithm

![Noise](image4)

Fig. 5 Noise of delta rule algorithm
The length of \( w(n) \) is chosen to be 96. The logistic function is used for the simulation of delta rule algorithm. All the weights are initialized to zero and all the results are average of 100 cycles.

All the simulations were done using MATLAB. The output estimation error of LMS algorithm is shown in the Fig. 2. The Error value of RLS algorithm is shown in the Fig. 3. The True and estimated output of RLS algorithm of RLS and noise of delta rule algorithm is shown in the Fig. 4 and 5, respectively. RLS algorithm converges very quickly than the other two algorithms and the residual noise is also less.

**CONCLUSIONS**— In this study, we have compared the performance of LMS, RLS and delta rule algorithm for a white Gaussian noise as reference. From the results shown in the above section, we can conclude that RLS algorithm is better in performance than both the algorithms. But the order of the complexity of the RLS algorithm is much more than the order of the complexity of the other two algorithms. The delta rule algorithm requires slightly more number of computations than the LMS algorithm and the residual noise of the delta rule algorithm is less than the residual noise of LMS algorithm. The delta rule algorithm is more efficient when both then noise reduction and computational complexity are taken in to consideration.

**REFERENCES:**


