Comparative Analysis of T-Sugeno and Mamdani Type Fuzzy Logic Controller for PMSM Drives

DEEPA BAJPAI, AHJIT MANDAL
DISHA INSTITUTE OF MANAGEMENT AND TECHNOLOGY, RAIPUR, INDIA
deepabajpai26612@gmail.com, mob.no. 09755878261

Abstract— An electric drive performance is paramount for crucial motion application and greatly influenced by capabilities of controller. For high performance application, vector control technique is normally applied with the permanent magnet induction motor (PMSM) drive. Instead of conventional PID controller fuzzy logic controller (FLC) has been widely used for such application. In this paper, presented for two different inference system namely T-Sugeno and Mamdani FLCs for the performance of vector controlled PMSM drives. The performance of the drive has been investigated for speed control at different loading condition. For T-Sugeno type FLC the performance of drive system is found superior as compared to the Mamdani type FLC in terms of rise time, settling time, under shoots and over shoots. The complete viability of above mentioned vector control strategy is implemented in the MATLAB/Simulink environment and a performance comparison of proposed drive system with different inference system based fuzzy logic controller used in fuzzy logic controller as speed controller for PMSM at no load and with fixed step load has been presented.

Keywords— PMSM drives, Vector control, FLC, Inference system, T-Sugeno Type FLC, Mamdani Type FLC, Speed Controller.

I. INTRODUCTION

From the last three decades AC machine drives are becoming more and more popular, especially Induction Motor Drives (IMD) and Permanent Magnet Synchronous Motor (PMSM), but with some special features, the PMSM drives are ready to meet sophisticated requirements such as fast dynamic response, high power factor, and wide operating speed range like high performance applications, as a result, a gradual gain in the use of PMSM drives will surely be witness in the future market in low and mid power applications.

In recent years, for its superior performance in speed control application FLC is distinguished and captured the attention of researchers. FLC’s have the advantage to handle the system nonlinearities [8], and its control performance is not much affected by system parameter variation [3-4].

The objective of this paper is to use the Takagi-sugeno type and mamdani type FLC and camper the performance of FLC for PMSM drives speed control. The stator current vector is represented in the stator flux reference frame, the T-S fuzzy controller calculate the quadrature component of stator current vector. For the proposed controller the rule based is defined in function of speed error and change in speed error using triangular membership function.

The simulation result shown the performance of T-S is good in terms of settling time ($t_s$), rise time ($t_r$) and torque ripple as compare to mamdani FLC. For two different condition at no load and at full load performance evaluation was carried out through simulation result. The system is dynamically simulated using Simulink/MATLAB Software [5-6].

II. PERMANENT MAGNET SYNCHRONOUS MOTOR DRIVE SYSTEM

The motor drive consists of four main components, the PM motor, inverter, control unit and the position sensor. The components are connected as shown in Fig.1.
In this work for high performance the indirect vector control technique is incorporated. The actual rotor speed $\omega_r$ measured and compared with the $\omega_r^*$. The reference torque $T_r^*$ is calculated as the output, when the resulting error generated from the comparison of the two speeds processed in the controller. A limiter is used to limit the reference torque $T_r^*$, in order to generate the q-axis reference current $i_{qs}^*$. The d-axis reference current set to zero. Both d-axis and q-axis stator current generate three phase reference current ($i_a^*, i_b^*$ and $i_c^*$) through Park’s Transformation which are compared with sensed winding current ($i_a, i_b$ and $i_c$) of the IM. The control signals generated after the comparing the sensed current and reference current will fire the power semiconductor devices of the three-phase voltage source inverter (VSI) to produce the actual voltage to be fed to the induction motor.

In synchronously rotating reference frame the mathematical model for a three-phase y-connected squirrel-cage induction motor under steady state condition and load is given as [7-10].

$$
\begin{align*}
[I^c_{qs}] & = \begin{bmatrix}
R_s & \omega_s L_s & 0 & \omega_s L_m \\
-R_s L_s & R_s & -\omega_s L_m & 0 \\
0 & \omega_s L_m & R_s & \omega_s L_r \\
-\omega_s L_m & 0 & -\omega_s L_r & R_r
\end{bmatrix} \begin{bmatrix} v_{qs}^c \\ v_{ds}^c \\ v_{qr}^c \\ v_{dr}^c \end{bmatrix} \\
(I_{ds}^c) & = 0 \\
(I_{qs}^c) & = 0
\end{align*}
$$

(1)

$$
T_r = \frac{3}{2} p L_m (i_q^r i_d^r - i_d^r i_q^r)
$$

(2)

$$
T_r - T_L = J \frac{d\omega_r}{dt} + B \omega_r
$$

(3)

$$
\frac{d\theta_r}{dt} = \omega_r
$$

(4)

Where $i_{ds}^c, i_{qs}^c$ are d,q-axis stator current respectively, are $v_{ds}^c, v_{qs}^c$ are d,q-axis stator voltages respectively, $i_{dr}^c, i_{qr}^c$ are d,q-axis rotor current respectively $R_s, R_r$ are stator and rotor resistance per phase respectively, $L_s, L_r$ are the self inductances of the stator and rotor respectively, $L_m$ is the mutual inductance, $\omega_e$ is the speed of the rotating magnetic field, $\omega_r$ is the rotor speed, $p$ is the number of poles, $T_r$ is the developed electromagnetic torque, $T_L$ is the load torque, $J$ is the inertia, $B$ is the rotor damping coefficient and $\theta_r$ is the rotor position. The key feature of the vector control is to keep the magnetizing current at a constant rated value by setting $i_{dr}^c = 0$. Thus, by adjusting only the torque –producing current component the torque demand can be controlled. With this assumption, the mathematical formulation can be rewritten as.
Where \( \omega_{sl} \) is the slip speed and \( \psi_{dr} \) is the d-axis rotor flux linkage. The indirect vector controlled drive system with FLC assisted speed controller model is represented from equation (1) to equation (7).

III. Vector Control PMSM Drive

The vector control separates the torque and flux channels in the machine through its stator excitation inputs. The vector control for PMSM is very similar to the vector control of induction motor drives. In this section, the vector control of the three-phase PMSM is derived from its dynamic model. Considering the currents as inputs, the three-phase currents are:

\[
i_a = i_s \sin(\omega t + \delta) \quad (8)
\]

\[
i_b = i_s \sin(\omega t + \delta - \frac{2\pi}{3}) \quad (9)
\]

\[
i_c = i_s \sin(\omega t + \delta + \frac{2\pi}{3}) \quad (12)
\]

Where \( \delta \) is the angle between the rotor field and stator current pasters.

The previous currents obtained are the stator currents that must be transformed to the rotor reference frame with the rotor speed \( \omega_0 \), using Park’s transformation. The q and d axis Currents are constants in the rotor reference frames since \( \delta \) is a constant for a given load torque. As these constants, they are similar to the armature and field currents in the separately excited dc machine. The q axis current is distinctly equivalent to the armature Current of the dc machine; the d axis current is field current, but not in its entirety. It is only a Partial field current; the other part is contributed by the equivalent current source representing the permanent magnet field. For this reason the q axis current is called the torque producing component of the stator current and the d axis current is called the flux producing component of the stator current.

Using park’s transformation this stator current must be transformed to rotor reference frame [11-12]

\[
\begin{bmatrix}
i_q \\
i_d \\
i_o
\end{bmatrix}
= \frac{2}{3}
\begin{bmatrix}
\cos \theta_r & \cos(\theta_r - 120) & \cos(\theta_r + 120) \\
\sin \theta_r & \sin(\theta_r - 120) & \sin(\theta_r + 120)
\end{bmatrix}
\begin{bmatrix}
i_a \\
i_b \\
i_c
\end{bmatrix}
\]

Putting the equation (7), (8) and (9) in (10) and solving, then we get

\[
\begin{bmatrix}
i_q \\
i_d
\end{bmatrix}
= (i_s)
\begin{bmatrix}
\sin \delta \\
\cos \delta
\end{bmatrix}
\]

(11)
Using equation (7) and (11) the electromagnetic torque is obtained as given below

\[ T_e = \frac{3}{2} \frac{2}{2} \frac{1}{2} (L_d - L_q) i_s^2 \sin \delta + \lambda_f i_q \sin \delta \]  \hspace{1cm} (12)

In order to achieve dc motor like behavior, the control needs knowledge of position of the instantaneous rotor flux or rotor position of PM motor. Knowing the position, the three phases current can be calculated. Its calculation using the current matrix depends on the control desired.

\textit{a. Constant Torque Operation.}

\textit{b. Flux weakening Operation.}

These options are based in the physical limitation of the motor and the inverter. The limit is established by the rated speed of the motor, at which speed the constant torque operation finishes and flux weakening starts shown in Fig.2.

\textbf{IV. Constant Torque Operation}

In this control strategy the d-axis current is kept zero, while the vector current is align with the q-axis in order to maintain the torque angle equal with 90°. This is one of the most used control strategy because of the simplicity, especially for SPMSM. In case of IPMSM, with a high saliency ratio it is not recommended to use this control strategy because of the reluctance torque produced.

The torque equation can be rewritten as:

\[ T_e = \frac{3}{2} \frac{P}{2} \lambda_f i_q \]  \hspace{1cm} (13)

So,

\[ T_e = K_t i_q \text{ where } K_t = \left( \frac{2}{2} \right) \frac{P}{2} \lambda_f \]  \hspace{1cm} (14)

\textbf{Fig.2 PMSM Characteristics in Constant Torque and Field-Weakening Regions}
Note that the torque equation (14) resembles with that of the dc machine where the torque is only dependent on quadrature axis current when we consider the field flux constant and hence provide its equivalent operation.

V. FLC DESIGNING

Fig.3 show the general block diagram of FLC. The main objective of the designed FLC is to maintain the performance obtained by ‘standard design’ while reducing the complexity of fuzzy rule base design. FLC has mainly four internal component from which input has to be processed to come out as output.

![Fig.3 Block diagram of FLC](image)

These component are –

**Fuzzification** - is the conversion of crisp numerical values into fuzzy linguistic quantifiers. Fuzzification is performed using membership functions. Each membership function evaluates how will the linguistic variable may be described by a particular fuzzy qualifier.

**Inference Engine** - The inference engine uses the fuzzy vectors to evaluate the fuzzy rules and producing an output for each rule. Mandani type fuzzy inference engine is used for this particular work.

**Defuzzification** - in this process the combined output fuzzy set produced from the inference engine into a crisp output value of real-world meaning. Center of gravity defuzzification.

VI. FUZZY INFERENCE SYSTEM

Fuzzy inference is the process of formulating the mapping from given input(s) to output(s) using fuzzy logic. This mapping provides a basis from which decisions can be made, or patterns discerned. It has found successful applications in a wide variety of fields, such as automatic control, data classification, decision analysis, expert systems, time series prediction, robotics, and pattern recognition. Because of its multidisciplinary nature, the fuzzy inference system is known by numerous others names, such as fuzzy-rule-based system, fuzzy expert system, fuzzy model, fuzzy associative memory, fuzzy logic controller and simply (and ambiguously) fuzzy system [1].

A fuzzy inference system with crisp inputs and outputs implements a nonlinear mapping from its inputs space to output space. This mapping is accomplished by a number of fuzzy if-then rules, each of which describes the local behavior of the mapping. In particular, the antecedent of a rule defines a fuzzy region in the input space, while the consequent specifies the output in the fuzzy region. Basically a fuzzy inference system is composed of five functional blocks as shown in Fig.4.
The Structure of the Fuzzy Inference system is described as

- A rule base containing a number of fuzzy if-then rules.
- A database which defines the membership functions of the fuzzy sets used in fuzzy rules.
- A decision-making unit which performs the inference operations on the rules.
- A fuzzification interface which transforms the crisp inputs into degrees of match with linguistic values.
- A defuzzification interface which transforms the fuzzy results of the inference into a crisp output.

The rule base and the database are jointly referred to as the knowledge base. Fuzzy if-then rules or fuzzy conditional statements are expressions of the form: If x is A Then y is B, where, x and y are input and output linguistic variables. A and B are labels of the fuzzy sets characterized by appropriate membership functions. A is the premise and B is the consequent parts of the fuzzy rule. Fuzzy values A and B are described by the membership functions. The forms of membership functions are different and problem depended. The steps of fuzzy reasoning (inference operations upon fuzzy IF–THEN rules) performed by FISs are described as follows

- Compare the input variables with the membership functions on the antecedent part to obtain the membership values of each linguistic label (this step is often called fuzzification).
- Combine (usually multiplication or min) the membership values on the premise part to get firing strength (weight) of each rule.
- Generate the qualified consequents (either fuzzy or crisp) of each rule depending on the firing strength.
- Aggregate the qualified consequents to produce a crisp output (This step is called defuzzification).

The most common types of fuzzy reasoning that have been introduced in the literature and applied to different applications are Mamdani and Sugeno type models [1], [8]. The most fundamental difference between Mamdani-type FIS and Sugeno-type FIS is the way the crisp output is generated from the fuzzy inputs. Mamdani-type FIS uses the technique of defuzzification of a fuzzy output, while Sugeno-type FIS uses weighted average to compute the crisp output. Hence, Mamdani FIS has output membership functions whereas Sugeno FIS has no output membership functions. Mamdani type is widely accepted for capturing expert knowledge [2]. It allows describing the expertise in more intuitive, more humanlike manner. However, Mamdani-type entails a substantial computational burden. On the other hand, Sugeno method is computationally efficient and works well with optimization and adaptive
techniques, which makes it very attractive in different applications. Mamdani-type FIS is less flexible in system design in comparison to Sugeno-type FIS [1-2].

(A) Mamdani Inference System

The most commonly used fuzzy inference technique is the so-called Mamdani method (Mamdani & Assilian, 1975) which was proposed, by Mamdani and Assilian, as the very first attempt to control a steam engine and boiler combination by synthesizing a set of linguistic control rules obtained from experienced human operators. Their work was inspired by an equally influential publication by Zadeh (Zadeh, 1973). Interest in fuzzy control has continued ever since, and the literature on the subject has grown rapidly. A survey of the field with fairly extensive references may be found in (Lee, 1990) or, more recently, in (Sala et al., 2005). In Mamdani’s model the fuzzy implication is modeled by Mamdani’s minimum operator, the conjunction operator is min, the t-norm from compositional rule is min and for the aggregation of the rules the max operator is used. In order to explain the working with this model of FLC will be considered the example from (Rakic, 2010) where a simple two-input one-output problem that includes three rules is examined.

In this thesis, mamdani inference used with center of gravity defuzzification method and 49 rules, Fig. 5 (a), (b), (c), and (d) shows the input1, input2 and output, which are speed error, change in speed error and d-axis stator current respectively and surface plot.

![Fig. 5 Membership Functions for Input1, Input2, Output and Surface Plot.](image-url)
(B). Sugeno-Type Fuzzy Inference

The fuzzy inference process we’ve been referring to so far is known as Mamdani’s fuzzy inference method, the most common methodology. In this section, we discuss the so-called Sugeno, or Takagi-Sugeno-Kang, method of fuzzy inference. Introduced in 1985 [Sug85], it is similar to the Mamdani method in many respects. The first two parts of the fuzzy inference process, fuzzifying the inputs and applying the fuzzy operator, are exactly the same. The main difference between Mamdani and Sugeno is that the Sugeno output membership functions are either linear or constant.

A typical rule in a Sugeno fuzzy model has the form

If Input 1 = \( x \) and Input 2 = \( y \), then Output is \( z = ax + by + c \)

For a zero-order Sugeno model, the output level \( z \) is a constant \( (a=b=0) \).

The output level \( z_i \) of each rule is weighted by the firing strength \( w_i \) of the rule. For example, for an AND rule with Input 1 = \( x \) and Input 2 = \( y \), the firing strength is

\[
W_i = \text{AndMethod}(F_1(x), F_2(y))
\]

where \( F_1, F_2 \) are the membership functions for inputs 1 and 2. The final output of the system is the weighted average of all rule outputs, computed as:

\[
\text{FinalOutput} = \frac{\sum_{i=1}^{N} w_i z_i}{\sum_{i=1}^{N} w_i}
\]

where \( w_i = \Pi x_i \) and \( z_i \) is the corresponding output.

In this work T-S inference used with witage average defuzzification method and 49 rules for speed control PMSM drives. Fig.6 (a), (b), (c) and (d) shows the input1, input2, output which are the speed error, change in spe

In speed control PMSM drives. Fig.6 (a), (b), (c) and (d) shows the input1, input2, output which are the speed error, change in speed error and d-axis stator current respectively and surface plot.

![Membership function plots](http://example.com/membership.png)

![plot points](http://example.com/plot.png)
VII. Result and Discussion

For the comparison of the performance of T-S and Mamdani type FLC for speed control of PMSM drives simulink model was developed. For the performance evaluation of the proposed fuzzy logic controllers based indirect vector control PMSM drive, simulated under no load and sudden step change in load condition. The parameters values are tabulated in Appendix A. The motor is operated in constant torque mode. In the designed model for performance improvement of PMSM drive system, two controllers have been integrated: One as outer speed controller and other as inner current controller.

The simulation runs for a period of 0.4 second with reference speed of 52.3 rad/sec. and load torque 1.05 N-m.

Case 1 No load with reference speed 52.3 rad/sec.

Fig. 7(a) Speed Response of Mamdani Type and T-Sugeno Type FLC at No-Load with Reference speed is 52.3 rad/sec.

From the Fig. 7(a) it is clear that the settling time is reduced, when used the Sugeno type FLC as compared to Mamdani type FLC. Here settling time is 0.018 sec. for Sugeno type FLC and 0.024 for Mamdani type FLC at no load condition with reference speed is 52.3 rad/sec.
Fig. 7(b) Torque Response of Mamdani Type and T-Sugeno Type FLC at No-Load with Reference speed is 52.3 rad/sec.

In Fig. 7(b) red color shows the torque response of Sugeno type FLC and blue color shows the Mamdani type FLC torque response. It is clear that the starting torque is higher, lesser settling time, rise time and ripples, when used the Sugeno type FLC as compared to Mamdani type FLC. Here settling time is 0.018 sec. for Sugeno type FLC and 0.024 for Mamdani type FLC.

Case II: Transient loading with step load and reference speed is 52.3 rad/sec.

Fig. 8(a) Speed Response of Mamdani Type and T-Sugeno Type FLC at Transient loading condition (Step-Load) with Reference speed is 52.3 rad/sec.

Transient speed response is shown in the Fig.8(a) red color shows the response of Sugeno type FLC and blue color shows the response of Mamdani type FLC. The step load is suddenly applied at 0.1 sec, speed is reduced from 52.3 rad/sec. to 52.04 rad/sec. After the loading undershoot is present in speed response of Mamdani type FLC and no undershoots and overshoots are present in speed response of Sugeno type FLC. Final value of speed is reached by both type of FLC is 52.04 rad/sec.

Fig. 8(b) Torque Response of Mamdani Type and T-Sugeno Type FLC at Transient loading condition (Step-Load) with Reference speed is 52.3 rad/sec.
Fig. 8(b) shows the torque response of both type of FLC. Red color shows the Sugeno type FLC response and blue color Mamdani type FLC response. Torque ripples, rise time and settling time are lesser in response of Sugeno type FLC as compared to Mamdani type FLC. Larger starting torque is provide by Sugeno type FLC. Final value of torque is 1N-m reached by both type of FLC.

Case III – Dynamic loading condition with Step-Load applied from the starting of the machine and reference speed is 52.3 rad/sec.

**Fig. 9(a) Speed Response of Mamdani Type and T-Sugeno Type FLC at Dynamic Loading Condition with Reference speed is 52.3 rad/sec.**

Fig.9(a) shows the response of both the FLC at dynamic loading condition. Here step-load is applied from the starting time on the motor with reference speed is 52.3 rad/sec. Here settling time is .049 sec. when Sugeno type FLC used and 0.069 sec. when Mamdani type FLC is used for speed control. Final value of speed is reached by both type of FLC is 52.04 rad/sec.

**Fig. 9(b) Torque Response of Mamdani Type and T-Sugeno Type FLC at Dynamic Loading Condition with Reference speed is 52.3 rad/sec.**

In Fig.9(b) torque response is shown. From the Fig. rise time, settling time and ripple are lesser in Sugeno type FLC response as compared to Mamdani type FLC. Here settling time is 0.49 sec. for Sugeno type FLC and 0.069 sec. for Mamdani type FLC. Final value of torque is 1 N-m reached by both type of FLC and starting torque is also higher when Sugeno type FLC is use for speed control.

**VIII. ACKNOWLEDGEMENT**

I would like to express my deep sense of profound gratitude to my honorable, esteemed guide, Assistance Prof. Abhijit Mandal, Assistant Professor, Electrical and Electronics Engineering Department for his guidance and constant support. Over the time he has introduced me to the academic world. His perspective on my work has inspired me to go on. I am glad to work with him.
I would also like to convey my sincere gratitude to **Prof. Amit Goswami**, HOD, Electrical and Electronics Engineering Department for his continuous encouragement and valuable advices.

I would like to thank **Dr. S. P. Dubey** sir for their encouragement and support in completing this project work.

I cannot end without thanking my in-laws, my husband and my daughter on whose encouragement, support, and love, I have relied throughout my studies.

I would like to thank to all those who directly or indirectly supported me in carrying out this project work successfully.

**IX. Conclusion**

This paper is mainly emphasized on the study of performance of PMSM drive systems using two different inference system namely Mamdani and T-Sugeno. In order to run PM motor at the desired speed, a closed loop with vector control PMSM drive was successfully designed and operated in constant torque mode. The feasibility of the above mentioned integrated control strategy is modeled and verified in the MATLAB/Simulink environment for effectiveness of the study.

From the obtained results we observed that, during both no load and step load conditions T-S type FLC reduces the settling time and rise time and does not exhibit any overshoot and undershoot. Starting torque is higher when Sugeno type FLC is used for speed control. Higher starting current is one and only drawback of Sugeno type FLC. While comparing with the Mamdani type FLC and this technique has superior performance. The simulation results are presented in forward motoring under no-load, sudden change in step load operating conditions.

So the proposed model with T-S type FLC as speed controller improved performances as compared to mamdani type FLC controller that have been taken in consideration in this work.

**REFERENCES:**


