Dynamic vibration analysis of ballastless track: 
Istanbul Aksaray-Airport LRT System

Veysel Arlı, Turgut Öztürk, Zübeyde Öztürk

Department of Track & Fixed Facilities, Istanbul Transportation Co., 34220 Istanbul, Turkey
Department of Civil Engineering, Istanbul Technical University, 34469 Istanbul, Turkey

ABSTRACT

In this study, ballastless superstructure with a steel base-plate of light metro track, which lies between Aksaray and the Airport in Istanbul, was analyzed with a numeric method using ANSYS 9.0 finite element software. Since the railroad superstructure has a symmetric structure with respect to the track axis, a rail system that consists of a single rail and a half sleeper was modeled to simplify the computations in this analysis. In the discrete supported track model, the rail was modeled as an Euler-Bernoulli beam and steel base-plate, and it was assumed to be a rigid mass. Dynamic computation results and graphs were determined by performing harmonic analysis with the aid of ANSYS software. A field vibration survey was conducted to determine the natural frequencies of the railroad and the dynamic receptance behavior, and to validate the finite element model according to the measurement results. The frequency receptance behavior of the rail and the support was measured by applying a hammer impact load on the rail head in the field, and the finite element model of the ballastless track with steel base-plate was verified.

ARTICLE INFO

Article history:
Received 13 May 2015
Accepted 18 July 2015

Keywords:
Finite element method
Numeric analysis
Dynamic analysis
Ballastless track
Vibration analysis

1. Introduction

When all systems that contain potential and kinetic energy are disturbed with dynamic loads, they start to vibrate. Dynamic loads and vibration occur due to irregularity and roughness during the contact between the rail and the wheel on a track superstructure, and the vibrations are conveyed both to the vehicle and the superstructure in all three axial directions. When resonance occurs, undesirable degradations and deformations affect vehicle stability, passenger comfort, and road and vehicle components; thus, the excitation frequency, or the natural frequency, of the system should be altered to prevent resonance. Since changing the natural frequency of the system as an afterthought is technically and economically difficult, it is appropriate to do it during the design stage. Therefore, numerical analyses must be performed that use finite element models and determine the dynamic effects during the resonance. In addition, field vibration surveys should be conducted, and the natural frequencies of the track should be measured to support and validate the dynamic analysis.

There are various excitation sources in the vehicles that cause oscillations, vibrations and noise on the railroad and in the environment. These include geometric defects, irregularities on the surface of rails and the wheel, fishplate rail joints, switch and crossing, and changing track rigidness. If the defect fits the sine curve, the excitation frequency of the defect, depending on the wavelength ($\lambda$) and vehicle velocity ($V$), can be calculated using Eq. (1).

\[ f = \frac{V}{\lambda}, \] (1)

As seen in the equation above, frequency decreases as the wave length increases; geometric track defects with large wave lengths cause low frequency vibrations, and rail corrugations with short wave length cause high frequency vibrations. The relationship between railroad vehicles and the track is a very complicated system that
includes many degrees of freedom. Lots of modeling, field surveying and laboratory work has examined dynamic vehicle-track interaction. Since, the vehicle-track system is an ambiguous one that displays variations; these studies were simplified by making basic assumptions. Thus, while the rail-wheel contact surface may change due to rail or wheel profile defects, and granular materials, such as ballast and soil, display plastic behavior, most of the research on railroad dynamics assumes that materials show linear elastic behavior, and material properties are obtained with field and laboratory tests.

The models should be as simple as possible and reliable enough to serve their purpose. Linear models, which produce quick solutions in frequency domain, are usually used for simplicity. The reliability of the model is dependent on disregarded, but important, effects: false loading or defect assumptions, or wrong system parameters. Methods should be applied to avoid these errors, and the model should be validated with experimental studies. In short, there is no model solely sufficient for a system, and as a result, various models are used for various purposes.

The first step in modeling complex dynamic systems is to establish discrete models for components such as the wheel, elastic layer or sleeper. These models are combined to define sub-systems, and then the entire vehicle-track system is modeled by combining the sub-systems. When modeling the railroad superstructure, generally the frequency interval 20–1500 Hz is used for vibration analyses. The primary cause of problems pertaining to track components and the rolling surfaces of wheel and rails is the vertical forces. Vertical forces are loads with high frequency, and they appear due to irregularities with short wave lengths; they become critical at frequencies below 1500 Hz.

The vehicle excitation load is usually modeled in four different ways (stationary load model, moving load model, moving irregularity model, moving mass model) (Diana et al., 1994). The most appropriate model for comparing the computed and measured values of excitation forces on the tracks, which are stimulated with a periodic (e.g., vibrator) or short-terminated (impact hammer) force applied on a fixed point, is the stationary load model. While the railroad superstructure is usually modeled as a two-dimensional infinite beam on an elastic foundation that consists of separate rails, the wheel load is modeled as a force that moves discretely. This model is linear elastic, and the effect of neighboring wheels is computed with the superposition method. Supporting the rails is basically done with two approaches, which require either discrete or continuous support.

A track model with continuous support was first used by Timoshenko in 1926, followed by studies by Heteny (1946), Sato (1977), Grassie (1980), and Tassily (1988). On the other hand, discrete support track model was used by Inglis (1938), Grassie et al. (1982), Nielsen (1990), Ripke (1991), Knothe and Grassie (1993). The most commonly used method is to position the rails on parallel spring and damping elements. This spring-damping element models the elastic rail layer, as the supports sit on the elastic foundation, which is another spring-damping element system.

In this study, the dynamic computations of the ballast-less superstructure of the Istanbul Aksaray–Airport light track were carried out with a numerical method and the use of ANSYS 9.0 finite element software, and graphical results were obtained. In modeling with finite elements methods, the wheel was applied both above and between two supports. A field vibration survey was conducted to determine the natural frequencies of the railroad and the dynamic receptance behavior and to verify the finite element model according to the measurement results. This study is different and unique compared to previous studies in terms of its approach and results. In this study, comparisons are made for a railroad using a finite elements method and field surveying results, along with a parametric study. The effect of different track parameters on dynamic behavior was analyzed as well. It is also important to investigate the effect of different track parameters on vibrations to achieve the most appropriate design in the vehicle-track-environment system in terms of vibration, particularly during the design stage (Dahlberg, 2002; Diana et al., 1994; Grassie, 1993, 1996; Popp and Schiehlen, 2003).

2. Dynamic Vibration Analyses with Using a Numerical Method

In this section, numerical dynamic vibration analysis is applied to selected track models; a ballastless (with concrete paving) superstructure with a steel base plate, which is used in rail systems in Istanbul downtown, was selected for the track model. Hence, the track model with discrete support was analyzed using ANSYS finite element software. In the track model with discrete support, rails are modeled as Euler-Bernoulli beams and steel base-plates as rigid masses, as seen in Fig. 1. Harmonic analysis was conducted using ANSYS software, and the results and graphs of dynamic computations were obtained. A harmonic load would cause harmonic receptance on the structural system. Harmonic receptance analysis enables the continuous dynamic behavior of the vehicle to be determined; as a result, the ability of the design to successfully withstand resonance, fatigue, and other harm alike effects of constrained vibration is revealed. Harmonic receptance analysis is a technique that is used to determine steady-state receptance against sinusoidal (harmonically) changing loads of a linear structure. The purpose of this analysis is to estimate the receptance of the structure at various frequencies and plot it. Transient vibrations, which emerge at the start of stimulation, are not included in the harmonic receptance analysis, (Sato et al., 1998; Knothe and Grassie, 1993; Grassie and Kalousek, 1993).

The railroad superstructure is modeled with the elements contained in the elements library of ANSYS 9.0, which is finite elements software. Since railroad structure has a symmetrical structure compared to the track axis, a track system consisting of a single rail and half sleeper was modeled for ease during calculations. In the finite element track model, which is displayed in Fig. 1, rails are established as two-dimensional beam elements with finite length, and sleeper masses are connected to
the nodes of the beam element. Rails were modeled with BEAM3 elastic beam elements with two degrees of freedom (vertical displacement and rotation), sleepers were modeled with equivalent singular mass elements and elastic layers, and the ballast layer was modeled with COMBIN14 spring-damping elements. A total of 1863 elements were used for 30 sleeper intervals, and 152 nodes were formed in the model. Since there are two degrees of freedom for each node, this corresponds to 304 degrees of freedom, and the parameters of the model are presented in Table 1 (Arlı, 2009).

![Fig. 1. Finite elements track model.](image)

**Table 1. Parameters of the track model used in the study.**

<table>
<thead>
<tr>
<th>Track parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia of the rail</td>
<td>$I = 1819 \times 10^{-8}$ (m$^4$)</td>
</tr>
<tr>
<td>Cross section of the rail</td>
<td>$A = 6297 \times 10^{-6}$ (m$^2$)</td>
</tr>
<tr>
<td>Shear factor of the rail</td>
<td>$K = 0.35$</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>$E = 2.1 \times 10^11$ (N/m$^2$)</td>
</tr>
<tr>
<td>Density</td>
<td>$\rho = 7850$ (kg/m$^3$)</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu = 0.3$</td>
</tr>
<tr>
<td>Base-plate distance</td>
<td>$a = 0.75$ (m)</td>
</tr>
<tr>
<td>Steel base-plate mass</td>
<td>$m = 8$ (kg)</td>
</tr>
<tr>
<td>Stiffness coefficient of the elastic layer under rail</td>
<td>$k_r = 97 \times 10^6$ (N/m)</td>
</tr>
<tr>
<td>Damping coefficient of the elastic layer under rail</td>
<td>$c_r = 32 \times 10^3$ (Ns/m)</td>
</tr>
<tr>
<td>Stiffness coefficient of the elastic layer under base-plate</td>
<td>$k_b = 90 \times 10^6$ (N/m)</td>
</tr>
<tr>
<td>Damping coefficient of the elastic layer under base-plate</td>
<td>$c_b = 4.1 \times 10^3$ (Ns/m)</td>
</tr>
</tbody>
</table>

In the numerical analysis, the effects of various track parameters on dynamic behavior are computed for the frequency interval 0-1500 Hz, and dynamic receptance graphs were obtained. While modeling with the finite elements method, the wheel load was applied both above the support and between two supports. When the wheel was exactly on the sleeper, the dynamic receptances of the rail and sleeper were determined, and it was observed that their receptances were different for this case. The dynamic receptance graph of the track was obtained for the unit wheel load. In the dynamic receptance graph of the harmonic analysis, peak values show a resonance event and accordingly show the natural frequencies of the track. Track and support natural frequencies were calculated out of these graphs (Table 2).

**Table 2. Natural frequencies of ballastless track model.**

<table>
<thead>
<tr>
<th>Track parameter</th>
<th>Track natural freq.</th>
<th>Support natural freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ballastless track model</td>
<td>216 Hz</td>
<td>756 Hz</td>
</tr>
<tr>
<td>Base-plate mass (10 kg)</td>
<td>213 Hz</td>
<td>756 Hz</td>
</tr>
<tr>
<td>Base-plate distance (0.60 m)</td>
<td>246 Hz</td>
<td>1161 Hz</td>
</tr>
<tr>
<td>Elastic layer type under baseplate (1403-N)</td>
<td>282 Hz</td>
<td>762 Hz</td>
</tr>
</tbody>
</table>

Since the harmonic analysis was carried out for every 500 points within the 0-1500 Hz frequency interval, graphic results have a frequency resolution of 3 Hz. The number of points could be increased for more precise analysis, but the analysis duration would also increase in this case.

According to the dynamic analysis, when the natural frequencies of the ballastless track are the same as the excitation load, the dynamic effects increase with resonance. Natural frequencies for the ballastless track models were obtained from frequency-dynamic receptance graphs. The first peak value on the dynamic receptance graph of a rail and support at the support point shows the track natural frequency, while the secondary peak values show the rail natural frequency. In the dynamic receptance graph of the rail between two supports, two peak values are observed, and the first value specifies the track natural frequency while the second one specifies the support natural frequency. Some vital results attained according to Table 1 are:

- The track natural frequency varies depending on base-plate distance and the elastic layer under the base-plate,
- The support natural frequency varies depending on the support interval and elastic layer type under the rail base plate and the base-plate mass has no effect.

In the dynamic receptance graphs of the ballastless track model, it is observed that the ballastless track model has a peak value, and the receptance curves of the rail and base-plate are the same. The dynamic receptance behavior of the rail and base-plate is not the same for the elastic layer under the base-plate, which is more rigid but has a higher damping coefficient (1403-N type, stiffness coefficient $k = 171 \times 10^6$ N/m and damping coefficient $c = 17.1 \times 10^3$ Ns/m), and the receptance of the base-plate is less than the rail. In other words, the rail and base-plate do not move together under the wheel load, and the rail has greater vertical displacement than the base-plate. Because the damping coefficient of the elastic layer is greater, the dynamic receptance of rail and base-plate is less than the ballastless track model (Figs. 2-9).

In Figs. 2 and 3, dynamic receptance graphs of the ballastless track model at the support point and between two supports are displayed, respectively. These graphs show that the dynamic receptance of the rail between two supports is greater than the dynamic receptance of the rail in the ballastless track model; in other words, more rail displacement occurs. The effect of each parameter (the base-plate mass, base-plate distance and elastic layer type) on dynamic behavior was investigated. The
most significant parameter is the elastic layer under the base-plate, and dynamic receptance is substantially reduced as the damping coefficient of the elastic layer increased. In addition, the dynamic receptance, or rail displacement, is also reduced when the support distance is changed from 0.75 m to 0.6 m. Increasing the base-plate mass slightly (10 kg instead of 8 kg) does not immediately affect the dynamic receptance, (Figs. 4-9).

Fig. 2. Dynamic receptance-frequency graph of the rail and the base-plate at the support point.

Fig. 3. Dynamic receptance-frequency graph of the rail between two supports.

Fig. 4. Dynamic receptance-frequency graph of the rail and the base-plate ($m=10$ kg) at the support point.

Fig. 5. Dynamic receptance-frequency graph of the rail between two supports base-plate ($m=10$ kg).

Fig. 6. Dynamic receptance-frequency graph of the rail and the base-plate at support point, 1403-N type.

Fig. 7. Dynamic receptance-frequency graph of the rail between two supports elastic layer 1403-N type.
functions, namely, receptance functions with FFT (Fast Fourier Transform) analysis. While finding the receptance function of the railroad, it is assumed that it is independent from the vehicle load. In other words, it is assumed that the stiffness and damping properties of the track are independent from the vehicle load.

Coherence is used to estimate the repeatability and reliability of acceleration and load measurements. If this value is close to 1, strong coherence exists; if it is between 0 and 0.8, the coherence is not strong, and it is not appropriate to use the results. While resonance frequencies produce correlation values very close to 1, correlation values of anti-resonance frequencies are generally close to zero. The small hammer test generates insufficient correlation below 40 Hz, while the big hammer test generates insufficient correlation above 1400 Hz. For measurements, a Brüel&Kjaer PULSE multi-channel measuring system, Dytran-5803A sledge hammer and power sensor, and a Brüel&Kjaer Type 4396 accelerometer were used. Vibration signals were recorded and analyzed with the aid of a computer.

The measurements were recorded in the vertical direction with an accelerometer, which was magnetically attached under the rails and above the base-plate and sleeper, on the track specified at (km 10 + 190). A coherence graph was examined for each hammer hit (0-1600 Hz freq.), and the vibration receptance behavior was recorded when there was a proper coherence value. If the proper value was not attained, then impact trials were repeated. The measurements on 21.07.2007 were conducted between 0100 and 0500, which are non-operating hours, to avoid excitation vibrations due to the train. The frequency response and coherence graphs of the rail and the support (steel base-plate) on tracks with and without ballast were obtained. Usually, measurement results between 0 and 20 Hz are not accepted since they have low coherence values. Moreover, it is perceived that since coherence values for several frequency intervals are less than 0.8, they would not be accepted. The natural frequency of the track was obtained as 210 Hz from the dynamic computation results, a single peak value is observed from the dynamic receptance graph based on measurement results, and this value indicates the track natural frequency. It is difficult to identify the natural frequency of the rail because there is no significant peak (Arlı 2009).

4. Conclusions

The ballastless superstructure with a steel base-plate of the light metro track, which lies between Aksaray and the Airport in Istanbul, was analyzed with a numeric method using ANSYS 9.0 finite element software in this study. Harmonic analysis was carried out with ANSYS software, and dynamic computation results and graphs were obtained, yielding the following findings:

- In the numerical analysis, the effect of various track parameters on dynamic behavior was estimated for the 0-1500 Hz frequency interval. The natural frequency of the ballastless track varies depending on base-plate distance and the type of elastic layer under the base-plate.
The support natural frequency changes depending on rail type, support distance, and type of elastic layer under the rail/base-plate; base-plate mass has no effect.

- When modeling with the finite elements method, the wheel load was applied both above the support and between two supports. The first peak value on the dynamic receptance graph of the rail and at the support point shows the track natural frequency, while the secondary peak values show the rail natural frequency. In the dynamic receptance graph of the rail between two supports, two peak values are observed, and the first value specifies the track natural frequency, while the second one specifies the support natural frequency.

- A field vibration survey was conducted to determine the natural frequencies of the railroad and the dynamic receptance behavior, and to verify the finite element model according to the measurement results. The frequency receptance behavior of the rail and the support (steel base-plate) was measured by applying a hammer impact load on the rail head in the track. A single peak value is observed on the dynamic receptance graph based on measurement results, and this value indicates the track natural frequency. But it is difficult to identify the natural frequency of the rail.

The finite elements model of the ballastless track with a steel base-plate was validated. Survey results for the ballastless track were remarkably consistent with the finite elements model, and the natural frequency and dynamic receptance values for both of them were found to be very close to each other. This is because there are not any granular materials that have unknown dynamic properties, like the ballast and soil in this superstructure; only elastic layers under rails and steel base-plate are present. Based on the above findings, it is possible to establish dynamic properties fully consistent with measurements, particularly for ballastless track models. Consequently, there is a good chance of precisely determining the dynamic behavior of the track for vehicle and track parameters.

This study is the first in Turkey that combines railroad dynamic analysis and modeling and surveying, and it can be considered as a significant development for urban railroad systems and high speed railroad projects, which have grown in the last several years. It is crucial to analyze the effect of various track parameters (rail type, elastic layer, support distance, etc.) on the dynamic behavior of the railroad, as the ballastless superstructure is preferred for urban rail systems (subway and tramway systems). It is possible to explicate with finite element models validated by field surveys. Deciding on the most fitting design in terms of vibration and determining the natural frequencies of the track during the planning stage is necessary to reduce the negative environmental effects of urban rail systems, which are located very close to areas sensitive to vibration and noise, such as historical structures, residences, hospitals, and schools. Furthermore, future studies should focus on determining the excitation frequencies caused by trains and natural frequencies of nearby buildings.

**REFERENCES**


